



## ADVANCED NUMERICAL TECHNIQUES IN ROCK SLOPE STABILITY ANALYSIS – APPLICATIONS AND LIMITATIONS

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### ABSTRACT

Stability analyses are routinely performed in order to assess the safe and functional design of an excavated slope (e.g. open pit mining, road cuts, etc.), and/or the equilibrium conditions of a natural slope. The analysis technique chosen depends on both site conditions and the potential mode of failure, with careful consideration being given to the varying strengths, weaknesses and limitations inherent in each methodology. This paper presents a review of numerical techniques used in rock slope stability analysis emphasising recent developments in numerical modelling, including advances in computer visualisation and the use of continuum and discontinuum numerical modelling codes.

### INTRODUCTION

The engineer today is presented with a vast range of methods for the stability analysis of rock and mixed rock-soil slopes; these range from simple infinite slope and planar failure limit equilibrium techniques to sophisticated coupled finite-/distinct-element codes. It is less than 25 years since most rock slope stability calculations were performed either graphically or using a hand-held calculator, the exception being advanced analyses involving critical surface searching routines performed on a mainframe computer and Fortran cards. The great majority of early stability analysis programs were in-house with very little software being available commercially. Today, every engineer has access to a personal computer that can undertake with relative ease complex numerical analyses of rock slopes.

Given the wide scope of numerical applications available today, it has become essential for the engineer to fully understand the varying strengths and limitations inherent in each of the different methodologies. For example, limit equilibrium methods still remain the most commonly adopted solution method in rock slope engineering, even though most failures involve complex internal deformation and fracturing which bears little resemblance to the 2-D rigid block assumptions required by most limit equilibrium back-analyses. Initiation or trigger mechanisms may involve sliding movements which can be analysed as a limit equilibrium problem, but this is followed by or preceded by creep, progressive deformation, and extensive internal

disruption of the slope mass. The factors initiating eventual sliding may be complex and not easily allowed for in simple static analysis. Notwithstanding the above comments, limit equilibrium analyses may be highly relevant to simple block failure along discontinuities. It is the authors' view that limit equilibrium techniques should be used in conjunction with numerical modelling to maximize the advantages of both.

The engineer today, if he is to demonstrate due-diligence, must show he has used both all the tools at his disposal and, more importantly, the correct tools. The argument for the use of all relevant available slope analysis techniques in a design or back-analysis is crystallized by the observation of Chen (1), *"In the early days, slope failure was always written off as an act of God. Today, attorneys can always find someone to blame and someone to pay for the damage – especially when the damage involves loss of life or property"*. The design of a slope using a limit equilibrium analysis alone may be completely inadequate if the slope fails by complex mechanisms (e.g. progressive creep, internal deformation and brittle fracture, liquefaction of weaker soil layers, etc.). Furthermore, within slope engineering design and analysis, increased use is being made of hazard appraisal and risk assessment concepts. A risk assessment must address both the consequence of slope failure and the hazard or probability of failure; both require an understanding of the failure mechanism in order that the spatial and temporal probabilities can be addressed.

## **CONVENTIONAL METHODS OF ROCK SLOPE ANALYSIS**

Table 1 provides a summary of those techniques that are routinely applied in conventional slope analyses together with their inherent advantages and limitations. As such, the first step in any rock slope stability analysis must be a detailed evaluation of the lithology and rock mass structure. From this follows the necessity to determine if the orientation of the existing discontinuity sets could lead to block instability. This assessment may be carried out by means of stereographic techniques and kinematic analysis. For example, the program DIPS (2) allows for the visualisation and determination of the kinematic feasibility of rock slopes using friction cones, daylight and toppling envelopes, in addition to graphical and statistical analysis of the discontinuity properties. It is essential that the engineer is aware that such approaches recognise potential sliding failures involving single discontinuities or discontinuity intersections. They do not cater for failure involving multiple joints/joint sets or internal deformation and fracture. Discontinuity data and joint set intersections defined in DIPS, however, can be imported into companion limit equilibrium codes (e.g. SWEDGE (2)) to assess the factor of safety against sliding (Figure 1). These programs often incorporate probabilistic tools, in which variations in joint set properties and added support measures can be assessed for their influence on the factor of safety.

All limiting equilibrium techniques share a common approach based on a comparison of resisting forces/moments mobilized and the disturbing forces/moments. Methods vary, however, in the assumptions adopted in order to achieve a determinate solution. Graphical analysis using stereonet techniques can also be carried out using block theory techniques to assess critical keyblocks. The stability of such keyblocks can then be assessed using limit equilibrium methods such as in the SAFEX program (3) and KBSLOPE (4).

Table 1. Conventional methods of analysis (after (5)).

Analysis method	Critical input parameters	Advantages	Limitations
Stereographic and Kinematic	Critical slope and discontinuity geometry; representative shear strength characteristics.	Relatively simple to use and give an initial indication of failure potential. Some methods allow identification and analysis of critical keyblocks. Links are possible with other analysis methods. Can be combined with statistical techniques to indicate probability of failure and associated volumes.	Only really suitable for preliminary design or design of non-critical slopes. Need to determine critical discontinuities that requires engineering judgement. Must be used with representative discontinuity/joint shear strength data. Primarily evaluates critical orientations, neglecting other important joint properties.
Limit Equilibrium	Representative geometry and material characteristics; soil or rock mass shear strength parameters (cohesion and friction); discontinuity shear strength characteristics; groundwater conditions; reinforcement characteristics and external support data.	Wide variety of software available for different failure modes (planar, wedge, toppling, etc.). Mostly deterministic but increased use of probabilistic analysis. Can analyse factor of safety sensitivity to changes in slope geometry and material behaviour. Capable of modelling 2-D and 3-D slopes with multiple materials, reinforcement and groundwater profiles.	Factor of safety calculations give no indication of instability mechanisms. Numerous techniques available all with varying assumptions. Strains and intact failure not allowed for. Do not consider <i>in situ</i> stress state. Probabilistic analysis requires well-defined input data to allow meaningful evaluation. Simple probabilistic analyses may not allow for sample/data covariance.
Rockfall Simulation	Representative slope geometry; rock block sizes and shapes; coefficient of restitution.	Practical tool for siting structures. Can utilise probabilistic analysis. 2-D and 3-D codes available	Limited experience in use relative to empirical design charts.

Considerable advances in commercially available limit equilibrium computer codes have taken place in recent years. These include:

- Integration of 2-D limit equilibrium codes with finite-element groundwater flow and stress analyses (e.g. GEO-SLOPE's SIGMA/W, SEEP/W and SLOPE/W (6)).
- Development of 3-D limit equilibrium methods (e.g. CLARA (7); 3D-SLOPE (8)).
- Development of probabilistic limit equilibrium techniques.
- Ability to allow for varied support and reinforcement.
- Incorporation of unsaturated soil shear strength criteria.
- Greatly improved visualisation, and pre- and post-processing graphics.

These codes are extremely relevant in the analysis of soil slopes and highly altered rock slopes, where sliding takes place on discrete well-defined surfaces. Figure 2 illustrates the use of the 2-D limit equilibrium program SLOPE/W in the back-analysis of a failure in a kaolinised granite slope. Where it is necessary to include the stress state within the rock mass and the influence of complex deformation and brittle fracture, numerical modelling techniques should be used (e.g. Figure 2).

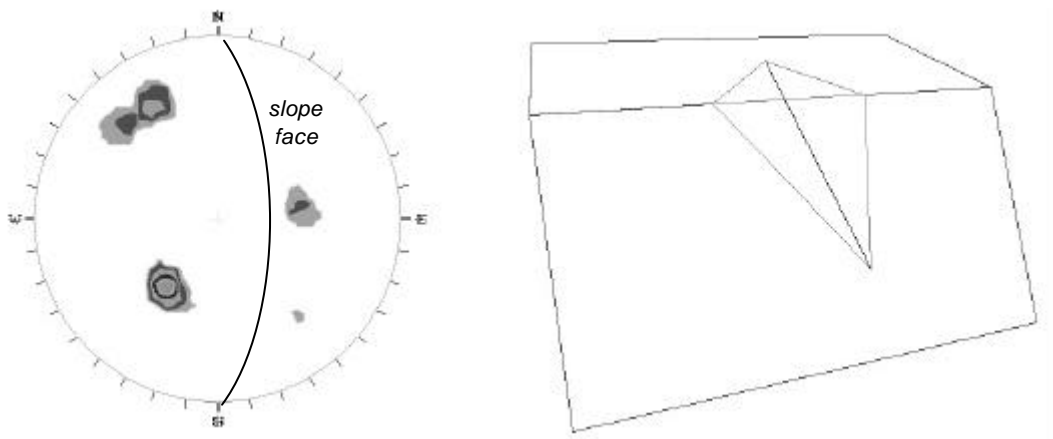


Figure 1. SWEDGE analysis (RIGHT) based on DIPS stereonet input (LEFT).

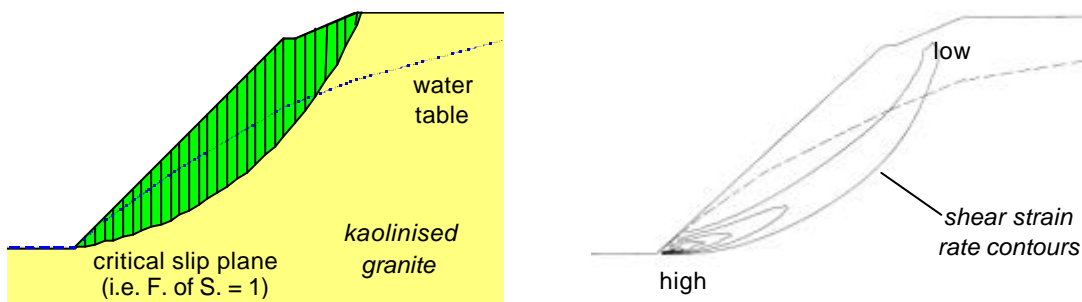


Figure 2. Analysis of China clay slope using limit equilibrium to find the critical slip plane (LEFT) and finite-difference to model shear strain development (RIGHT).

Rockfall simulators, another conventional form of analysis, include tools used to assess hazards of individual falling blocks. Programs such as ROCKFALL (2) analyse the trajectory of falling blocks based on changes in velocity as rock blocks roll and bounce over a given slope geometry. Other factors solved for include block velocity, bounce height and endpoint distance, which can be analysed statistically over a repeated number of simulations to aid in a risk assessment. Rockfall simulators can also assist in determining remedial measures by calculating the effectiveness and kinetic energy of impact on barriers. Similar developments that deal with failed rock blocks and rapid slides include Hungr's (9) DAN code, which proposes a dynamic analysis tool suited for the prediction of flow and runout behaviour.

## NUMERICAL METHODS OF SLOPE ANALYSIS

Many rock slope stability problems involve complexities relating to geometry, material anisotropy, non-linear behaviour, *in situ* stresses and the presence of several coupled processes (e.g. pore pressures, seismic loading, etc.). Advances in computing power

and the availability of relatively inexpensive commercial numerical modelling codes means that the simulation of potential rock slope failure mechanisms could, and in many cases should, form a standard component of a rock slope investigation.

Numerical methods of analysis used for rock slope stability may be conveniently divided into three approaches: continuum, discontinuum and hybrid modelling. Table 2 provides a summary of existing numerical techniques.

Table 2. Numerical methods of analysis (after (5)).

Analysis method	Critical input parameters	Advantages	Limitations
Continuum Modelling (e.g. finite-element, finite-difference)	Representative slope geometry; constitutive criteria (e.g. elastic, elasto-plastic, creep etc.); groundwater characteristics; shear strength of surfaces; <i>in situ</i> stress state.	Allows for material deformation and failure. Can model complex behaviour and mechanisms. Capability of 3-D modelling. Can model effects of groundwater and pore pressures. Able to assess effects of parameter variations on instability. Recent advances in computing hardware allow complex models to be solved on PC's with reasonable run times. Can incorporate creep deformation. Can incorporate dynamic analysis.	Users must be well trained, experienced and observe good modelling practice. Need to be aware of model/software limitations (e.g. boundary effects, mesh aspect ratios, symmetry, hardware memory restrictions). Availability of input data generally poor. Required input parameters not routinely measured. Inability to model effects of highly jointed rock. Can be difficult to perform sensitivity analysis due to run time constraints.
Discontinuum Modelling (e.g. distinct-element, discrete-element)	Representative slope and discontinuity geometry; intact constitutive criteria; discontinuity stiffness and shear strength; groundwater characteristics; <i>in situ</i> stress state.	Allows for block deformation and movement of blocks relative to each other. Can model complex behaviour and mechanisms (combined material and discontinuity behaviour coupled with hydro-mechanical and dynamic analysis). Able to assess effects of parameter variations on instability.	As above, experienced user required to observe good modelling practice. General limitations similar to those listed above. Need to be aware of scale effects. Need to simulate representative discontinuity geometry (spacing, persistence, etc.). Limited data on joint properties available (e.g. $j_{kn}$ , $j_{ks}$ ).
Hybrid/Coupled Modelling	Combination of input parameters listed above for stand-alone models.	Coupled finite-element/distinct-element models able to simulate intact fracture propagation and fragmentation of jointed and bedded media.	Complex problems require high memory capacity. Comparatively little practical experience in use. Requires ongoing calibration and constraints.

## Continuum Modelling

Continuum modelling is best suited for the analysis of slopes that are comprised of massive, intact rock, weak rocks, and soil-like or heavily fractured rock masses. Most continuum codes incorporate a facility for including discrete fractures such as faults and bedding planes but are inappropriate for the analysis of blocky mediums. The continuum approaches used in rock slope stability include the finite-difference and finite-element methods. The salient advantages and limitations are discussed by Hoek *et al.* (10), and both have found widespread use in rock slope analysis.

In recent years the vast majority of published continuum rock slope analyses have used the 2-D finite-difference code, FLAC (11). This code allows a wide choice of constitutive models to characterize the rock mass and incorporates time dependent behaviour, coupled hydro-mechanical and dynamic modelling. An example of the use of FLAC in the modelling of buckling type failures in a surface coal mine slope is shown in Figure 3.

Two-dimensional continuum codes assume plane strain conditions, which are frequently not valid in inhomogeneous rock slopes with varying structure, lithology and topography. The recent advent of 3-D continuum codes such as FLAC3D (11) and VISAGE (12) enables the engineer to undertake 3-D analyses of rock slopes on a desktop computer. An example of a FLAC3D analysis of a china clay slope, which incorporated distinct zones of alteration along strike, is shown in Figure 4.

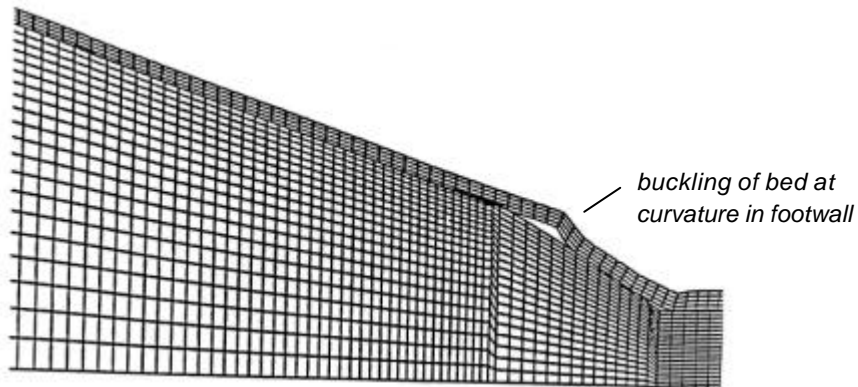


Figure 3. FLAC model of buckling failure in a surface coal mine slope.

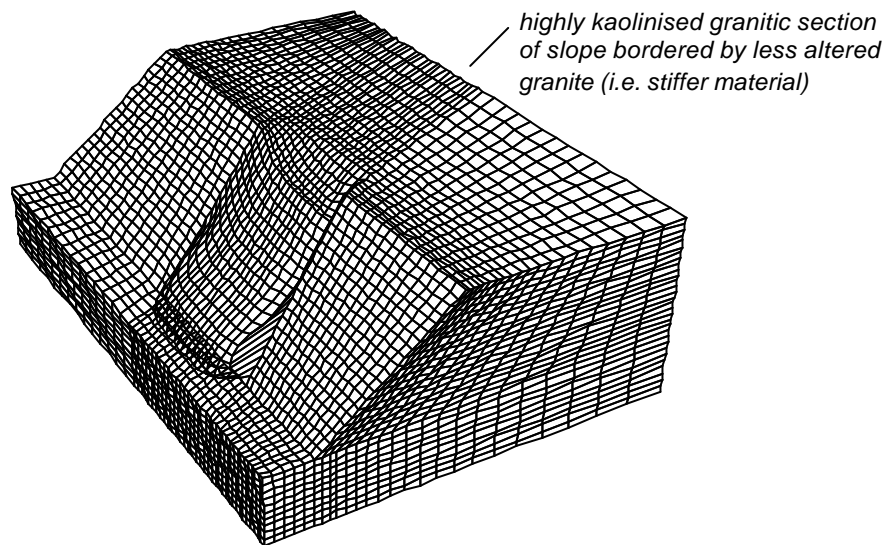


Figure 4. FLAC3D model of china clay slope.

Although 2-D and 3-D continuum codes are extremely useful in characterizing rock slope failure mechanisms it is the responsibility of the engineer to verify whether they are representative of the rock mass under consideration. Where a rock slope comprises multiple joint sets, which control the mechanism of failure, then a discontinuum modelling approach may be considered more appropriate.

## Discontinuum Modelling

Discontinuum methods treat the rock slope as a discontinuous rock mass by considering it as an assemblage of rigid or deformable blocks. The analysis includes sliding along and opening/closure of rock discontinuities controlled principally by the joint normal and joint shear stiffness. Discontinuum modelling constitutes the most commonly applied numerical approach to rock slope analysis, the most popular method being the distinct-element method (13). Distinct-element codes such as UDEC (11) use a force-displacement law specifying interaction between the deformable joint bounded blocks and Newton's second law of motion, providing displacements induced within the rock slope.

UDEC is particularly well suited to problems involving jointed media and has been used extensively in the investigation of both landslides and surface mine slopes. The influence of external factors such as underground mining, earthquakes and groundwater pressure on block sliding and deformation can also be simulated. Figure 5 shows an analysis of the Frank Slide, a major rockslide that occurred in Alberta, Canada. This modelling investigation is described in detail by Benko and Stead (14) and illustrates the possible role of underground coal mining at the foot of the mountain slope on the initiation of the rockslide. Figure 6 illustrates the use UDEC in the modelling of a major toppling instability at the Luscar Mine, Alberta, Canada. This analysis was able to simulate the progressive development of a basal flexure surface as mining proceeded with depth from the surface (15). By undertaking a program of numerical analyses on both observed stable and unstable slopes, the modelling was able to provide valuable information for future mine planning.

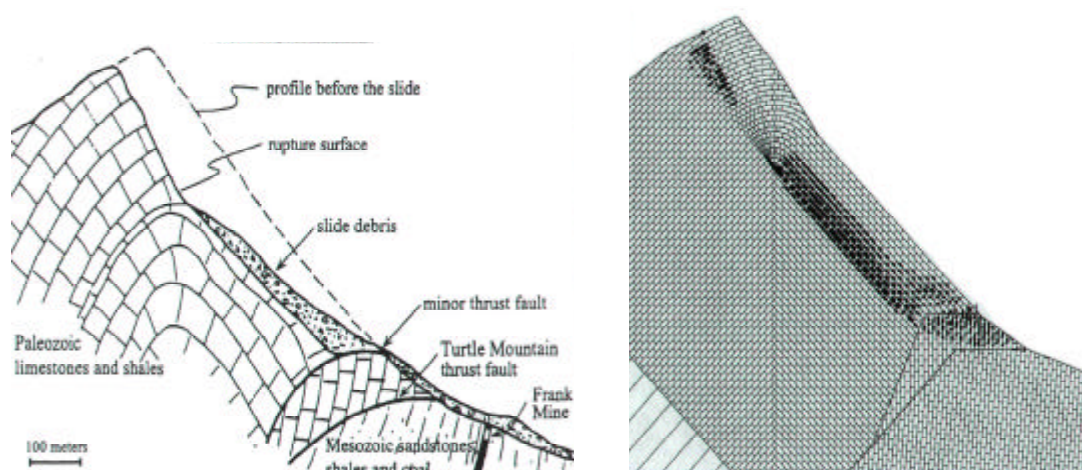


Figure 5. Schematic cross-section of Frank Slide (LEFT) and UDEC model showing shear along bedding and joints (RIGHT).

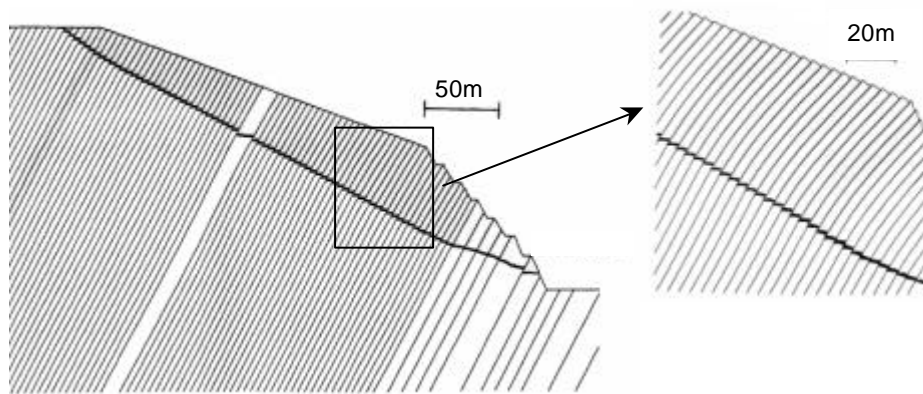


Figure 6. UDEC model of flexural toppling in a surface coal mine slope.

The engineer must again be cautious that the structural input into the distinct-element analysis is representative. Hencher *et al.* (16) illustrated the importance of bedding spacing on predicted failure mechanism. Stead and Eberhardt (17) showed the importance of discontinuity orientation on failure modes observed in surface coal mine slopes. It is stressed that tailoring the structure of the model to accommodate the low random access memory of a laptop computer, for example by using unrepresentative discontinuity spacing, may lead to unrepresentative results. Simulations must always be verified with field observations and wherever possible instrumented slope data. This becomes even truer with the development of 3-D discontinuum codes such as 3DEC (11). Only when a confident portrayal of the 3-D characteristics of a slope has been obtained, can the results be considered representative. This in turn requires the undertaking of an extensive and in-depth site investigation beforehand.

The discontinuous deformation analysis, DDA, developed by Shi (18) has also been used with considerable success in the modelling of discontinuous rock masses, both in terms of rockslides (19) and rockfalls (20). An important recent development in discontinuum codes is the application of distinct-element methodologies and particle flow codes, e.g. PFC<sup>2D/3D</sup> (11). This code allows the rock mass to be represented as a series of spherical particles that interact through frictional sliding contacts. Clusters of particles may be bonded together through specified bond strengths in order to simulate joint bounded blocks. One major advantage of such an approach is that high stresses induced in the rock slope will break the bonds between the particles simulating, in an approximate manner, the intact fracture of the rock.

The use of discontinuum methods in association with continuum methods has been shown by several authors to provide an instructive approach to rock slope analysis. Board *et al.* (21) illustrate the analysis of complex deformation within the 650m high Chuquicamata pit slope, Chile, using a combined approach which utilises both FLAC and UDEC analyses. Similarly, Benko and Stead (14) used an approach adopting FLAC for the initial investigation of Frank Slide and UDEC for the in-depth analysis. The latter study integrated results from limit equilibrium, continuum and discontinuum analyses using each technique as a tool to provide a step in the overall rock slope analysis.



## Hybrid Techniques

Hybrid approaches are increasingly being adopted in rock slope analysis. This may include combined analyses using limit equilibrium stability analysis and finite-element groundwater flow and stress analysis such as adopted in the GEO-SLOPE suite of software (6). Hybrid numerical models have been used for a considerable time in underground rock engineering including coupled boundary-/finite-element and coupled boundary-/distinct-element solutions. Recent advances include coupled particle flow and finite-difference analyses using FLAC3D and PFC<sup>3D</sup> (22). These hybrid techniques already show significant potential in the investigation of such phenomena as piping slope failures, and the influence of high groundwater pressures on the failure of weak rock slopes. Coupled finite-/distinct-element codes are now available which incorporate adaptive remeshing. These methods use a finite-element mesh to represent either the rock slope or joint bounded block. This is coupled with a discrete -element model able to model deformation involving joints. If the stresses within the rock slope exceed the failure criteria within the finite-element model a crack is initiated. Remeshing allows the propagation of the cracks through the finite-element mesh to be simulated. Hybrid codes with adaptive remeshing routines, such as ELFEN (23), have been successfully applied to the simulation of intense fracturing associated with surface mine blasting, mineral grinding, retaining wall failure and underground rock caving (24). The authors are currently exploring the use of this code in the modelling of varied rock slope failure processes.

## FUTURE DEVELOPMENTS

The analysis of complex landslides can now be undertaken routinely using state-of-the-art numerical modelling codes on desktop computers. If the benefits of these methods are to be maximized then it is essential that field data collection techniques are more responsive to advances in design capabilities. Much of current data collection methodology has changed little over the last decade and is aimed towards limit equilibrium analysis. Data including rock mass characteristics, instrumentation and groundwater must be collected in order to allow more realistic modelling of rock slope failure mechanisms. The practising engineer and the research scientist must make efforts to think beyond the use of stand-alone desktop computers and embrace the rapidly developing technology of parallel computers. Several decades ago engineers in industry accepted the need to run slope analyses on mainframe computers, as they could not be done by hand or calculator. The analogy exists today that in order to fully exploit the developments in 3-D coupled models with adaptive remeshing we must use parallel processors where personal computers are no longer sufficient. Such an approach has now been accepted by industry in the 3-D modelling of underground potash mines and in the simulation of petroleum reservoirs.

The next decade holds enormous potential in our ability to model the complete failure process from initiation, through transport to deposition. This will provide a far more rigorous understanding on which to base risk assessment. The advent of virtual reality programming will allow the engineer to convey the results of simulations in a powerful and graphically efficient manner. It is essential however that quality/quantity of both input data and instrumentation data for modelling purposes be improved concomitantly in order to provide the requisite validation.

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