NUMERICAL ANALYSIS OF BRITTLE FRACTURE PROPAGATION AND STEP-PATH FAILURE IN MASSIVE ROCK SLOPES

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ABSTRACT
Although individually both continuum and discontinuum numerical methods provide useful means to analyze rock slope stability problems, complex failures typically involve mechanisms related to both deformation along existing discontinuities and brittle fracture of intact rock. One such example is the 1991 Randa rockslide in the Swiss Alps, where failure involved several complex mechanisms, which require the consideration of progressive failure and brittle fracture propagation. This paper presents results from a series of hybrid finite-/discrete-element models directed towards the explicit modelling of brittle fracturing in natural rock slopes simulating the formation of a step-path failure surface. Two series of models are presented, the first starting from the assumption of an initial continuum (i.e. pre-existing discrete fractures are not included), the second including pre-existing fractures of limited persistence. These results demonstrate that massive rock slope failure involves the initiation and propagation of brittle fractures driven by extension strain, which interact with natural pre-existing discontinuities to form basal and internal shear planes.

RÉSUMÉ
Même si individuellement les méthodes numériques dites de continuum et discontinu procurent une façon utile d'analyser les problèmes de stabilité des pentes rocheuses, les ruptures complexes impliquent normalement des mécanismes reliés à la déformation le long des discontinuités pré-éxistantes et aux fractures fragiles de roche intacte. Un exemple de ce type de rupture est le glissement rocheux Randa de 1991 dans les Alpes Suisse, où la rupture impliqua plusieurs mécanismes complexes, ce qui demanda une considération de la rupture progressive et de la propagation de fractures fragile. Cet article présente les résultats d'une série de modèles avec des éléments hybrides finis discrets. Ceux-ci sont employés vers la modélisation explicite de la fracturation fragile sur des pentes rocheuses naturelles simulant ainsi la formation de surface de rupture en échelon. Deux séries de modèles sont présentées, la première à pour point de départ la supposition que le matériel soit un continuum (i.e. les fractures discrètes préexistantes ne sont pas incluses). La deuxième série inclue les fractures préexistantes de persistance limitée. Ces résultats démontrent qu'une rupture de pente rocheuse massive implique l'initiation et la propagation de fracture fragile en tension conduite par la déformation en extension, et qui interagit avec les discontinuités naturelles préexistantes pour former des plans de cisaillements de base et internes.

1. INTRODUCTION

In massive brittle rock slopes, both natural and engineered (e.g. open pit mines), potential failure surfaces are often assumed to be fully persistent continuous planes. The justification for this is in part due to post-failure observations where tectonic structures are fitted to the failure surface to explain its origin in a geological context, and partly due to the constraints of the analysis technique employed (many of which require the input of fully persistent discontinuities, e.g. limit equilibrium wedge or planar analysis, distinct-element method, etc.). Terzaghi (1962), Robertson (1970), Einstein et al. (1983) and others, however, argue that the persistence of key discontinuity sets is in reality more limited and that a complex interaction between existing natural discontinuities and brittle fracture propagation through intact rock bridges is required to bring the slope to failure. This mode of failure is often referred to as a "step-path" failure.

The analysis of step-path failures and their incorporation into traditional rock slope analysis techniques is somewhat limited. Several solutions have been forwarded based on limit equilibrium techniques that extend Coulomb wedge theory to incorporate an apparent cohesion to account for intact rock bridges along the shear surface, and an apparent friction angle to account for the fact that some inclinations of potential failure have much lower strengths than others (e.g. Jennings, 1970). Eberhardt et al. (2004) argue that processes relating to the internal degradation of rock mass strength, brittle fracture damage, extension strain and internal deformation and shearing are also instrumental in the kinematics of step-path failure in massive rock slopes.

Numerical techniques have evolved to allow for the routine treatment of rock slope deformation and failure, but continue to treat the problem as that of a continuum (e.g. finite-element method) or a discontinuum (e.g.

discrete-element method). Step-path failure mechanisms incorporate a complex integration of continuum deformation, interaction along existing discontinuities and the creation of new fractures. To accommodate this complexity, recent developments in hybrid finite-/discrete-element codes allow for the modelling of both intact behaviour and the development of fractures (Rockfield, 2001).

This paper first presents a detailed literature review of more traditional approaches to step-path failure analysis (e.g. limit equilibrium, probabilistic, etc.). The paper then explores the application of hybrid finite-/discrete-element techniques to explicitly model and simulate the development and kinematic release of step-path failure through brittle fracturing. The 1991 Randa rockslide in southern Switzerland is used as a working example.

2. STEP-PATH ROCK SLOPE FAILURE

Step-path analyses of rock slopes was first discussed in detail by Jennings (1970) who used a limit equilibrium approach that incorporated shear failure along joints, shear through intact rock and tensile failure of rock bridges. He recognized the importance of continuity of jointing and introduced several important, but perhaps under-utilized concepts such as the coefficients of continuity and discontinuity. Jennings (1970) also considered the various models of stepping from upper to lower joints in the step path and the existence of both stepped surfaces in the dip and strike directions.

The next major advance in the limit equilibrium analysis of step-path geometries was a transition from deterministic to probabilistic techniques. McMahon (1979) and Read and Lye (1984) developed and applied probabilistic-Monte-Carlo random sampling techniques to simulate step-path geometries along open pit mining slopes at the Panguna Mine, Bougainville, Papua New Guinea. Baczynski (2000) modified McMahon's Bougainville work for the final pit optimization study at the Ok Tedi mine in Papua New Guinea, re-coding it for use on modern personal computers and incorporating recent developments in rock mass strength representation. The resulting code, STEPSIM4, simulates potential step-path failure surfaces along a system of adjacent ground condition "cells". Each cell is statistically associated with failure through one of several failure modes including sliding along adverse joints, stepping up along steeper dipping joints and direct shear through intact rock bridges. Repeating the simulation for a large number of potential failure paths (e.g. 2000-5000), STEPSIM4 provides a statistical distribution of shear strength along critical step paths.

Fracture mechanics approaches to step-path analysis have evolved from the treatment of simple small steep slopes containing single Mode I (i.e. tensile), Mode II (i.e. shear) and mixed mode I-II cracks (Tharp and Coffin, 1985; Singh and Sun, 1989). Tharp and Coffin (1985) demonstrated the importance of joint persistence (continuity) and compared fracture mechanics results with limit equilibrium analyses. Scavia (1990, 1995) and Scavia and Castelli (1996) investigated the mechanical behaviour of intact rock bridges between a series of parallel joints using the displacement discontinuity method. Sensitivity analyses were undertaken to investigate the variation of the stress intensity with the slope in relation to the rock bridge length and plane inclination.

Muller and Martel (2000), using a two-dimensional boundary element approach, analysed the initiation and development of translational failure surfaces. They demonstrated the importance of stress-concentrating notches in a slope's topography in the promotion of sliding failure. These authors suggested that small stress concentrations at the tips of slope parallel fractures could allow high angle fractures to open thereby linking parallel slide planes and lengthening the surface of the rupture. Such short linking fractures were suggested as potential "risers" in a step-path surface. Kemeny (2003) describes a time-dependent fracture model for brittle fractured rock. The model uses degradation of rock cohesion to simulate the time dependent breaking of intact rock mass bridges. This method is developed for a simple discrete planar failure surface with in-plane intact rock bridges of specified length and spacing. Analysis assuming time dependent sub critical crack growth shows significant reductions in joint cohesion and a consequent reduction in factor of safety. This research is particularly relevant to the time-dependent development of step-path failure surfaces and the long term stability of rock slopes.

3. HYBRID FINITE-/DISCRETE-ELEMENT MODELLING

Recent work by Stead et al. (2004) has demonstrated the simulation of step-path failures using a combined finite-discrete element approach with fracture propagation (Figure 1). Hybrid finite-/discrete-element modelling with fracture propagation has been used successfully in varied geotechnical fields including blasting (Munjiza et al., 1995), underground and experimental rock engineering (Klerck, 2000; Sellers and Klerk, 2000; Klerk et al., 2004), wellbore breakout (Crook et al., 2003) and rock slope instability (Stead et al., 2004; Eberhardt et al., 2004).

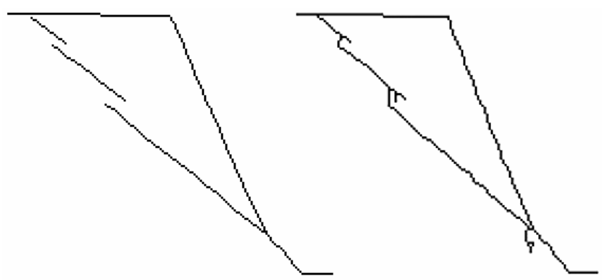

Figure 1. Simulation of step-path failure in a 100 m high rock slope (after Stead et al., 2004)

Models commence with a continuum representation of the problem geometry using finite elements. Discontinuities within the continuum domain can be included using discrete elements. Based on the distribution of stresses throughout the problem domain, progressive fracturing is allowed to occur until an equilibrium is reached, in the process forming discrete elements along the newly formed fracture-bounded blocks, which in turn are composed of deformable finite elements (Figure 2). The simulation of fracturing, damage and associated softening is accomplished using a fracture energy approach controlled by a designated constitutive fracture criterion. Two constitutive fracture models may be used, a Rankine rotating crack model, or a Mohr-Coulomb crack model with a Rankine cutoff. The fracture energy release rate in tension, G_f is assumed to control only the post peak process after the strength limit f_t has been reached (Munjiza et al., 1995). For mode I fracturing and linear softening slope (Crook et al., 2003), the strain softening relationship is given as:

$$\varepsilon_p^{f(e)} = \varepsilon_p^{f(m)} \cdot \frac{l_c^{(m)}}{l_c^{(e)}} \quad \text{and} \quad \varepsilon_p^{f(m)} = \frac{2G_f}{l_c^{(m)} f_t} \qquad [1]$$

where:

$l_c^{(m)}$ is the crack band width
$l_c^{(e)}$ is the element characteristic length
$\varepsilon_p^{f(m)}$ is the reference softening strain
$\varepsilon_p^{f(e)}$ is the inelastic elemental failure strain
f_t is the tensile strength
G_f is the fracture energy

In both constitutive models, fracturing is associated with extensional strain and thus occurs parallel to the localized loading direction.

4. THE 1991 RANDA ROCKSLIDE

The 1991 Randa rockslide, located 12 km north of the Matterhorn in the southern Swiss Alps, involved the failure of 30 million m^3 of massive crystalline rock in two successive episodes approximately three weeks apart (Figures 3 and 4). The failed rock mass was comprised of massive orthogneisses overlain by mica-rich paragneisses. Foliation dips favourably into the slope.

To explain the kinematics of the first slide event (April 18, 1991), Sartori et al. (2003) report its detachment along a more than 500 m long, persistent basal fault near the foot of the slope. The second event (May 9, 1991) is reported by Schindler et al. (1993) and others as having slid along a 40° surface formed along highly persistent shallow dipping joints. These persistent joints can be observed along the sliding surface but are more limited in persistence when encountered in surface outcrops (Willenberg et al., 2002). Schindler et al. (1993) also proposed a series of steep sub-vertical faults as dividing the slide mass into smaller units.

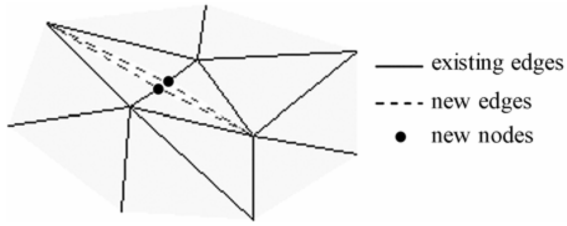

Figure 2. Fracture initiation within a continuum finite-element mesh (after Klerck, 2000)

Although the failure events coincided with a period of snow melt, analysis of climatic and seismic data showed no clear indications of an exceptional triggering event (Schindler et al., 1993). Eberhardt et al. (2001) instead suggested that time-dependent mechanisms relating to brittle strength degradation and progressive failure may more likely be the significant contributing factors that brought the slope to failure. Heavy snowmelt and precipitation prior to failure, although less than recorded in previous years, would have provided the final impetus resulting in failure, the slope having reached a state of strength degradation approaching its limit equilibrium state (i.e. cumulative deformation and fracture due to cyclic fatigue). Based on this conceptual model and corroborating field observations, Eberhardt et al. (2004) questioned the need for fully persistent joints to explain the failure and instead suggested joints of more limited persistence separated by intact rock bridges. These intact rock bridges would then have failed progressively over time forming a basal shear surface stepping up through the rock slope (Figure 5), eventually enabling kinematic release through internal shearing.

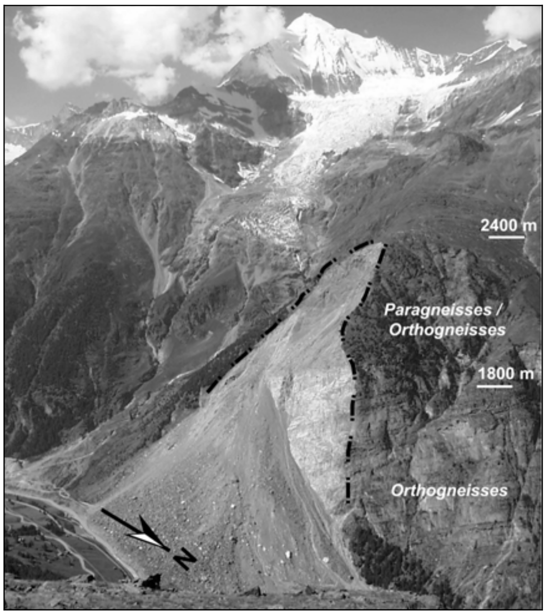

Figure 3. The 1991 Randa rockslide (photo courtesy of H. Willenberg)

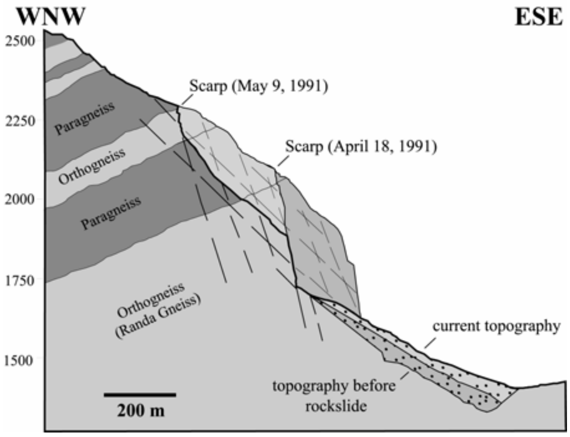

Figure 4. Cross-section of 1991 Randa rockslide (after Schindler et al. 1993)

Figure 5. Back scarp of 1991 Randa rockslide showing limited nature of joint persistence and step-path configuration

5. NUMERICAL ANALYSES ASSUMING NO PRE-EXISTING DISCONTINUITIES

Initial hybrid finite-/discrete-element modelling of the 1991 Randa rockslide (Eberhardt et al., 2004), demonstrated that starting from a continuum representation of the problem geometry (i.e. a geometry coincident with the surface topography), the outline and staged nature of the failure could be reproduced. In this study, the two previously mentioned constitutive fracture models were adopted and compared, the Rankine rotating crack model and the Mohr-Coulomb crack model (with Rankine tension cutoff).

When employing the Rankine rotating crack model, the fractures were observed to form parallel to the surface topography (i.e. parallel to the maximum principal stress). This model was able to closely reproduce the irregular outline of the 1991 Randa failure surface (Figure 6). When initiation processes were modelled using the Mohr Coulomb crack model, extensional strains induced through down-slope movements of the slope mass result in fracture initiation and propagation (Figure 7). This leads to the formation of numerous sub-vertical tension cracks (i.e. normal to the direction of downslope displacements). As the density of these fractures increases, the shear plane progressively develops perpendicular to them forming a curvilinear failure surface typical of more ductile/shear-driven failures. Furthermore, the internal slope deformation and damage developed with the Mohr-Coulomb crack model produces a staged failure closely agreeing with the observed boundaries delineating the two slide phases constituting the 1991 Randa rockslide (Figure 7).

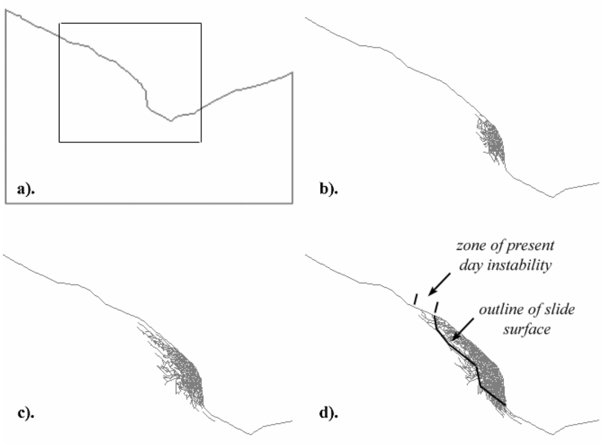

Figure 6. Hybrid finite-/discrete-element model employing a <u>Rankine rotating crack</u> constitutive fracture model, showing: a) full model geometry; and b-d) modelled progressive development of the failure. Superimposed is the outline of the 1991 Randa failure surface

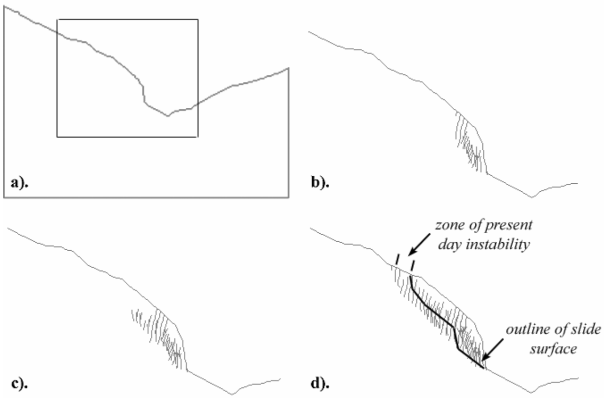

Figure 7. Hybrid finite-/discrete-element model employing a <u>Mohr-Coulomb</u> constitutive fracture model, showing: a) full model geometry; and b-d) modelled progressive development of the failure. Superimposed is the outline of the 1991 Randa failure surface

In both cases, the process was largely driven by the initial formation of brittle tensile fractures, eventually leading to shear failure and full mobilization once the rock mass coherence was significantly degraded. In other words, shear only became a factor after enough tensile fracture damage was incurred to allow mobilization.

6. NUMERICAL ANALYSES ASSUMING PRE-EXISTING DISCONTINUITIES

In the previous models, the role of pre-existing discontinuities was neglected. The final profile of the failure surface in reality, however, will be influenced by the interaction of the prevailing pre-existing discontinuity network and fractures generated through either tension or shear. In other words, the presence of any low dipping natural discontinuities would further aid the process, aligning to form a stepped shear plane that could accommodate further movements and eventually kinematic release of the slide mass.

To examine the potential influence of pre-existing tectonic structures, a staged approach was adopted for which the problem geometry was modified to include either:

- A single tectonic fracture located along the basal shear surface of the first Randa rockslide event.
- A single tectonic fracture located along the basal shear surface of the second Randa rockslide event.
- Two tectonic fractures located along the basal failure surfaces of the first and second Randa failure events.

As in the previous analyses, both the Rankine rotating crack and the Mohr-Coulomb with Rankine tension cutoff constitutive fracture criteria were used. Material properties common for each of the model runs include the rock unit weight, γ = 26 kN/m^3, elastic (Young's) modulus, E = 30 GPa, the Poisson's ratio, ν = 0.33 and fracture energy release rate, G = 200N/m. For the Rankine rotating crack models, the tensile strength was varied between 1.5 and 4 MPa. For the Mohr-Coulomb analyses, a cohesion of 3 MPa, friction angle of 40° and tensile strength of 2 MPa was assumed.

Figure 8 shows several stages in the development of the modelled failure assuming a single lower pre-existing tectonic fracture coinciding with the base of the first rockslide event. These results, obtained using the Rankine rotating crack fracture model, to some extent simulate the retrogressive nature of the rockslide but fail to accurately reproduce the final failure surface geometry. Initially, the model correctly predicts the disintegration and collapse of the rock mass at the front of the slope, but as the solution continues, over-predicts the depth of the basal shear plane relating to the second event. Similar results were obtained using the Mohr-Coulomb fracture criterion (Figure 9). One key difference though, was that the Mohr-Coulomb models appeared to better simulate the progressive development and failure of the lower (first) Randa rockslide event. In both cases, the stepped character of the failure surface between the first and second rockslide events was not properly simulated.

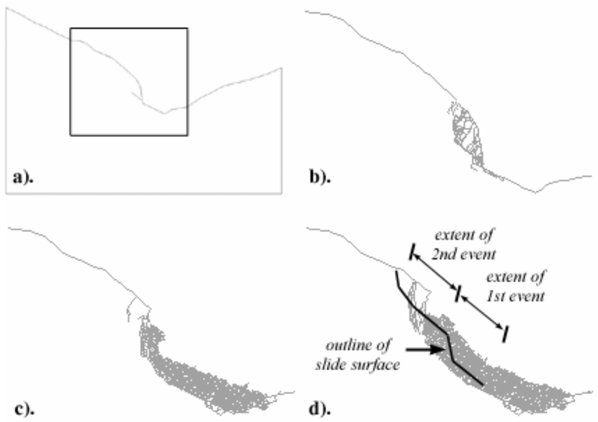

Figure 8. Step-path model showing: a) initial model geometry with pre-existing discontinuity located in the toe of slope; and b-d) modelled progressive development of the slide surface using a Rankine rotating crack fracture model. Superimposed is the outline of the 1991 Randa failure surface with boundaries of the first and second slide events

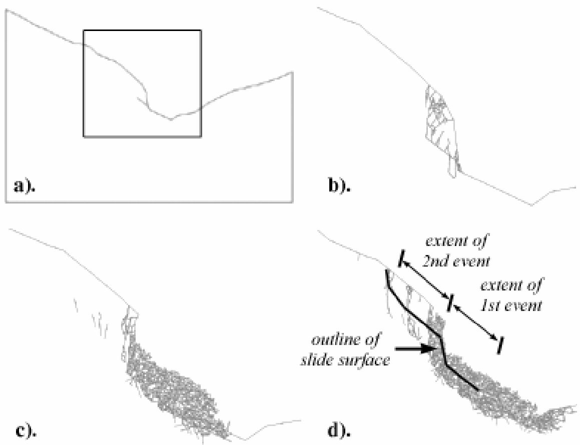

Figure 9. Step-path model showing: a) initial model geometry with pre-existing discontinuity located in the toe of slope; and b-d) modelled progressive development of the slide surface using a Mohr-Coulomb fracture model. Superimposed is the outline of the 1991 Randa failure surface with boundaries of the first and second slide events

Figures 10 and 11 show the results from analyses in which a pre-existing fracture coincident with the shallow dipping base of the second failure event is included (Figure 11 includes both the upper and lower pre-existing basal fractures). Both sets of results were obtained using the Rankine rotating crack constitutive fracture criteria. Interestingly, whether the pre-existing fracture outlining the base of the first slide event is included (Figure 11) or not (Figure 10), both closely reproduce the stepped geometry of the failure surface, and simulate the staged nature of the failure in the process. In the case of the latter (Figure 10), this is accomplished through the initiation and development of brittle fractures propagating to form a basal shear surface below the first slide block (without the aid of a lower pre-existing fracture to constrain it!!).

In both cases, the presence and low dip of the upper pre-existing fracture works to promote slip along its surface as down-slope gravitational forces pull on the rock mass lying above it. The resulting extensional strains that develop within the intact rock eventually lead to the initiation and propagation of subvertical wing cracks near the tips of the pre-existing structure (e.g. Figure 11c). These tensile fractures then continue to grow, ultimately forming both the back scarp of the second slide mass, and the stepped failure surface delimiting the two stages of failure, allowing kinematic release once they intersect the free surface.

Comparable results were obtained for the same geometries when the Mohr-Coulomb constitutive fracture model was used instead. One key difference was with respect to the Mohr-Coulomb fracture model's ability to accurately reproduce the staged nature of the failure when the lower pre-existing fracture at the base of the first slide block was omitted (i.e. using the same initial geometry as shown in Figure 10). Although the fractures generated in these models coalesce to form both a basal shear plane and subvertical fractures stepping up from it to connect with the pre-existing low-dipping fracture in the upper slope, the fractures continued to grow beyond this point working their way up and through the upper slope as opposed to allowing sliding along the pre-existing structure (Figure 12).

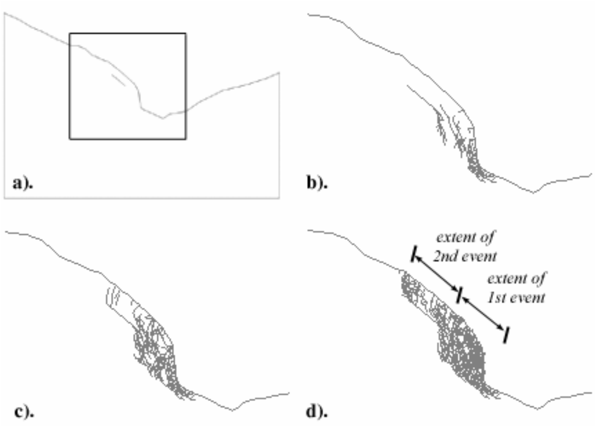

Figure 10. Step-path model showing: a) initial model geometry with pre-existing discontinuity located in the upper section of the slope; and b-d) modelled progressive development of the slide surface using a <u>Rankine rotating crack</u> fracture model. Superimposed are the limits of the first and second Randa slide events

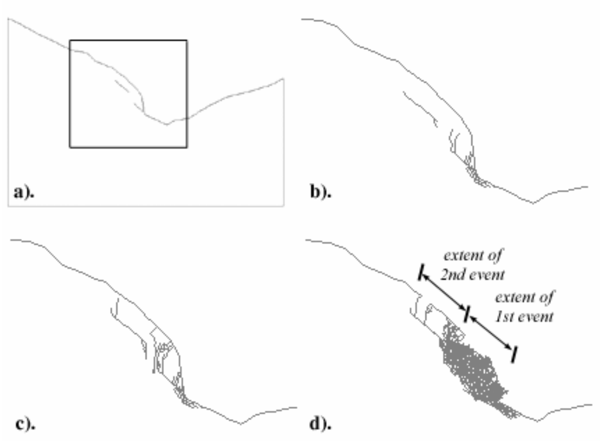

Figure 11. Step-path model showing: a) initial model geometry with pre-existing discontinuity located in both the toe and upper section of the slope; and b-d) modelled progressive development of the slide surface using a <u>Rankine rotating crack</u> fracture model. Superimposed are the limits of the first and second Randa slide events

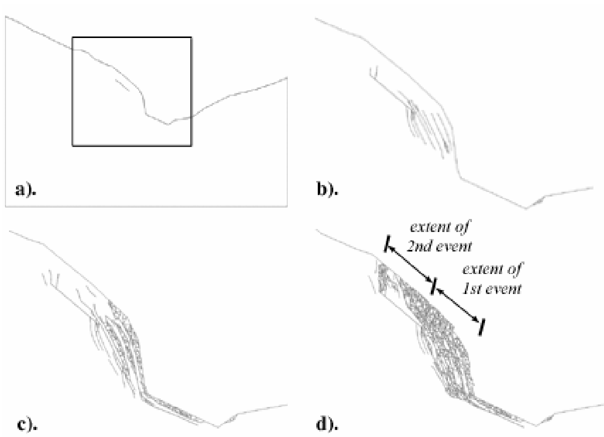

Figure 12. Step-path model showing: a) initial model geometry with pre-existing discontinuity located in the upper section of the slope; and b-d) modelled progressive development of the slide surface using a <u>Mohr-Coulomb</u> fracture model. Superimposed are the limits of the first and second Randa slide events

Further work on these models is currently underway to more rigorously test the initial findings and to investigate their sensitivity to the various input parameters. However, based on the preliminary results presented, it can be said that the Rankine rotating crack fracture criterion is only able to reproduce both the outline/geometry of the failure surface and staged nature of the failure when constraint is provided through the presence of a pre-existing tectonic fracture parallel to the upper slope. In this scenario the predominance of zones of tensile stresses/extension strains subvertical to this feature is well suited to the Rankine criterion. When no pre-existing fracture surface is located in the upper part of the rock slope the Mohr-

Coulomb fracture criterion (with fracturing developing in a more compressive stress environment) appears more relevant, particularly in the development of the first failure event.

As such, these results suggest that the first rockslide event involved the long term degradation of the cohesive/tensile strength in the lower slope through brittle fracture damage driven by a compressive stress regime; the compressive stress regime itself derives from the glacially carved over-steepened slope face and its relatively sharp intersection with the gently dipping valley floor. Following the progressive failure and collapse of the lower Randa block, a transition in the stress environment from a compressional/shear dominated regime to one dominated by extensile strains develops (i.e. through the loss of confinement). This is supported by the results of the Rankine rotating crack-based fracture simulations. Furthermore, these results agree with those presented by Stacey et al. (2003) expressing the importance of extensional strains in the failure of large rock slopes.

7. CONCLUSIONS

Although individually both continuum and discontinuum numerical methods (e.g. finite-element and distinct-element, respectively) provide useful means to analyze rock slope stability problems, complex step-path failures typically involve mechanisms related to both deformation along existing discontinuities and brittle fracture of intact rock. One such example is the 1991 Randa rockslide in the south western Swiss Alps, where failure involved several complex mechanisms, which require the consideration of progressive failure and brittle fracture propagation.

Through the application of hybrid finite-/discrete-element techniques, the explicit modelling of brittle fracture processes in natural rock slopes shows promise in enabling the modelling of progressive failure - from initiation through to the development of catastrophic failure, transport and deposition. When such techniques are applied to model simulations of the 1991 Randa Rockslide, the hybrid finite-/discrete-element solution reproduces both the outline of the failure surface and the staged nature of the failure.

The first series of models presented start from the assumption of an initial continuum (i.e. pre-existing discrete fractures are not included). Results show the generation of predominantly sub-vertical tensile fractures normal to the direction of downslope movement (i.e. driven by extensional strain). As the density of these fractures increase, the shear plane progressively develops perpendicular to them forming a curvilinear failure surface typical of more ductile failures.

The second series of models presented include pre-existing sub-horizontal tectonic fractures. These fractures help to constrain the stepped development of the failure surface, especially when the pre-existing tectonic structure is located in the upper slope. Preliminary results suggest that this is due to the difference in stress regimes controlling the two separate slide events that constitute the 1991 Randa rockslide. In the lower slope, the first slide event appears to be more closely related to a long term destabilization driven by strength degradation and brittle fracture damage within a compressive stress regime. In contrast, the upper slope failure (i.e. the second rockslide event) appears to be related to the development of extensile strains and subvertical tension fractures.

Together, these results show that massive rock slope failure involves the initiation and propagation of brittle tensile fractures driven by extensional strain, which interact with natural pre-existing discontinuities to form basal and internal shear planes. Through the adoption of new hybrid numerical modelling techniques, combining finite- and discrete-element methodologies, results presented in this paper demonstrate the value of explicitly modelling brittle fracture processes to better understand complex rock slope failure mechanisms.

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