Two ways to think about the dynamics of earthquake ruptures

(1) In terms of friction

(2) In terms of fracture mechanics

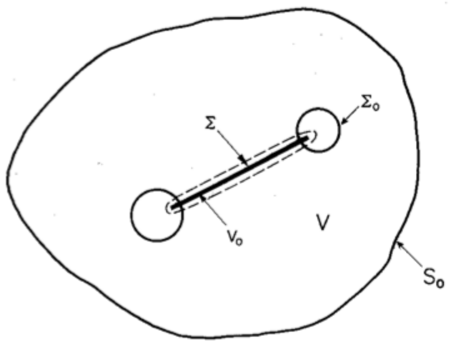


Fig. 4.2. The domains of integration for the dynamic energy balance.

Scholz describes conditions for rupture propagation (i.e. instability) via energy conservation. We did this for RS friction already.

$$W - U_e = U_s + U_k + U_f$$

EQ 4.1 (more or less)

$$W - U_e = U_s + U_k + U_f$$

- W: externally applied work on surface S_o
- U_e: elastic strain energy of the volume V
- U_s: energy spent at the edges to grow the crack (integrated over Σ_o)
- U_k: kinetic energy integrated over volume V
- U_f: energy spent on frictional sliding (integrated over $\sum$)

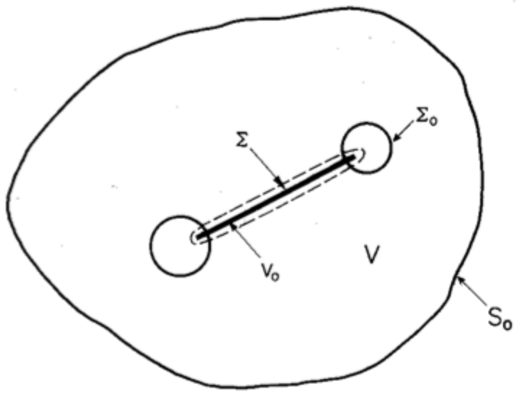


Fig. 4.2. The domains of integration for the dynamic energy balance.

EQ 4.3a through e.

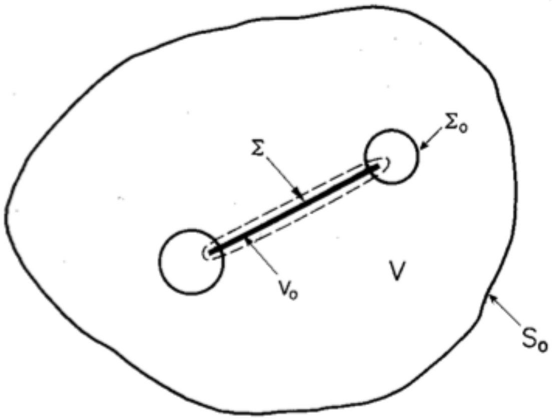


Fig. 4.2. The domains of integration for the dynamic energy balance.

$$W - U_e = U_s + U_k + U_f$$

U_f and U_e depend on stress and slip which we don’t know. Here, constant stress during sliding is assumed.

If we assume S_o is Earth’s surface (traction-free), applied work term W is 0.

Energy conservation applies over any ΔT.

$$W - U_e = U_s + U_k + U_f$$

U_f and U_e depend on stress and slip which we don’t know. Here, constant stress during sliding is assumed for U_f.

If we assume S_o is Earth’s surface (traction-free), applied work term W is 0.

Energy conservation applies over any ΔT.

$$\Big\downarrow$$

$$\Delta U_k = -\Delta U_e - \Delta U_{f_{const\ stress}} - \Delta U_s$$

Using Gauss’ theorem and the definition of strain, the elastic strain energy of the volume V can be transformed to a surface integral over crack area Σ.

(EQ 4.3e through 4.5)

$$\iiint_V (\nabla \cdot \mathbf{F})\, dV = \oiint_S (\mathbf{F} \cdot \mathbf{n})\, dS.$$

$$\epsilon = \bigtriangledown \cdot \mathbf{u}$$

$$\dot{U}_e = \frac{\partial}{\partial t}\frac{1}{2}\int_{V-V_o} \sigma_{ij}\epsilon_{ij}dV$$

$$\Delta U_e = \frac{-1}{2}\int_{\Sigma} \sigma_{ij}\cdot \mathbf{u}_j \; d\Sigma \longrightarrow \Delta U_e = \frac{-1}{2}(\sigma_1 + \sigma_2)\Delta u A$$

EQ 4.5

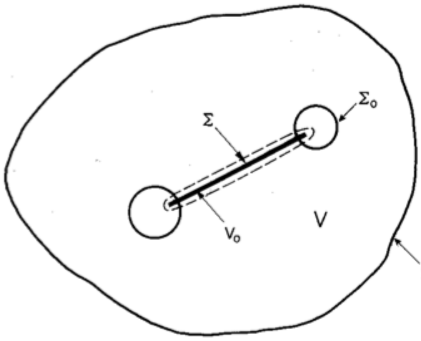


Fig. 4.2. The domains of integration for the dynamic energy balance.

$$\Delta U_k = -\Delta U_e - \Delta U_{f\,const\ stress} - \Delta U_s$$

Equation 4.6:

$$E_s = \frac{1}{2}(\sigma_1 + \sigma_2)\Delta u A - \sigma_f \Delta u A - 2\gamma A$$

radiated energy (waves)

change in elastic strain energy

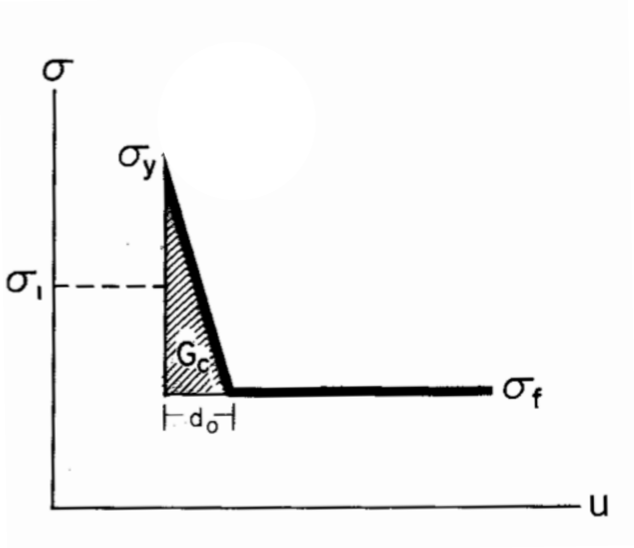
energy loss to frictional sliding

energy spent on growing the fracture at its edges

This reduces to 4.7 if we assume stress on the fault during sliding = stress on the fault after the earthquake is over, and if we assume that fracture energy is negligible

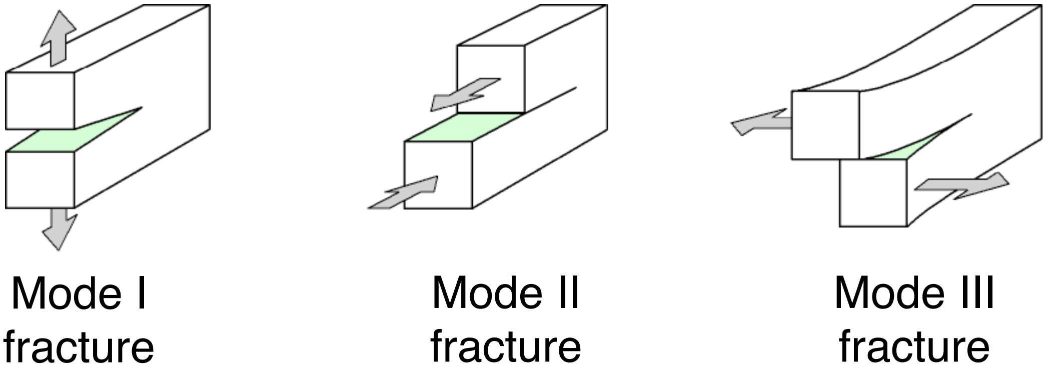
From this Scholz gets the result that radiated energy is a small % of the total energy budget in an earthquake (4.8). He's ignoring heat loss, and energy spent on damage (plastic deformation) in the surrounding rock (beyond the rupture tip area).

This energy balance also gets Scholz to a condition for rupture instability.
(1) Using EQ 4.5 and an expression for displacements along a shear crack (borrowed from Knopoff, 1958), EQ 4.10 is elastic strain energy.
(2) calculate radiated energy and set this equal to the fracture energy for both ends of the crack (2 G_c dL). This gives the absolute minimum crack length beyond which it will grow. Limiting velocity is shown to be S wave veloc (for mode III crack).



Result is EQ 4.15 $$L_c = \frac{\mu}{\pi}\frac{(\sigma_y - \sigma_f)d_o}{(\sigma_1 - \sigma_f)^2}$$

Some definitions



EQ 4.15 $$L_c = \frac{\mu}{\pi} \frac{(\sigma_y - \sigma_f) d_o}{(\sigma_1 - \sigma_f)^2}$$

Recall that we got a similar condition from rate-and-state-dependent friction (RS friction). It was:

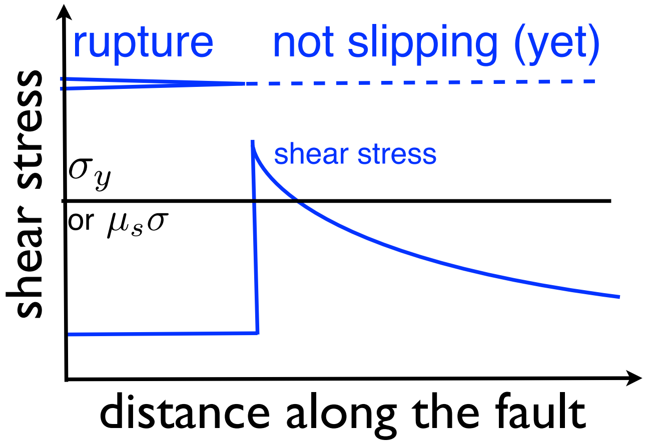
$$L > \frac{D_c G}{(1-\nu)(b-a)\sigma_n}$$

Rupture propagation is governed by dimensionless parameter S (EQ 4.20):

$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$

stress jump required to start slip

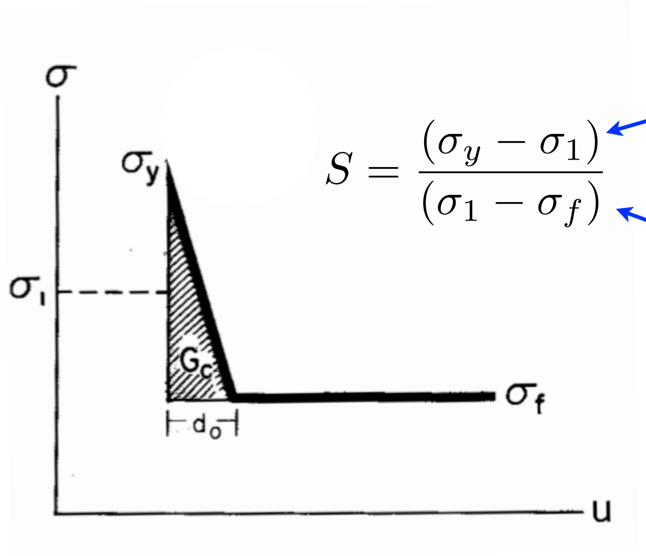
stress drop



shear stress on unbroken fault next to the rupture increases!

the shear stress increase is proportional to the earthquake stress drop (or the length of the crack) and (1/ the square root of distance to the rupture tip)

neighboring parts of the fault can be driven to failure by this domino effect **if they are already close to failure**



$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$

stress jump required to start slip

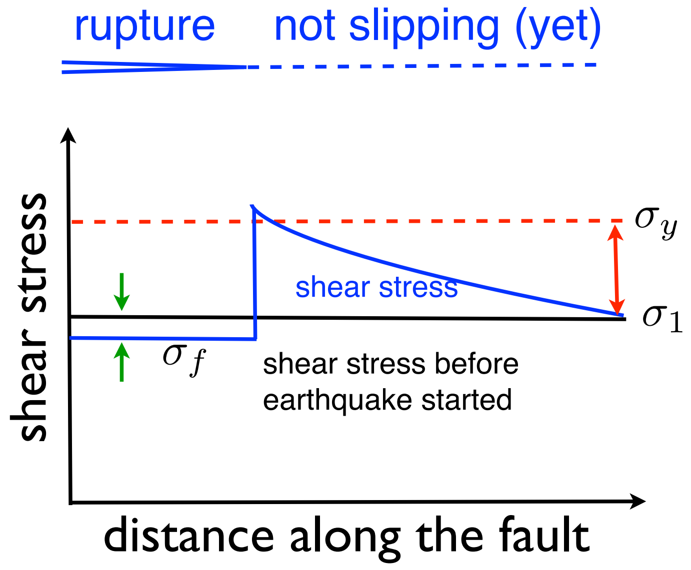
stress drop

S is “dimensionless strength parameter”

Large S slows or stops propagating rupture

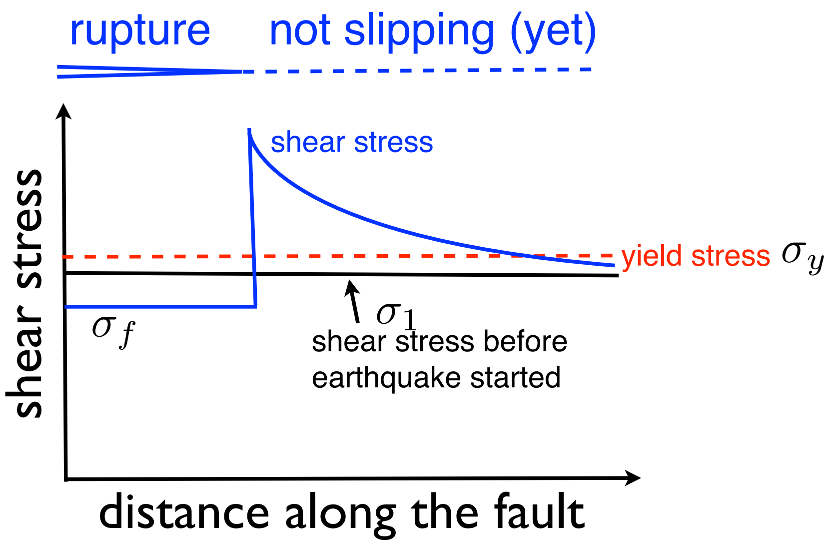
Small S encourages fast rupture propagation

So a “strong” fault patch might not necessarily be a patch with a large yield stress (or equivalently with a large frictional strength)



$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$

Conditions for rupture propagation - good or bad?

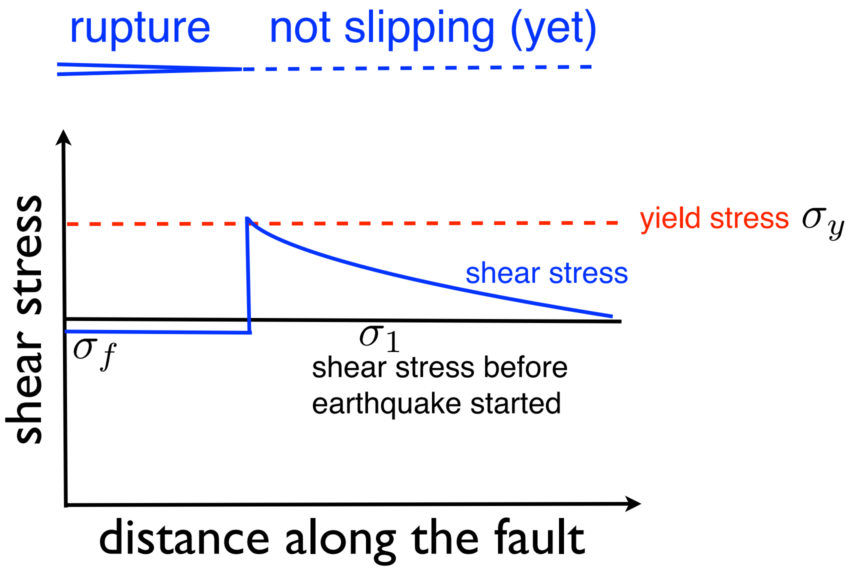


$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$

stress jump required to start slip

dynamic stress drop

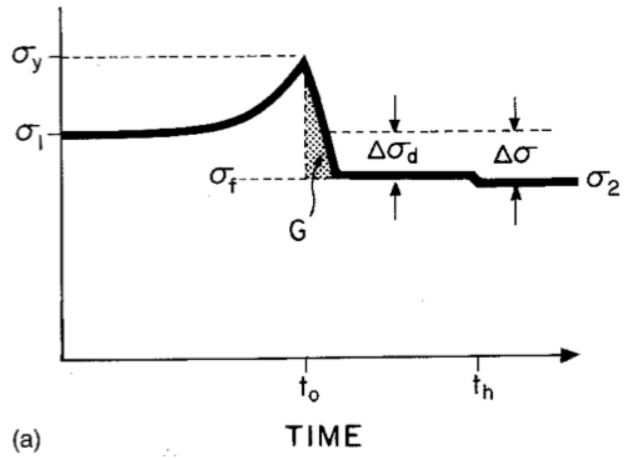
Good or bad conditions for rupture propagation?



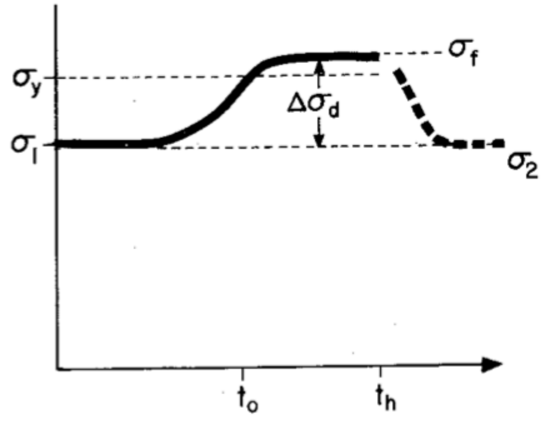
$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$

stress jump required to start slip

stress drop



overshoot



afterslip

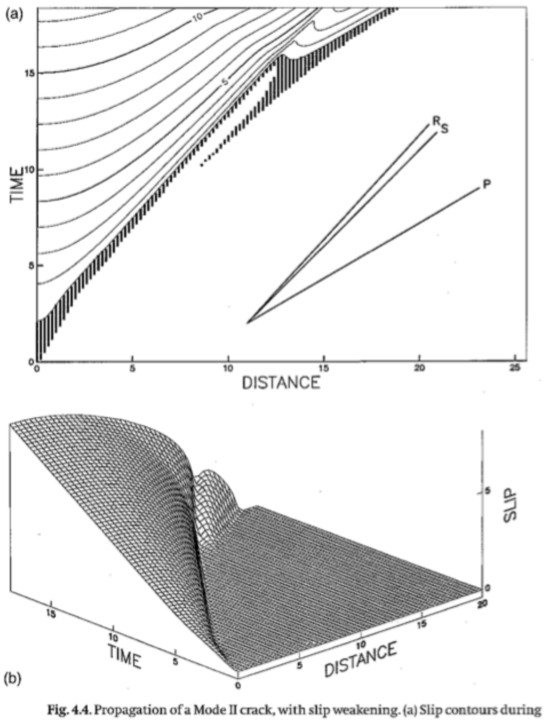
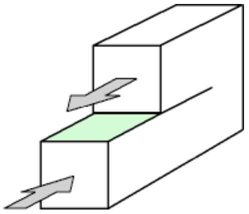


Fig. 4.4. Propagation of a Mode II crack, with slip weakening. (a) Slip contours during propagation of the crack. Dimensionless distance is x/L_c, time is $\beta t/L_c$, and the slip contour interval is $1.3L_c\Delta\sigma_d/\mu$. The shaded region indicates the breakdown region where points are slipping with $u < d_0$. Lines denoted P, S, and R indicate the P, S, and Rayleigh speeds for the medium. (b) Mesh perspective of dimensionless slip $(\mu/\Delta\sigma_d)(u/L_c)$ in the same coordinates. Azimuth of view is the Rayleigh wave direction. (c) Mesh perspective, similar to (b), of dimensionless tractions in the crack plane. (From Andrews, 1985.)

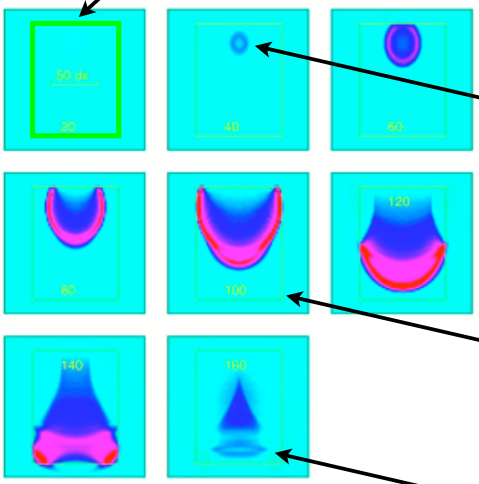
S = 0.8

“supershear” rupture propagation at small S



In a large earthquake, the slipping patch grows: the rupture propagates into previously unbroken parts of the fault

$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$



set model up to keep slip inside the green box. How?

start earthquake in model on a patch where...

earthquake rupture is growing. slip has already stopped at the source.

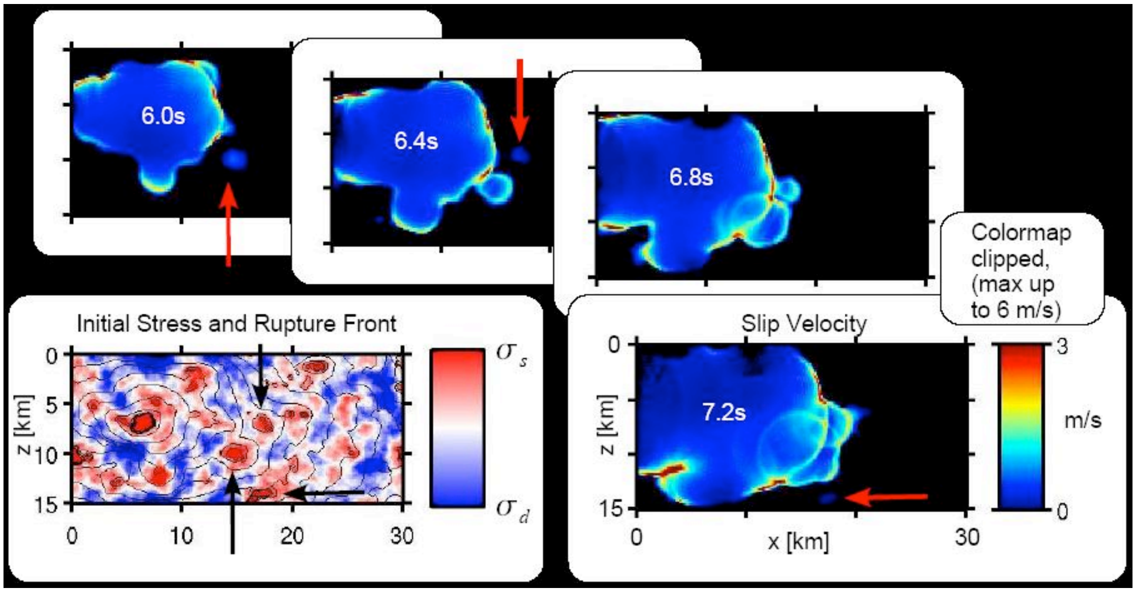
slip stays inside the green box

slip fizzles out.

from a numerical model Madariaga et al.

Frictional strength and shear stress are heterogeneous on real faults

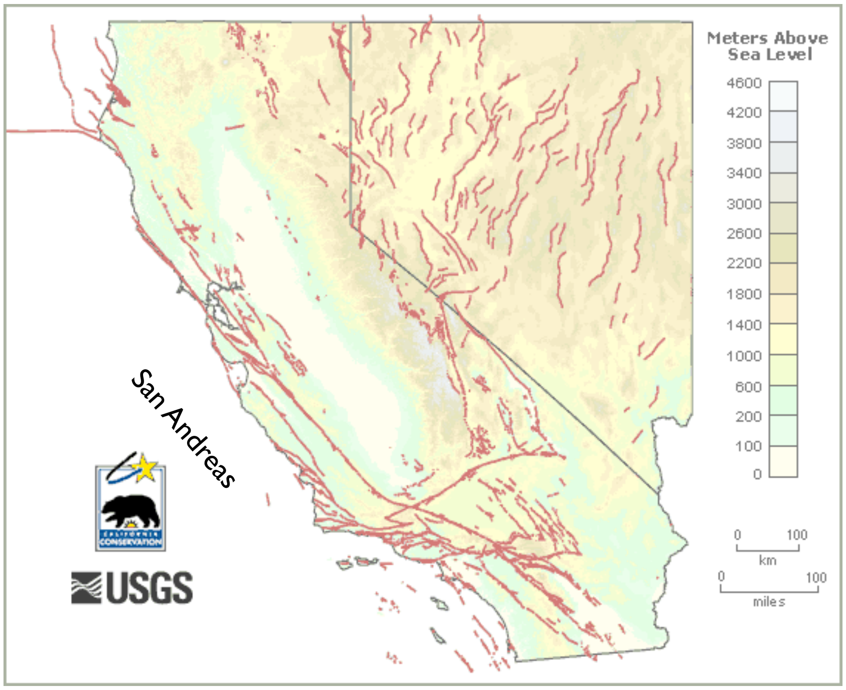
$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$



Rupture propagation model from J. Ampuero et al.

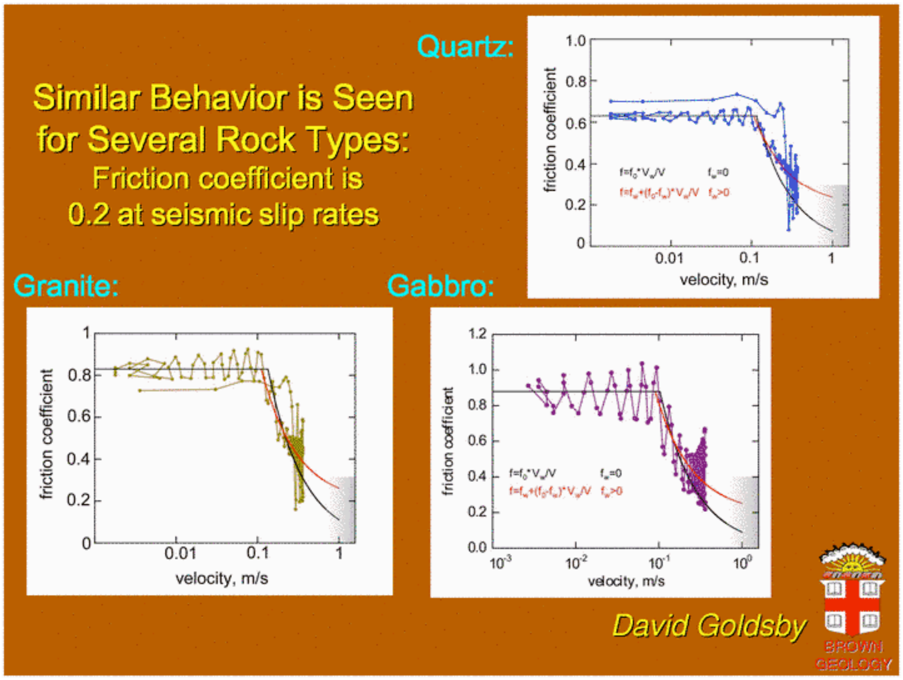
- Long, continuous fault with shear stress near the Coulomb threshold
- Large normal stress and velocity weakening friction --> big stress drop and also large stress "kick" to adjacent parts of the fault

$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \sigma_f)}$$



And yet another thing..."extreme weakening"

Lab experiments show that if slip speed gets up to about 0.1-0.2 m/s, dynamic friction may drop to near zero



Similar Behavior is Seen for Several Rock Types:

Friction coefficient is 0.2 at seismic slip rates

Quartz:

Granite:

Gabbro:

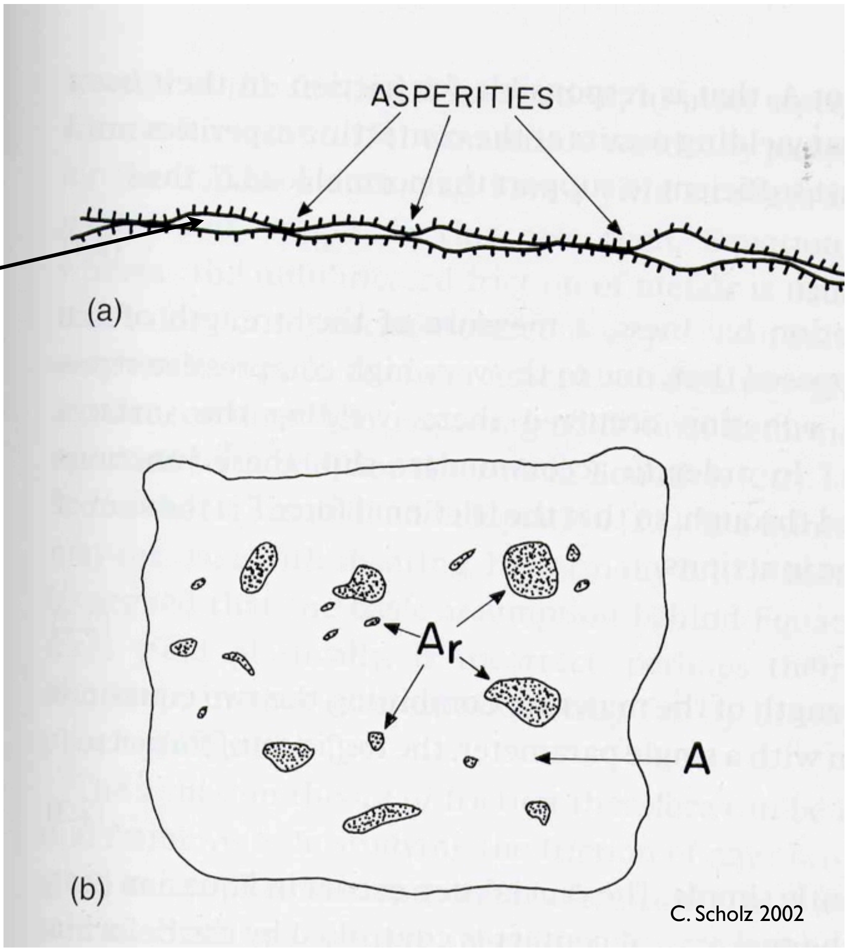
David Goldsby

the quake has already begun at this point, but this frictional strength drop will encourage the earthquake to keep going.

$$S = \frac{(\sigma_y - \sigma_1)}{(\sigma_1 - \boxed{\sigma_f})}$$

near zero

just add water at high pressure. what will happen?



C. Scholz 2002

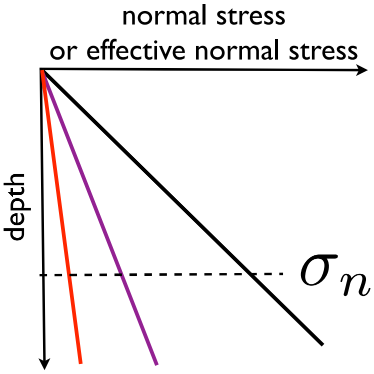
Pore pressure can dramatically reduce effective normal stress

$$|\sigma_e| = |\sigma_n| - P_p$$

At a depth of 10 km:

σ_e if water is not overpressured?

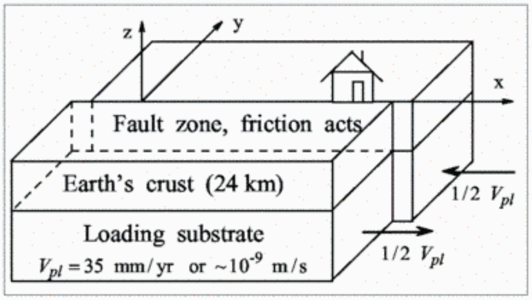
σ_e if pore pressure = 0.9 x lithostatic pressure?

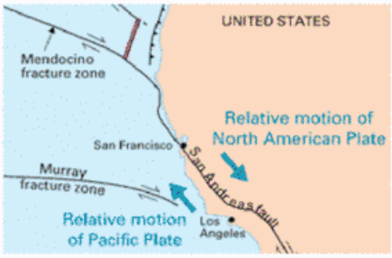


$$S = \frac{(\boxed{\sigma_y} - \sigma_1)}{(\sigma_1 - \boxed{\sigma_f})}$$

both may reduce

Example of modeling with rate and state law





http://pubs.usgs.gov/publications/text/dynamic.html

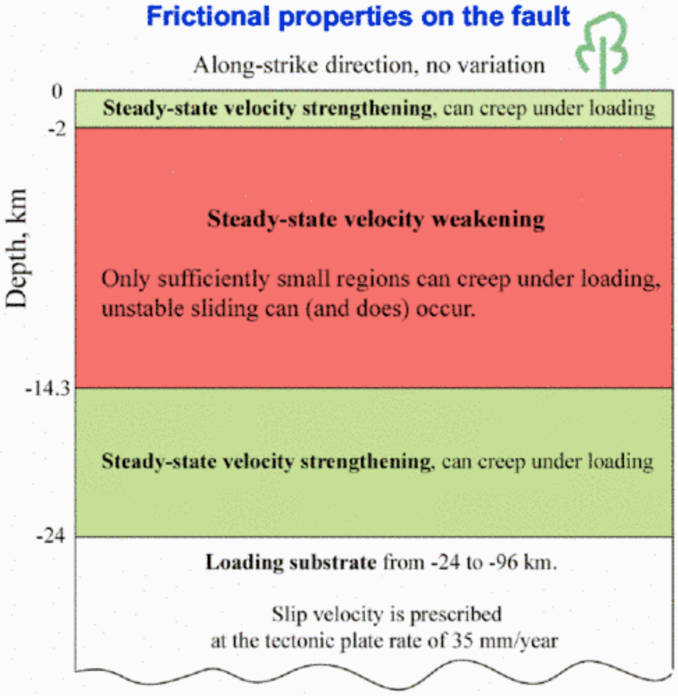
2D depth-variable model
Variations with z and y only,
no variation with x (Rice, 1993)

Goal: To simulate ***spontaneous*** slip accumulation on the interface by solving the system

Shear traction on the interface =

Friction strength of the interface

Frictional properties on the fault



Along-strike direction, no variation

Steady-state velocity strengthening, can creep under loading

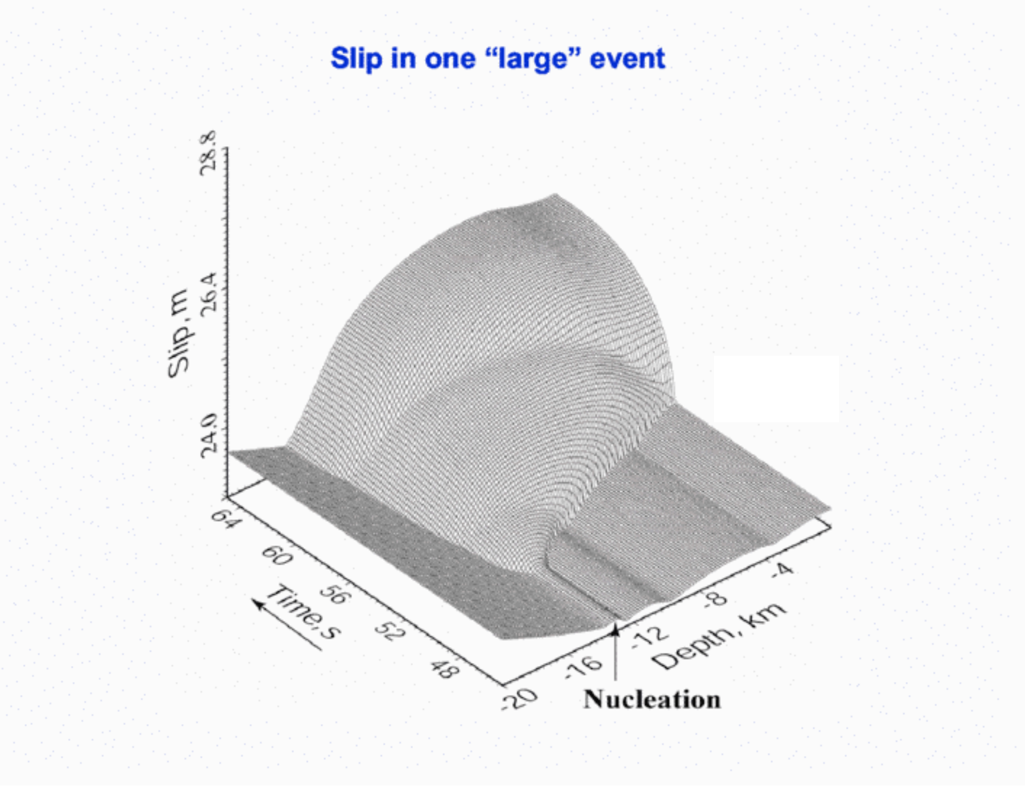
Steady-state velocity weakening

Only sufficiently small regions can creep under loading, unstable sliding can (and does) occur.

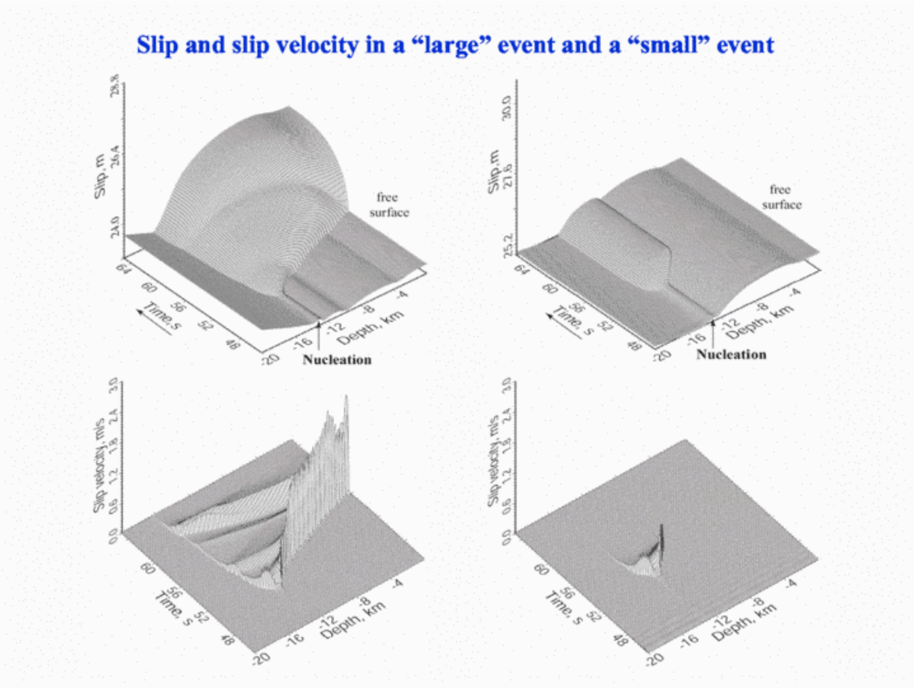
Steady-state velocity strengthening, can creep under loading

Loading substrate from -24 to -96 km.

Slip velocity is prescribed
at the tectonic plate rate of 35 mm/year



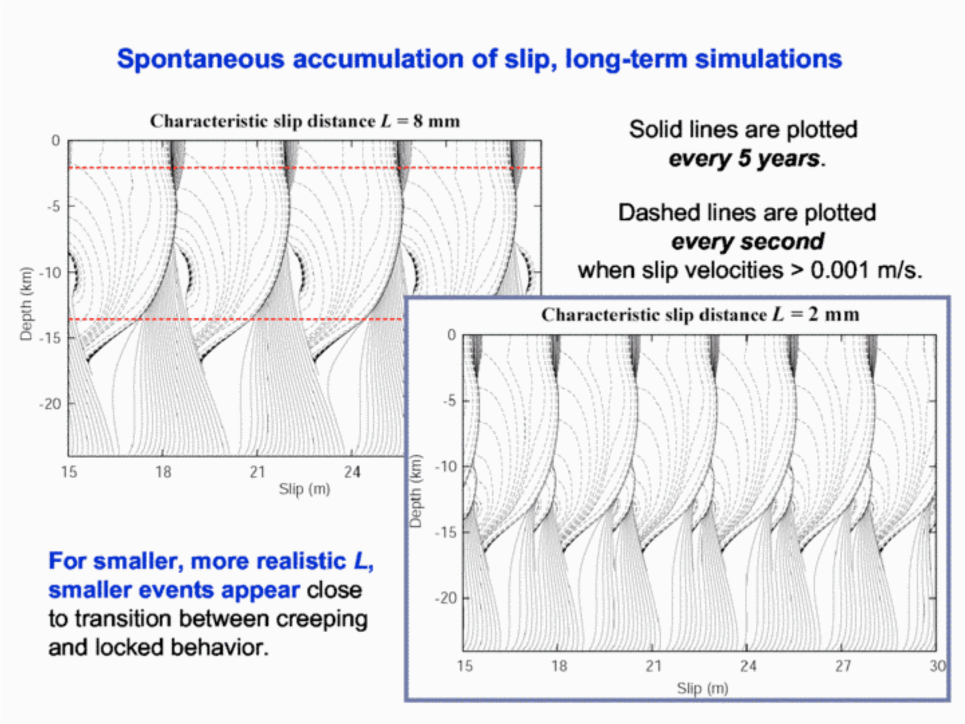
Nadia Lapusta



At the start, large and small earthquakes look the same

Pre-earthquake stress conditions can limit rupture

Nadia Lapusta



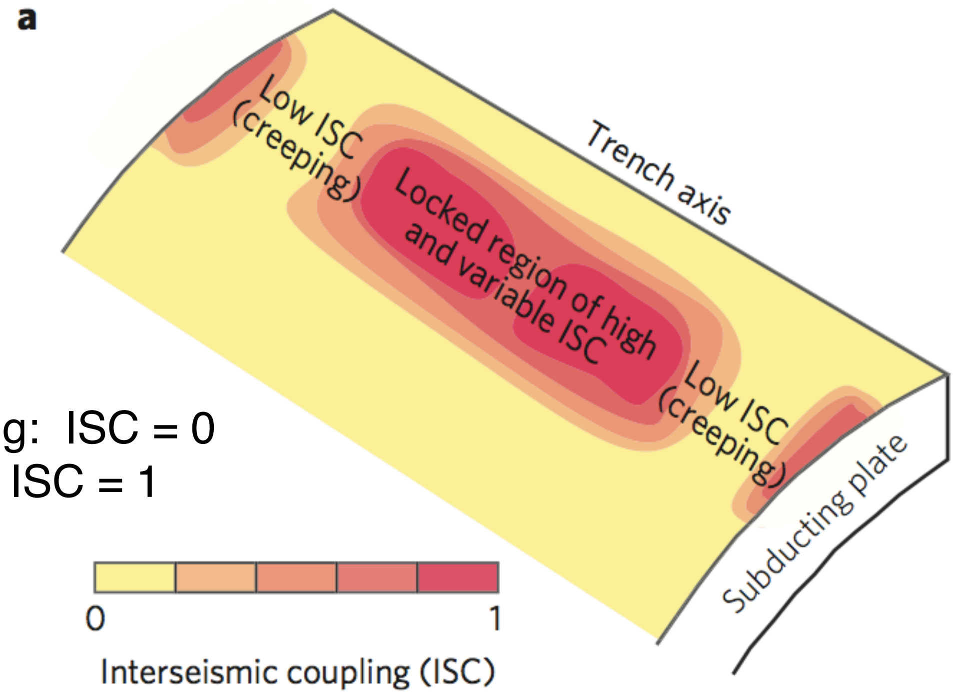
Spontaneous accumulation of slip, long-term simulations

Solid lines are plotted *every 5 years*.

Dashed lines are plotted *every second* when slip velocities > 0.001 m/s.

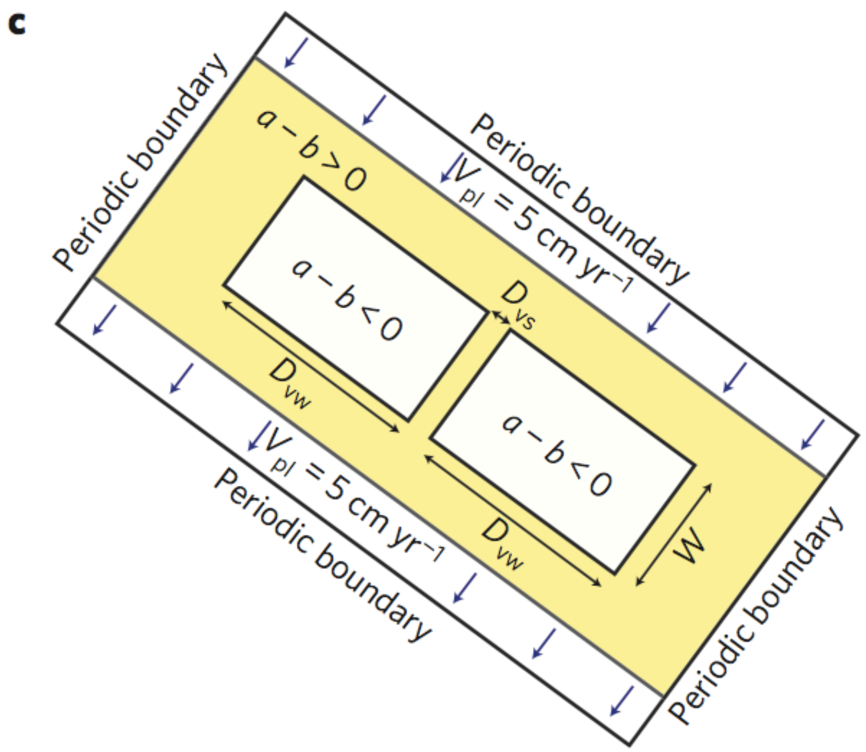
For smaller, more realistic *L*, smaller events appear close to transition between creeping and locked behavior.

Subduction zone fault surface, showing locked and creeping areas

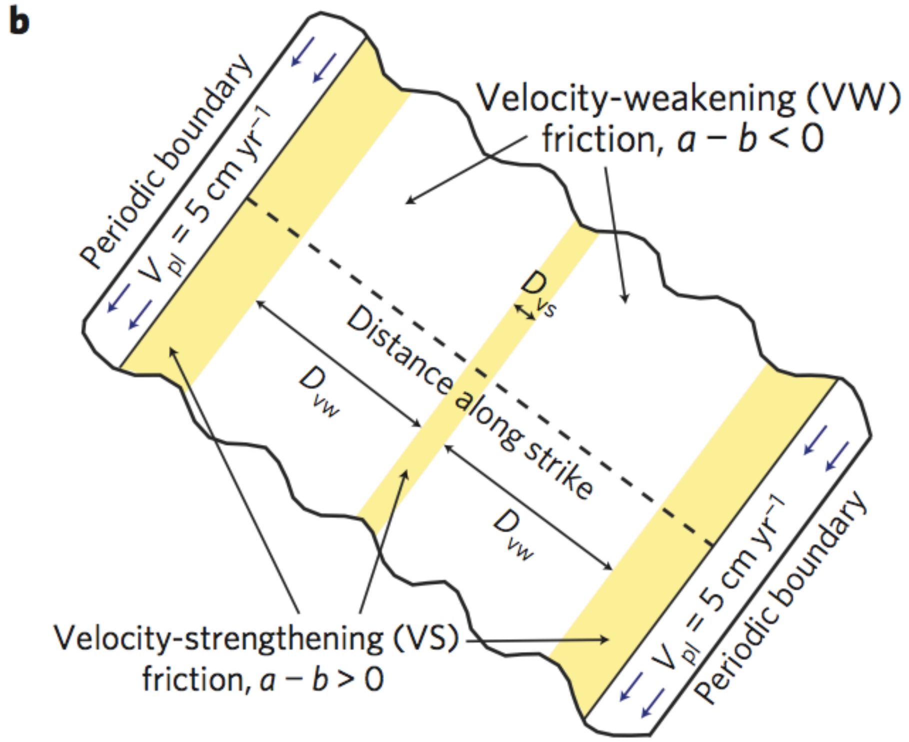


- creeping: ISC = 0
- locked: ISC = 1

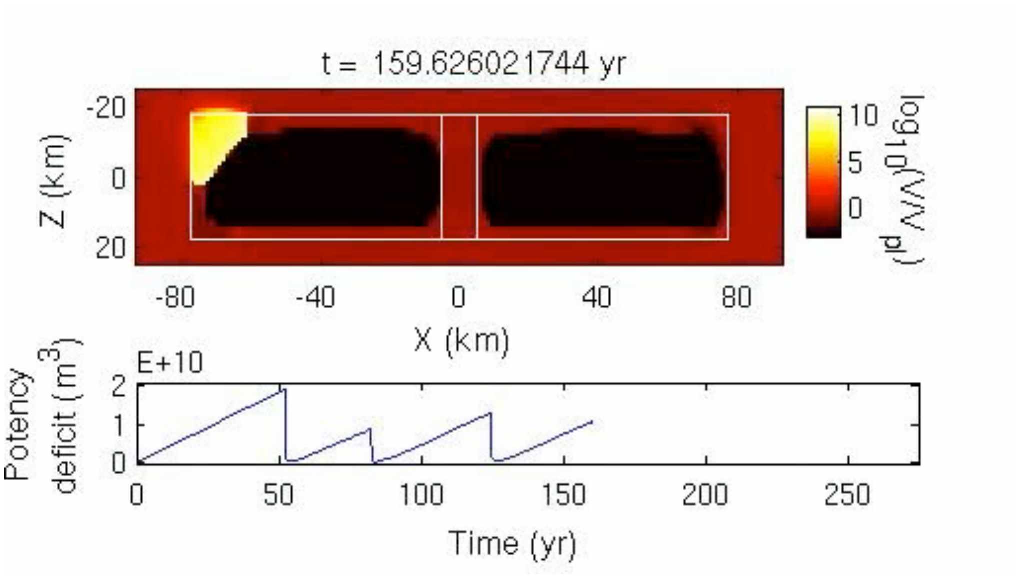
Model of a subduction zone fault surface, with velocity-weakening and velocity-strengthening areas



2D case - no variation in properties downdip

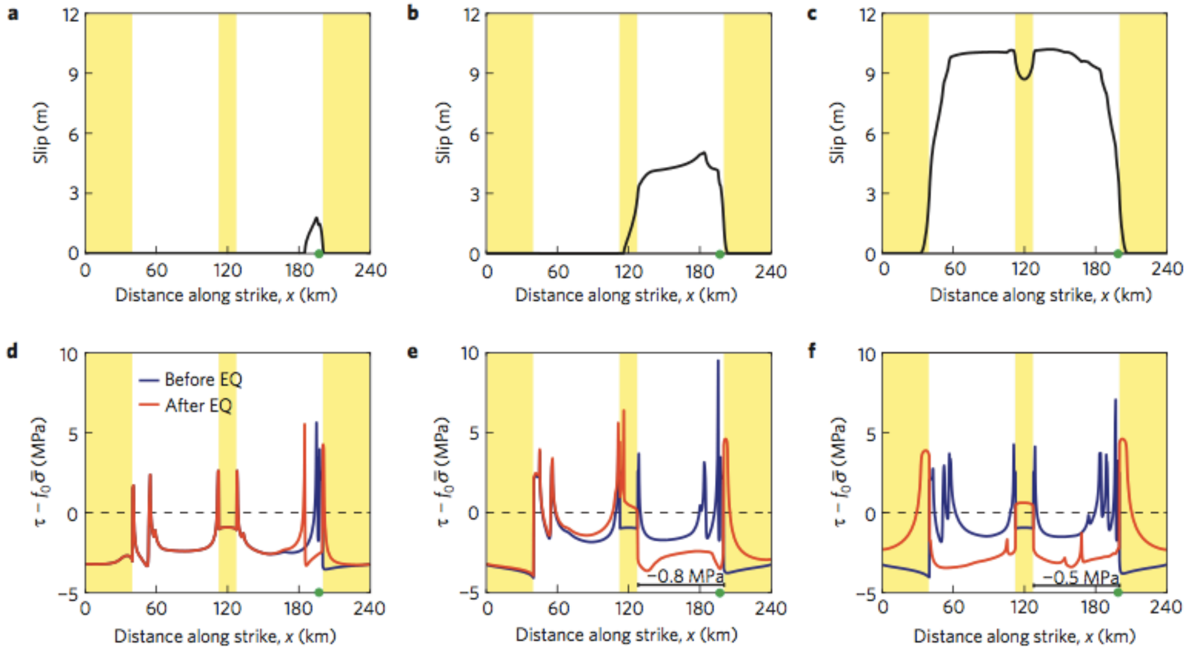


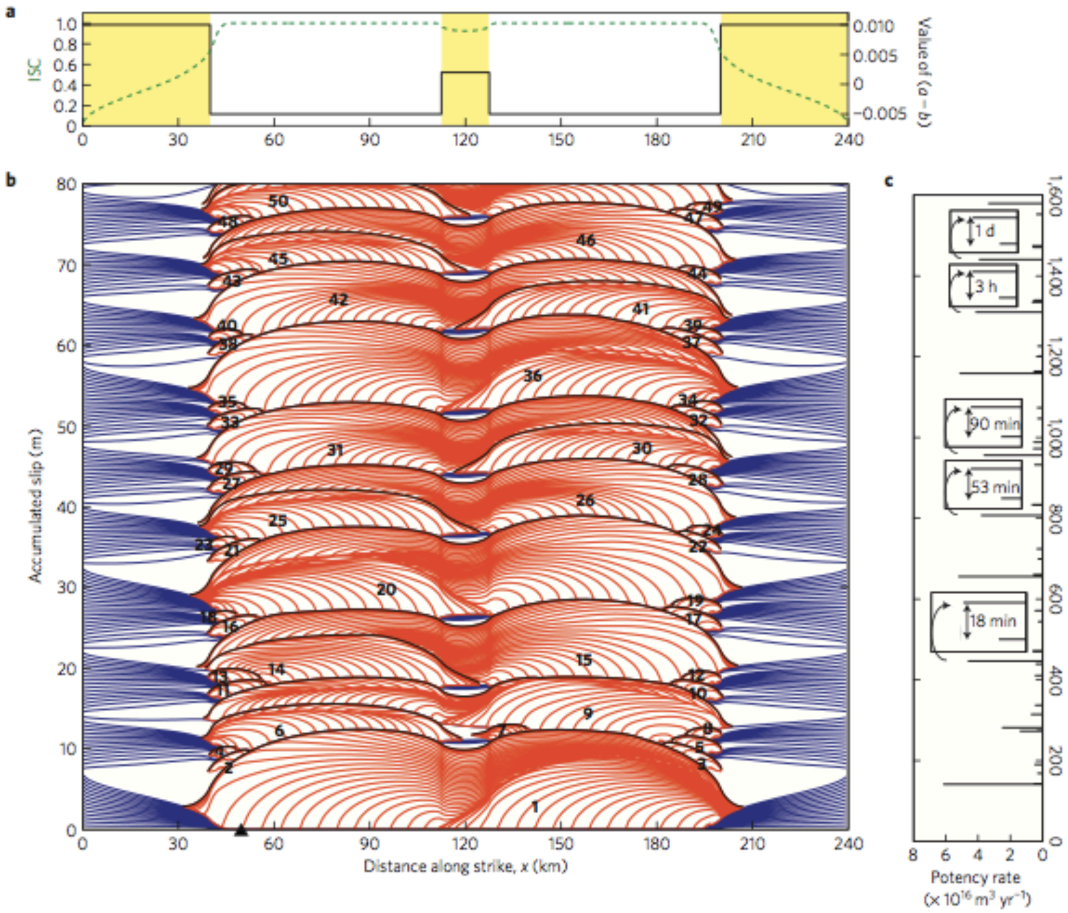
Movie showing modeled earthquakes and interseismic creep over many earthquake cycles

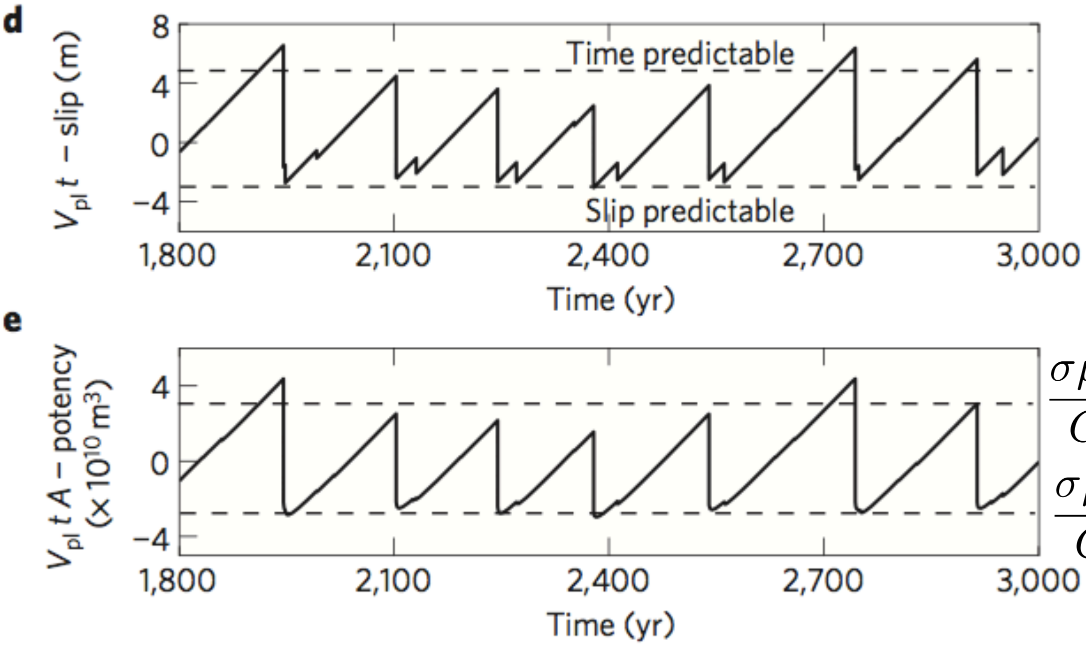


seismic potency is just seismic moment / shear modulus (= slip times area)

Small and large earthquakes. top row = slip distribution, bottom row = corresponding pre-and post-quake stress



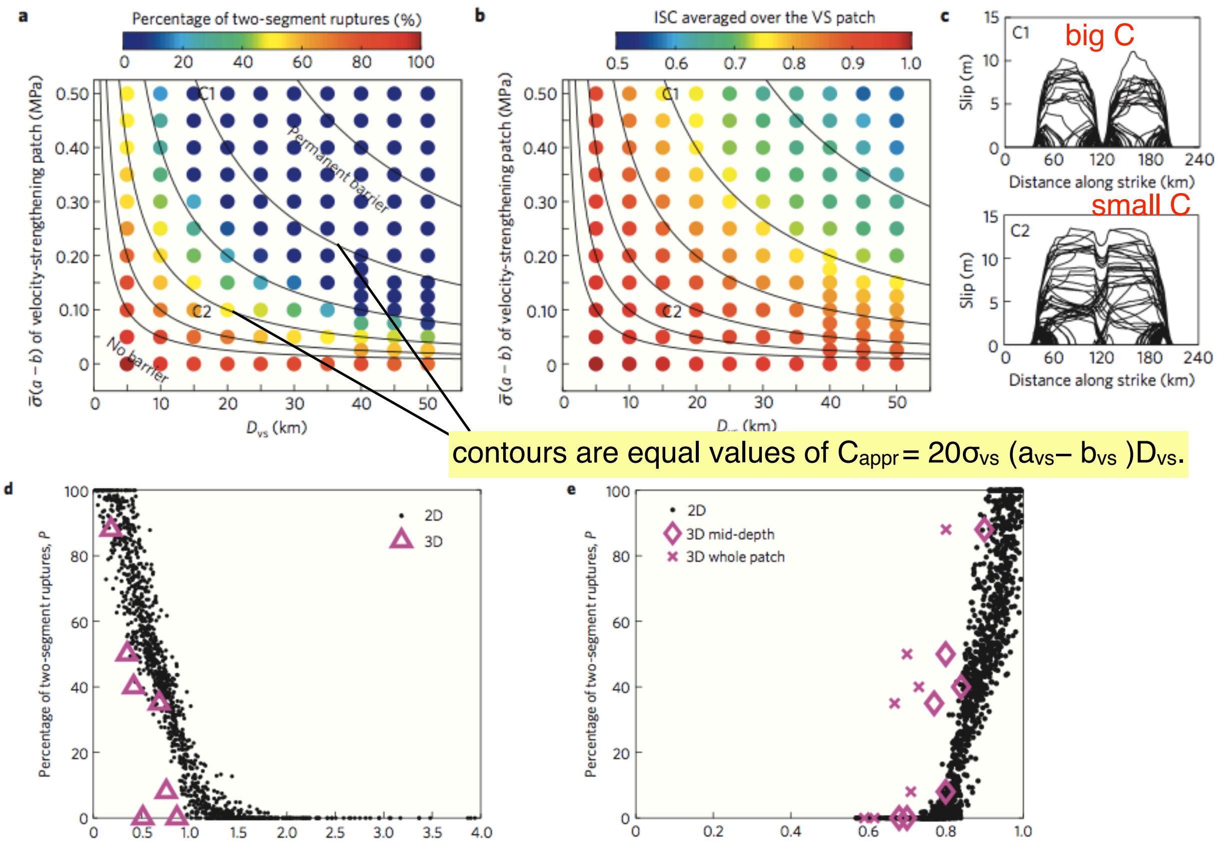




$$\frac{\sigma\mu_s}{G}$$

$$\frac{\sigma\mu_d}{G}$$

- define time-predictable and slip-predictable seismicity models
- seismic potency is just seismic moment / shear modulus (= slip times area)



big C

small C

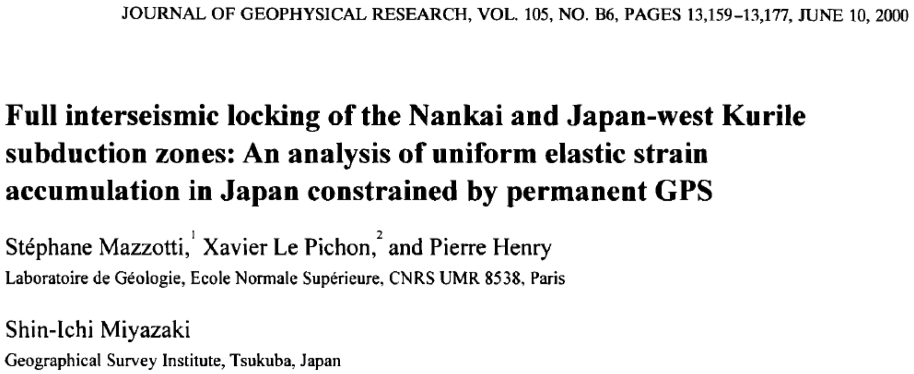
contours are equal values of $C_{appr} = 20\sigma_{vs}\,(a_{vs} - b_{vs})D_{vs}$.

B is “barrier efficiency”

$$B = \frac{\Delta\tau_{prop} D_{vs}}{\beta \Delta\tau_{vw} D_{vw}}$$

Upshot: by monitoring seismic coupling between earthquakes (via GPS for example) the future large slip patches might be delineated.

example:

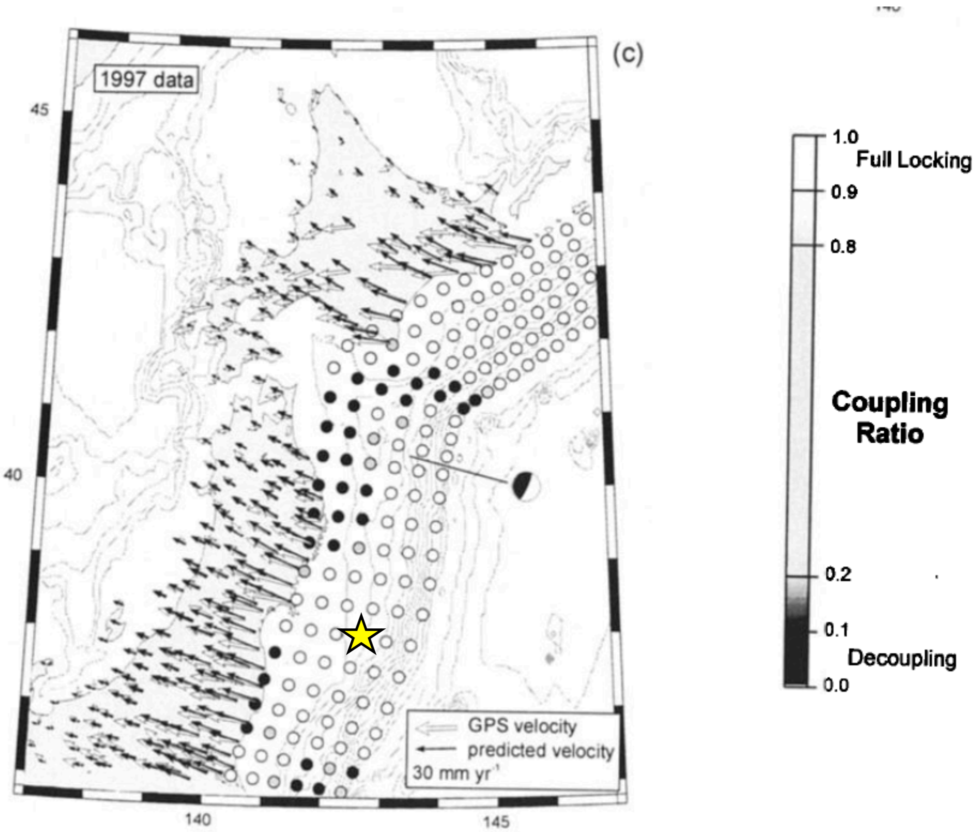


JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 105, NO. B6, PAGES 13,159–13,177, JUNE 10, 2000

Full interseismic locking of the Nankai and Japan-west Kurile subduction zones: An analysis of uniform elastic strain accumulation in Japan constrained by permanent GPS

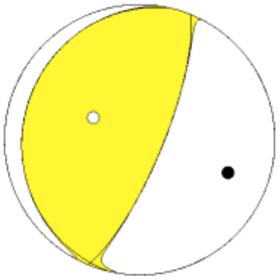
Stéphane Mazzotti,[1] Xavier Le Pichon,[2] and Pierre Henry
Laboratoire de Géologie, Ecole Normale Supérieure, CNRS UMR 8538, Paris

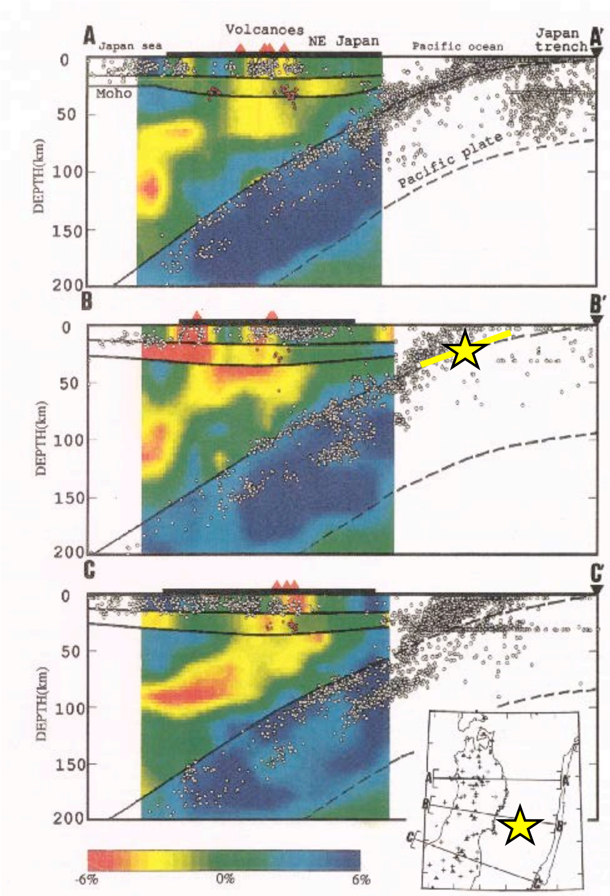
Shin-Ichi Miyazaki
Geographical Survey Institute, Tsukuba, Japan



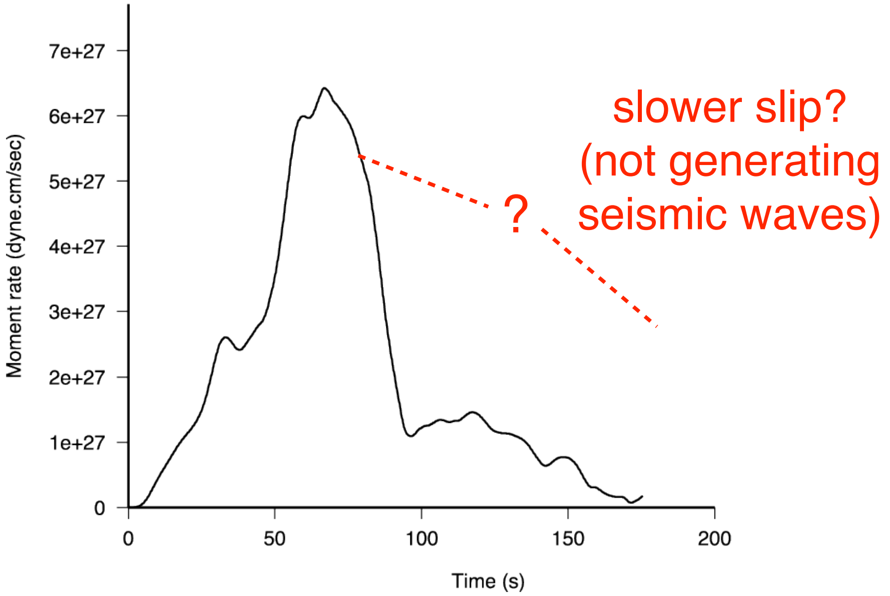
```
USGS/WPHASE CENTROID MOMENT TENSOR
11/03/11 05:46:23.00
Centroid:    38.321   142.969
Depth  24          No. of sta:256
Moment Tensor;    Scale 10**22 Nm
  Mrr= 1.82         Mtt=-0.13
  Mpp=-1.69         Mrt= 1.34
  Mrp= 3.17         Mtp=-0.56
 Principal axes:
  T  Val=   3.88  Plg=59  Azm=295
  N     =   0.03     2       201
  P     = -3.92     30       110

Best Double Couple:Mo=3.9*10**22
 NP1:Strike=193 Dip=14 Slip=  81
 NP2:        22     76        92
```

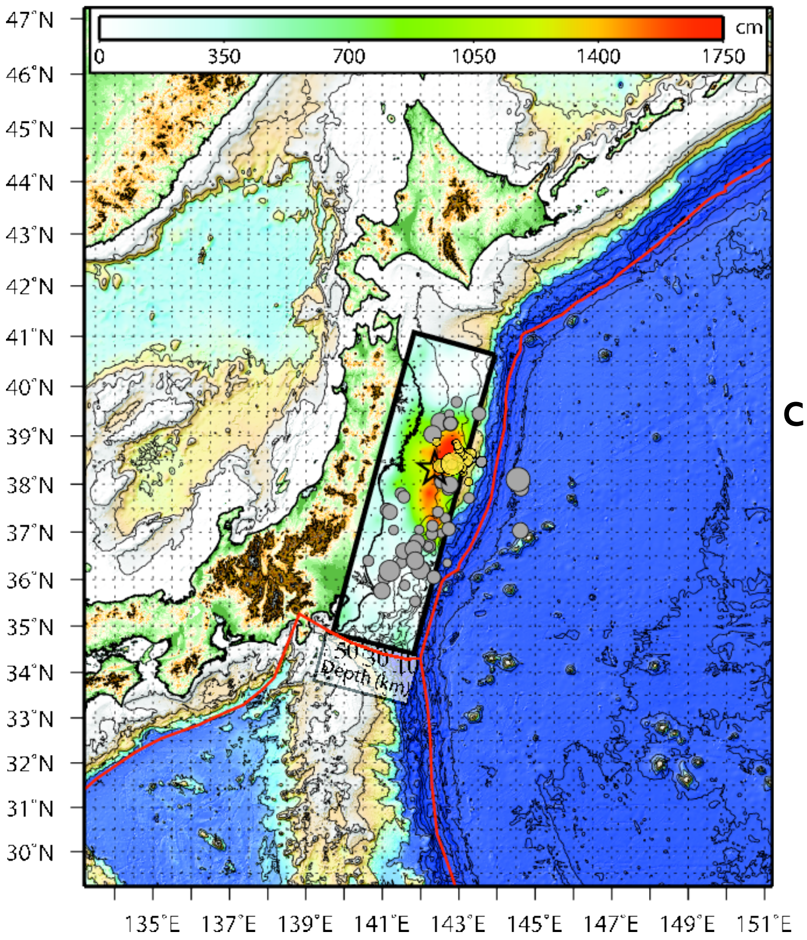




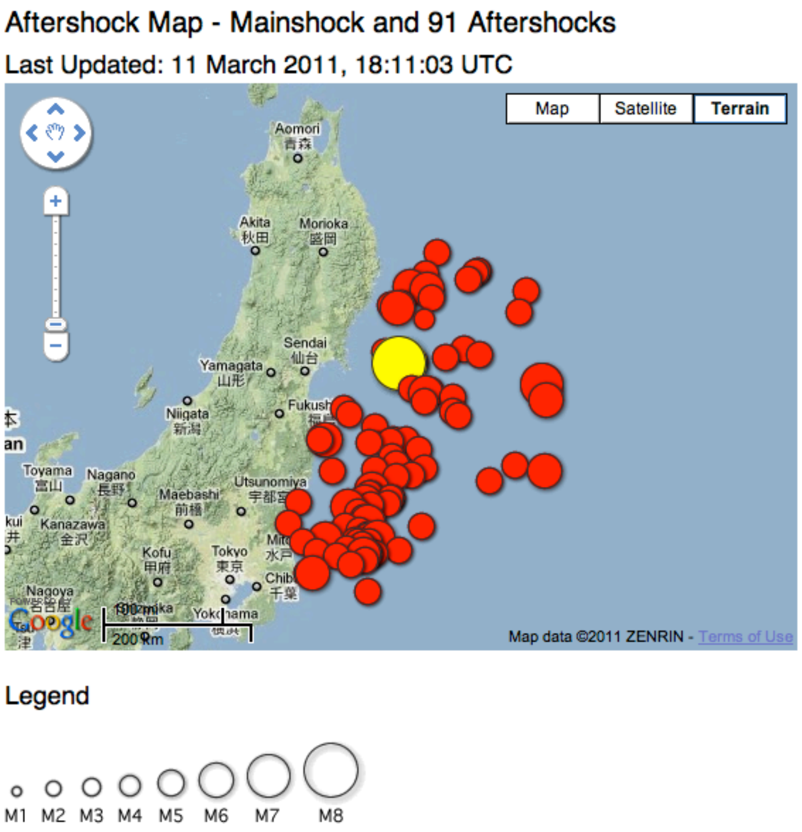
Most fault slip happened in 1st 100 seconds, though this estimate probably missed a lot (slower slip)



slower slip?
(not generating
seismic waves)



Distribution of coseismic slip and aftershocks



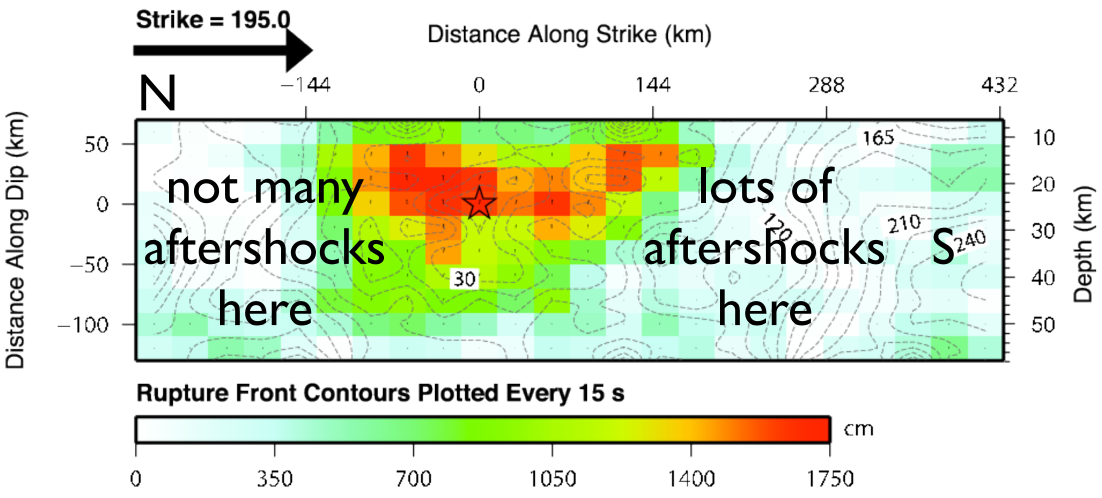
Aftershock Map - Mainshock and 91 Aftershocks

Last Updated: 11 March 2011, 18:11:03 UTC

Legend

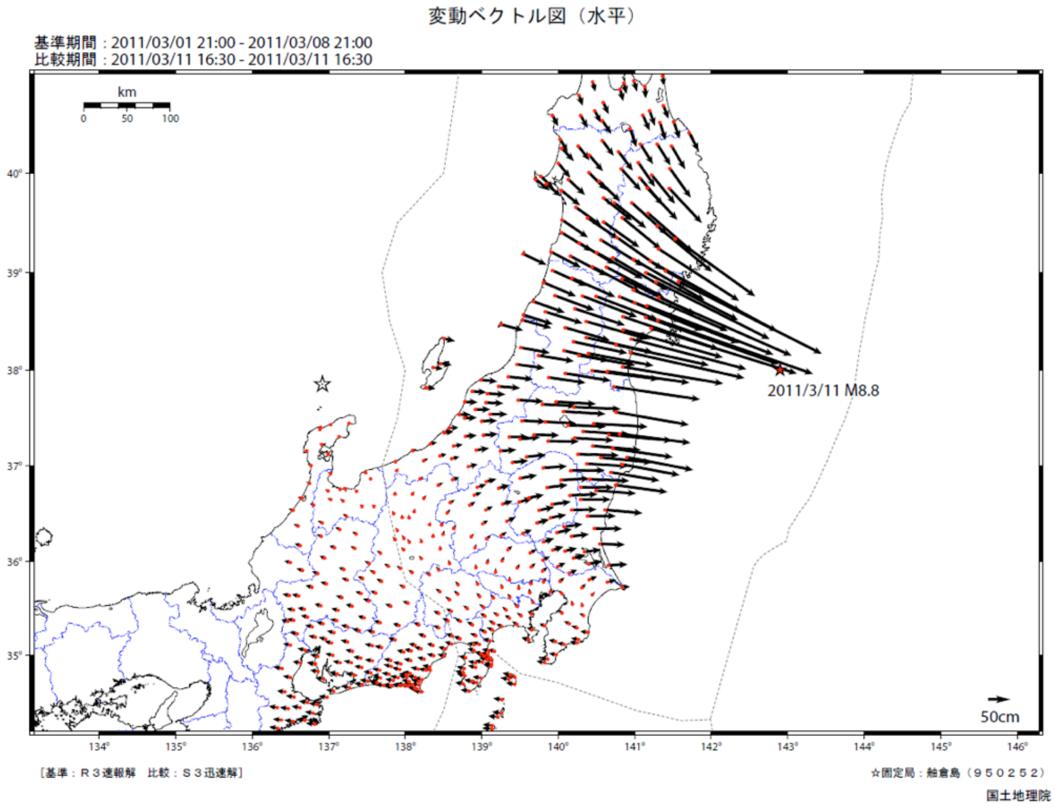
USGS

Fault slip estimated from modeling surface waves

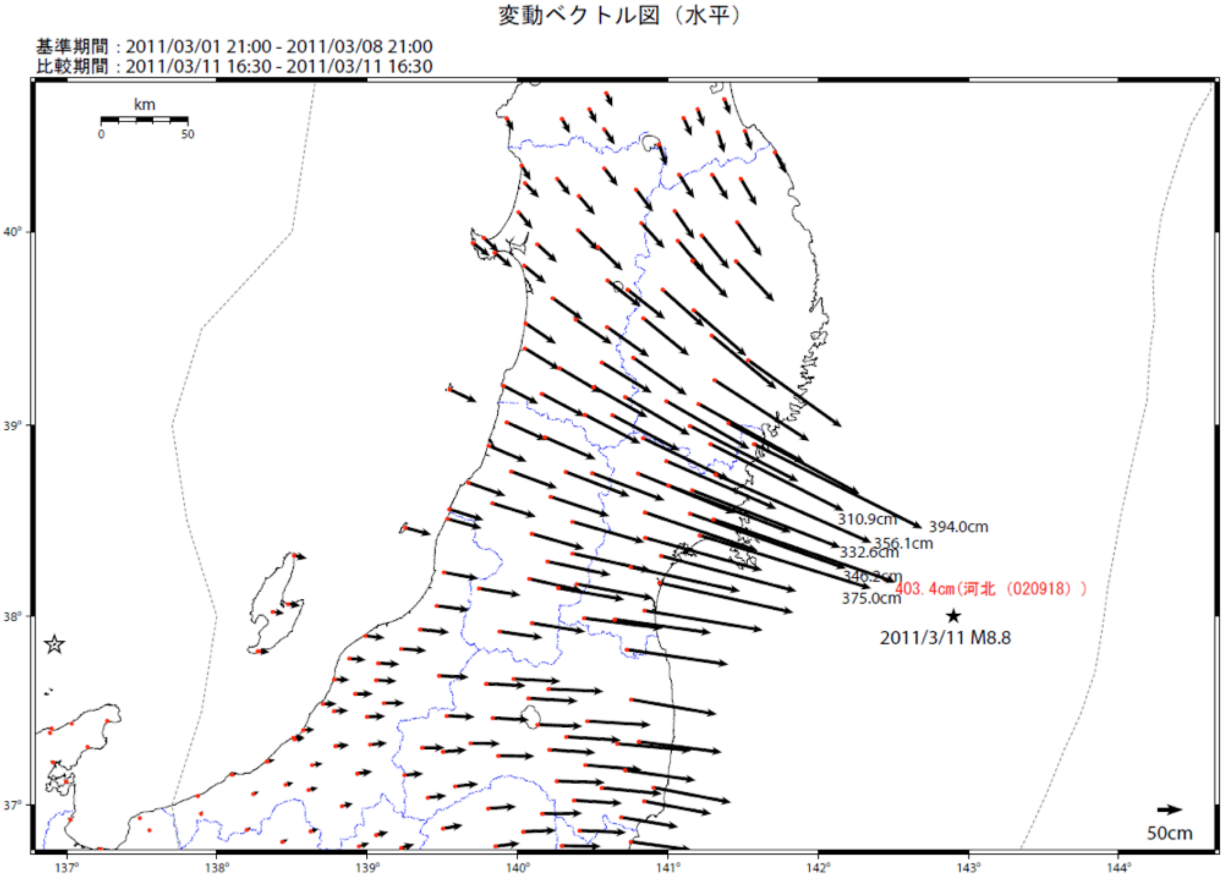


USGS
(Gavin Hayes)

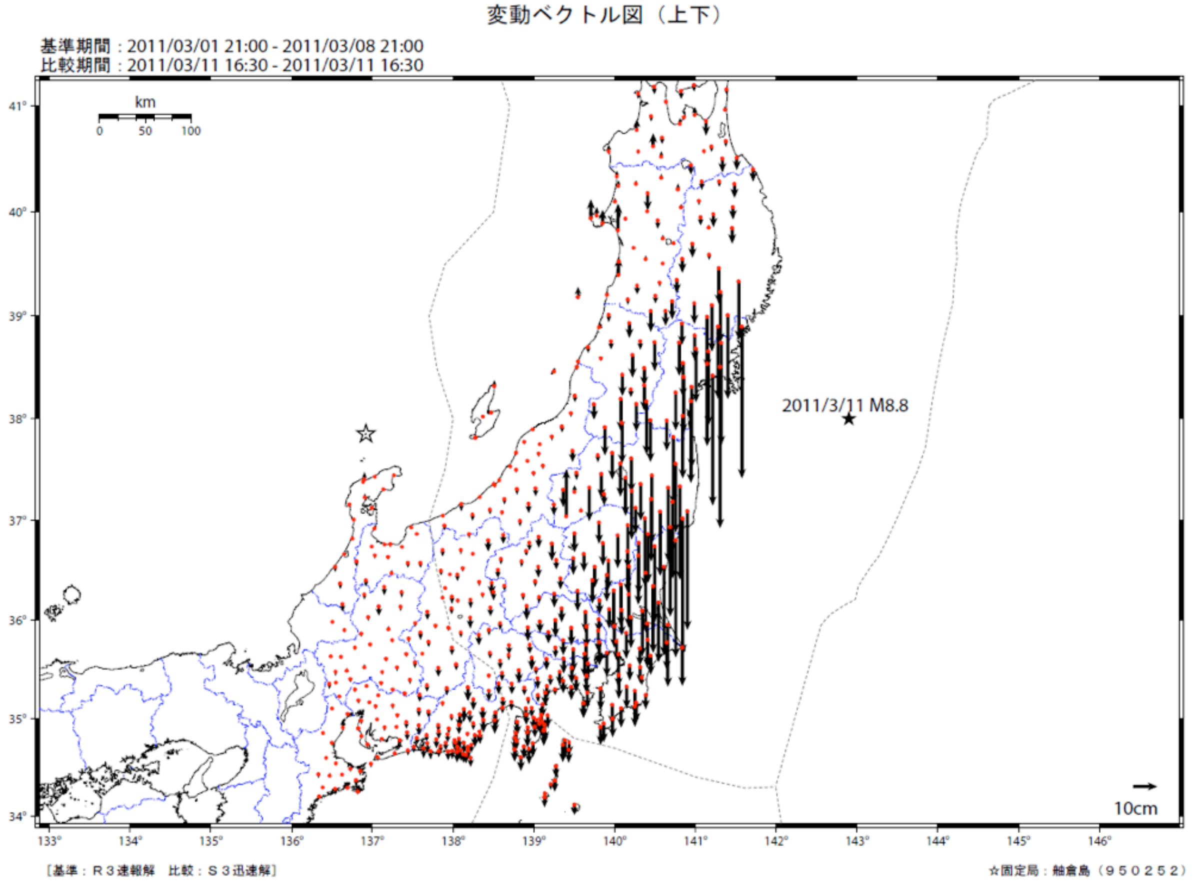
GPS Coseismic displacements



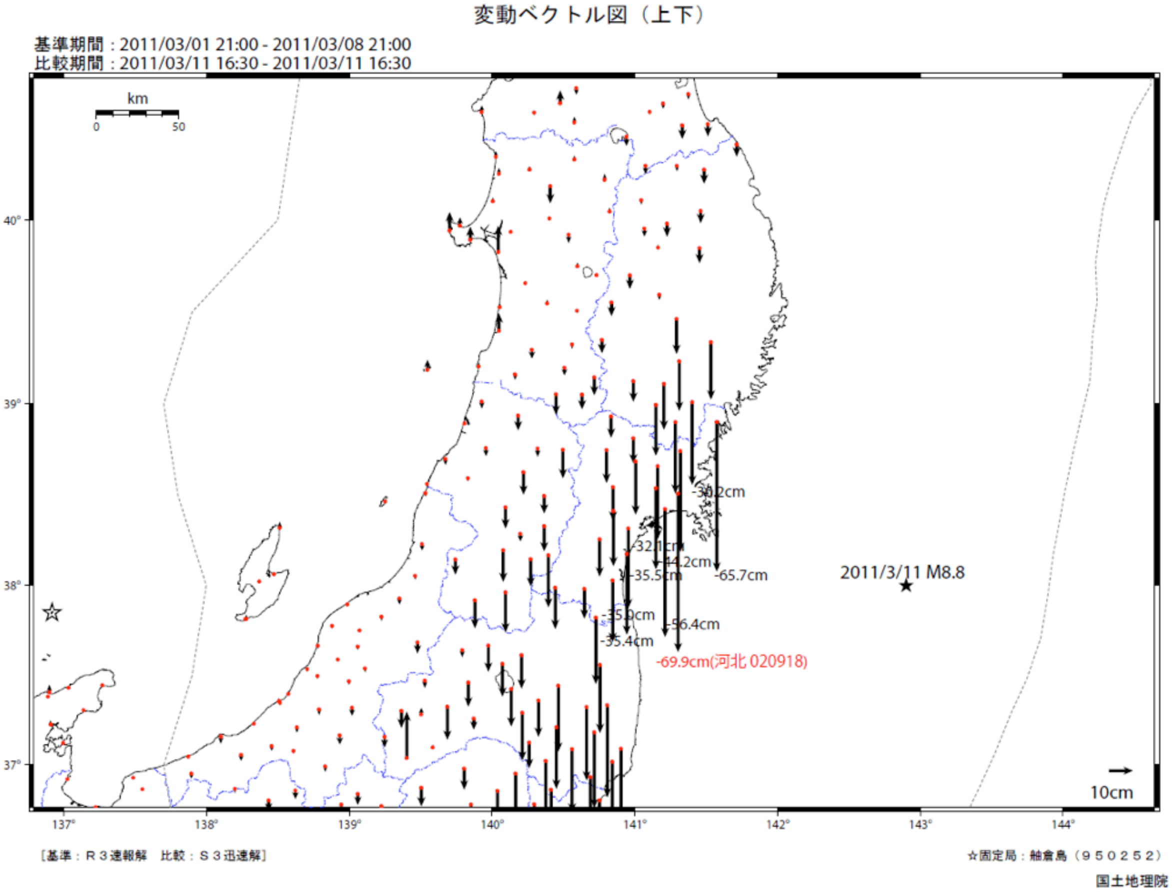
GPS Coseismic displacements - zoomed



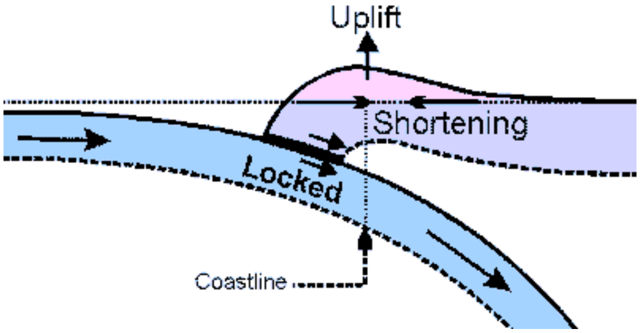
GPS Coseismic displacements (vertical)



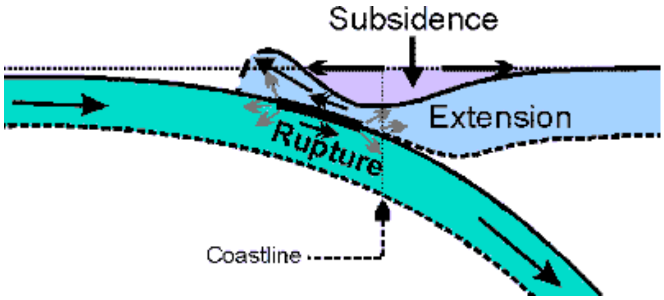
GPS Coseismic displacements (vertical): zoomed



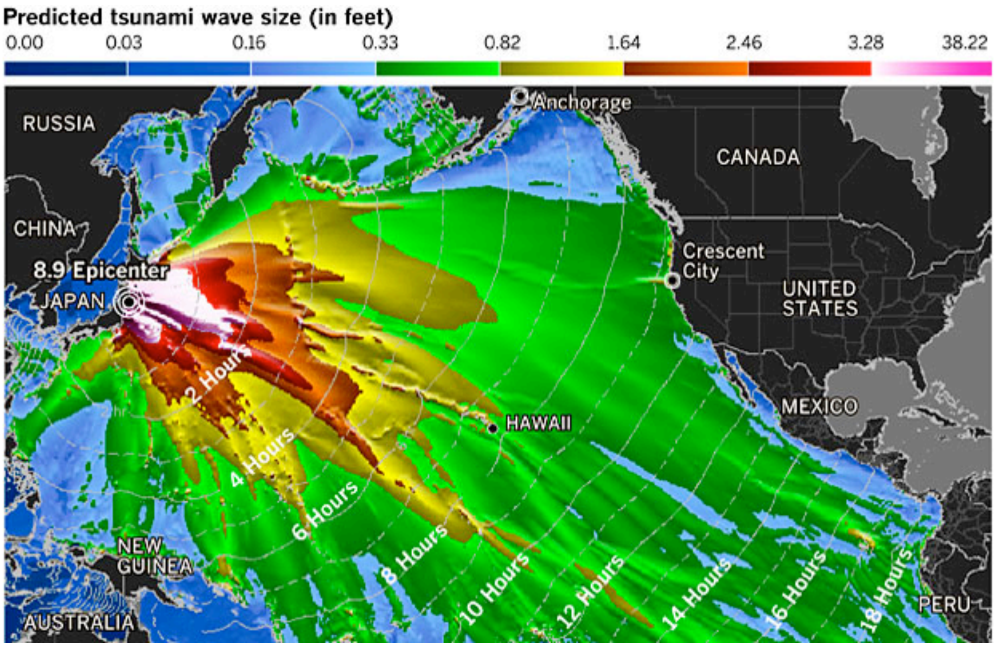
Subduction zone fault earthquake cycle



interseismic



coseismic

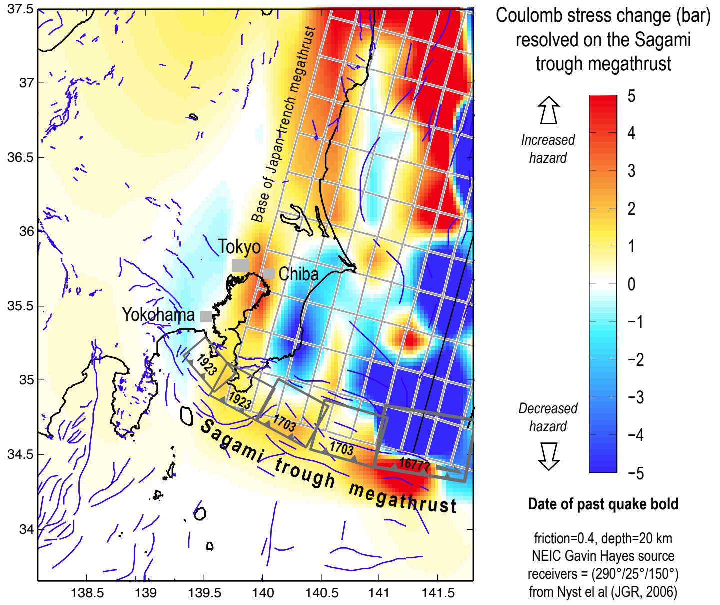


Tsunami: Much of Crescent City harbor destroyed; 4 people swept into sea, 1 feared dead [LA Times]

Waves at Crescent City = 2 m high

Coulomb stress change resolved onto Sagami thrust fault (from Ross Stein)

11 Mar 2011 M=8.9 Off-Tohoku earthquake may have increased stress by several bars on the Sagami megathrust, which last ruptured in 1923 M=7.9 Kanto earthquake (90,000 deaths)

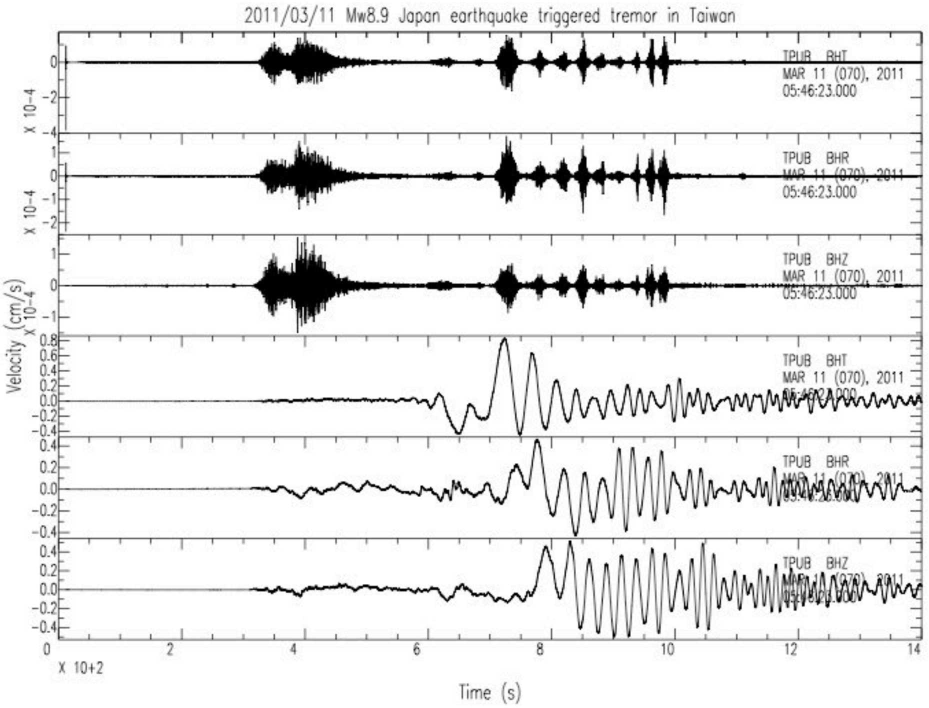


Date of past quake bold

friction=0.4, depth=20 km
NEIC Gavin Hayes source
receivers = (290°/25°/150°)
from Nyst el al (JGR, 2006)

Ross Stein & Volkan Sevilgen (USGS), Shinji Toda (Kyoto Univ.) rstein@usgs.gov 11 Mar 2011 1:14 PM PST

Long-period surface waves triggered tremor in southern Taiwan (Z. Peng) and probably elsewhere



5 Hz high-pass-filtered on the top, and broadband velocity trace on the bottom