EOSC 550: Assignment #1

Due: Wednesday, January 24, 2007

1. **An analytic solution to a deconvolution problem.**
Consider data that arise from the following convolutional problem in which the blurring function is a two-sided decaying exponential;

$$d(x) = \int_0^b e^{-\sigma|x-y|} m(y)dy \qquad 0 \leq x \leq b \tag{1}$$

The decay of the kernel function is controlled by σ.

   a. Assuming that $d(x)$ is known everywhere on the interval $[0, b]$ derive an analytic inverse for this problem.

   b. Show that this inverse is unstable in the presence of noise. You can apply the same methodology that we used in examining the RMS problem.

   c. From looking at your analytic expression, why would you expect the result to be unstable when applied to field data?

   d. What role does σ play in determining how unstable the inverse formula is?

2. Consider a simplified seismic velocity problem in which the average velocity (from surface to a receiver in the borehole) of the earth is measured as a function of depth in the borehole. As in the RMS problem we assume that conversions of depth to time have been made. The average velocity is given by

$$V(t) = \frac{1}{t}\int_0^t v(u)du \tag{2}$$

and we assume that the interval velocity is

$$v(t) = v_0 + c\, sin\, \omega t \tag{3}$$

   a. Compute the data $V(t)$ that would arise from the velocity model $v(t)$ in eq(3).

   b. Derive an analytic inverse expression that shows how $v(t)$ can be obtained from the data $V(t)$.

   c. Derive an analytic expression for $V'(t)$ for the data generated in Part (a). What is $V'(t)$ when t approaches zero? Check that the true velocity model is recovered from your analytic solution in Part (b).

   d. Let the velocity be

$$v(t) = 500 + 50\, sin\, \omega t \qquad m/sec \tag{4}$$

   with $\omega = 1000\pi$ and with $t \subset [0,\ 20]\ msec$. Divide the time region into 200 points and plot $V(t)$.

e. Use the analytic formula to evaluate the velocity $v(t)$ but carry out the computation of $V'(t)$ numerically. Use a one-sided difference

$$V'(t) = lim_{\epsilon \to 0} \frac{V(t+\epsilon) - V(t)}{\epsilon} \tag{5}$$

Plot the recovered velocity using a few values of ϵ. In particular note that if ϵ is too small, then you lose accuracy because of round-off error. If ϵ is too large you lose accuracy because the Taylor expansion is a poor approximation. (Matlab uses double precision as default, so that determines what is meant by "small". For "large" values I mean a substantial portion of one cycle of a sinusoid.)

f. Take the 200 data points and add Gaussian random noise (using the Matlab function randn) with zero mean and standard deviation $0.4m/sec$. Plot the true and noisy data. Use a linear interpolation between points to estimate the derivative. Plot the resultant velocity model and compare it with the true model. Comment upon the progressive deterioration of the recovered model with depth.

Comment: For all exercises provide a copy of your Matlab code with the written work.

3. **Optional: (no marks for this question, but you can use it start thinking about your project)** Provide an example of a linear or nonlinear problem that is of interest to you. For discussion purposes, present the problem as

$$d_j = F_j[m] \tag{6}$$

a. What is the quantity of interest, which we generically label as m in the above equation?

b. What are the data?

c. What are the equations that describe the relationship between the model m and the data?

d. For what academic/practical reasons is the above problem useful? That is, why are we interested in solving the inverse problem to recover m?

e. Comment, if you can, about the current state of the art in solving this inverse problem.