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Engineering Geology 83 (2006) 217–235
ENGINEERING GEOLOGY
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Developments in the characterization of complex rock slope deformation and failure using numerical modelling techniques

D. Stead [a,*], E. Eberhardt [b], J.S. Coggan [c]

[a] Department of Earth Sciences, Simon Fraser University, Burnaby, BC, Canada
[b] Geological Engineering/Earth and Ocean Sciences, University of British Columbia, Vancouver, BC, Canada
[c] Camborne School of Mines, University of Exeter in Cornwall, Penryn, Cornwall, U.K.

Accepted 24 June 2005
Available online 10 November 2005

Abstract

Recent advances in the characterization of complex rock slope deformation and failure using numerical techniques have demonstrated significant potential for furthering our understanding of both the mechanisms/processes involved and the associated risk. This paper illustrates how rock slope analyses may be undertaken using three levels of sophistication. Level I analyses include the conventional application of kinematic and limit equilibrium techniques with modifications to include probabilistic techniques, coupling of groundwater simulations and simplistic treatment of intact fracture and plastic yield. Such analyses are primarily suited to simple translational failures involving release on smooth basal, rear and lateral surfaces where the principle active damage mechanisms are progressive failure and/or asperity breakdown. Level II analyses involve the use of continuum and discontinuum numerical methods. In addition to simple translation, Level II techniques can be applied to complex translational rock slope deformations where step-path failure necessitates degradation and failure of intact rock bridges along basal, rear and lateral release surfaces. Active damage processes in this case comprise not only strength degradation along the release surface (e.g., asperity breakdown) but also a significant component of brittle intact rock fracture. Level III analyses involve the use of hybrid continuum–discontinuum codes with fracture simulation capabilities. These codes are applicable to a wide spectrum of rock slope failure modes, but are particularly well suited to complex translation/rotational instabilities where failure requires internal yielding, brittle fracturing and shearing (in addition to strength degradation along release surfaces). Through a series of rock slope analyses the application of varied numerical methods are discussed. Particular emphasis is given to state-of-the-art developments and potential use of Level III hybrid techniques.

Keywords: Rock slopes; Numerical modelling; Continuum; Discontinuum; Hybrid techniques; Brittle fracture damage

1. Introduction

Numerical modelling of rock slopes is now used routinely in the civil and mining engineering sectors as well as in academic research. Given the wide scope of numerical applications available today, it is essential for the engineer and geoscientist to fully understand the varying strengths and limitations inherent in each of the different methodologies. The use of limit equilibrium methods still remains the most common adopted solution method in surface rock engineering although in many cases, major rock slope instabilities often involve complex internal deformation and fracturing bearing little resemblance to the 2-D/3-D rigid block assump-

* Corresponding author.
E-mail address: dstead@sfu.ca (D. Stead).

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doi:10.1016/j.enggeo.2005.06.033

tions adopted in most limit equilibrium back-analyses. Initiation or trigger mechanisms may involve sliding movements that can, in the most idealized of cases, be analyzed as a limit equilibrium problem. The processes leading up to this initial slip are however invariably far more complex than a simple balance of disturbing and resisting forces.

In recognition of the controlling influence jointing has on complex rock slope deformation, numerical discontinuum techniques are being increasingly used in practice. It must be recognized however that conventional discontinuum models also have inherent limitations. Failure is frequently followed or preceded by creep, progressive deformation (fatigue damage processes), and extensive internal disruption of the slope mass (brittle/plastic damage). The factors controlling initiation and eventual sliding may be complex and are not easily allowed for in a simple static analysis. Addressing these challenges, the authors suggest that a new phase of slope stability analysis is warranted that utilizes recent advances in computing software and hardware development. In many cases, this may involve the combined use of limit equilibrium and numerical modelling techniques to maximize the advantages of both. As engineers are increasingly required to undertake landslide hazard appraisals and risk assessments, they must address both the consequence of slope failure and the hazard or probability of failure; a critical component of both is an understanding of the underlying processes/mechanisms driving the instability so that spatial and temporal probabilities of failure can be addressed. Limit equilibrium concepts alone cannot answer these questions. This paper will discuss and provide examples of the slope analysis tools that are available to the engineer, emphasising recent developments in numerical methods in the analysis of complex rock slope deformations.

2. Kinematic and limit equilibrium analysis of rock slopes

2.1. Conventional applications

Conventional rock slope analyses in current practice invariably begin with engineering geological investigations of the discontinuities, leading to kinematic and limit equilibrium stability assessments. Table 1, modified after Coggan et al. (1998), provides a summary of conventional methods, together with their advantages and limitations. Several commercial programs are available which may be used to assess rock slope stability using either daylight envelopes (e.g., Dips — Rocscience, 2004) or keyblock theory (e.g., SAFEX — Windsor and Thompson, 1993; Kblock — Pantechnica, 2001). These stereographic techniques can be used as input for deterministic or probabilistic limit equilibrium calculations to determine a factor of safety

Table 1
Conventional methods of analysis (modified after Coggan et al., 1998)

Analysis method	Critical input parameters	Advantages	Limitations
Stereographic and kinematic	Critical slope and discontinuity geometry; representative shear strength characteristics.	Simple to use and show failure potential. Some methods allow analysis of critical key-blocks. Can be used with statistical techniques to indicate probability of failure and associated volumes.	Suitable for preliminary design or for non-critical slopes, using mainly joint orientations. Identification of critical joints requires engineering judgement. Must be used with representative joint/discontinuity strength data.
Limit equilibrium	Representative geometry, material/joint shear strength, material unit weights, groundwater and external loading/support conditions.	Much software available for different failure modes (planar, circular, wedge, toppling, etc.). Mostly deterministic but some probabilistic analyses in 2-D and 3-D with multiple materials, reinforcement and groundwater profiles. Suitable for sensitivity analysis of FofS to most inputs.	FofS calculations must assume instability mechanisms and associated determinacy requirements. In situ stress, strains and intact material failure not considered. Simple probabilistic analyses may not allow for sample/data covariance.
Rockfall simulation	Representative slope geometry and surface condition. Rock block sizes, shapes, unit weights and coefficients of restitution.	Practical tool for siting structures and catch fences. Can utilize probabilistic analysis. 2-D and 3-D codes available.	Limited experience in use relative to empirical design charts.

or probability of failure for potential blocks/wedges. Such stereographic/limit equilibrium methods are normally constrained to joint bounded rigid blocks or wedges. Coupling of keyblock analyses with numerical models can be undertaken but is not common in practice. Where a multi-planar or complex failure surface/mechanism is encountered, then limit equilibrium techniques that calculate the factor of safety by methods of slices with circular, composite or fully specified failure surfaces are routinely used. The various solutions to these methods (e.g., Bishop's modified, Janbu's simplified, Morgenstern–Price, etc.) differ primarily in the assumptions required to make the problem statically determinate.

2.2. Further developments in conventional limit equilibrium techniques

Further extensions and developments in limit equilibrium approaches include:

- increased availability of probabilistic techniques, including use of geostatistics (e.g., Pascoe et al., 1998);
- introduction of 3-D methods, some with capabilities for including support (e.g.,Hungr et al., 1989; Lam and Fredlund, 1993);
- improved searching routines for critical failure surfaces;
- integrated groundwater–stress-limit equilibrium analysis;
- incorporation of unsaturated soil mechanics;
- incorporation of surface hydrology influences (e.g., Wilkinson et al., 2000);
- integration with GIS and risk assessment.

Limit equilibrium analyses of landslides may now be undertaken using 3-D commercial software for both wedge (e.g., SWEDGE — Rocscience, 2004) and circular/multiplanar failure mechanisms (CLARA-W — Hungr, 2002). These techniques, although useful, neglect internal fracture and deformation which are arguably a prerequisite for most 3-D failure geometries. Three-dimensional circular analysis/multiplanar analyses also assume rotation of a block around a common linear axis of revolution. Numerous field-based examples exist in which complex joint bounded 3-D blocks slide and rotate out of the slope with obvious implications for the applicability of both 2-D toppling and 3-D rock wedge limit equilibrium analyses.

The increasing use of risk assessment in engineering practice, and the need to deal with parameter uncertainty in slope analysis, has driven the development of probabilistic limit equilibrium tools. Integration of infinite slope probabilistic analyses with Geographic Information Systems (GIS) is becoming commonplace in landslide hazard assessment (Zhou et al., 2003; Pack et al., 2001). The quick and interactive means by which computer-based software operates makes it ideal for incorporating probabilistic algorithms in which variations in joint geometrical characteristics (e.g., dip, dip direction, etc.) can be assessed for its influence on the factor of safety (Fig. 1). Fuzzy logic routines designed to manage uncertainty in the input parameters can likewise be incorporated (Faure and Maiolino, 2000).

2.3. Advanced application of limit equilibrium techniques to complex rock slope failure

Fig. 2 shows a flow chart illustrating three levels of landslide analysis. Level I includes preliminary kine-

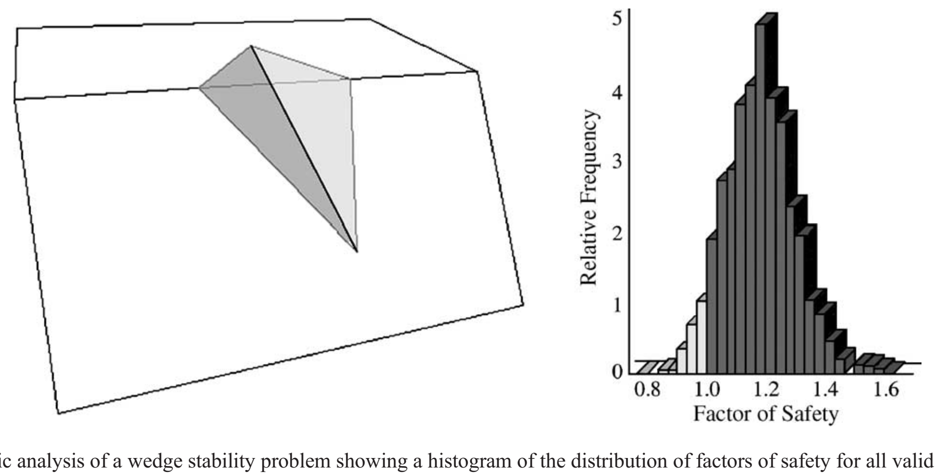

Fig. 1. Probabilistic analysis of a wedge stability problem showing a histogram of the distribution of factors of safety for all valid wedges generated by a Monte Carlo sampling of the input data.

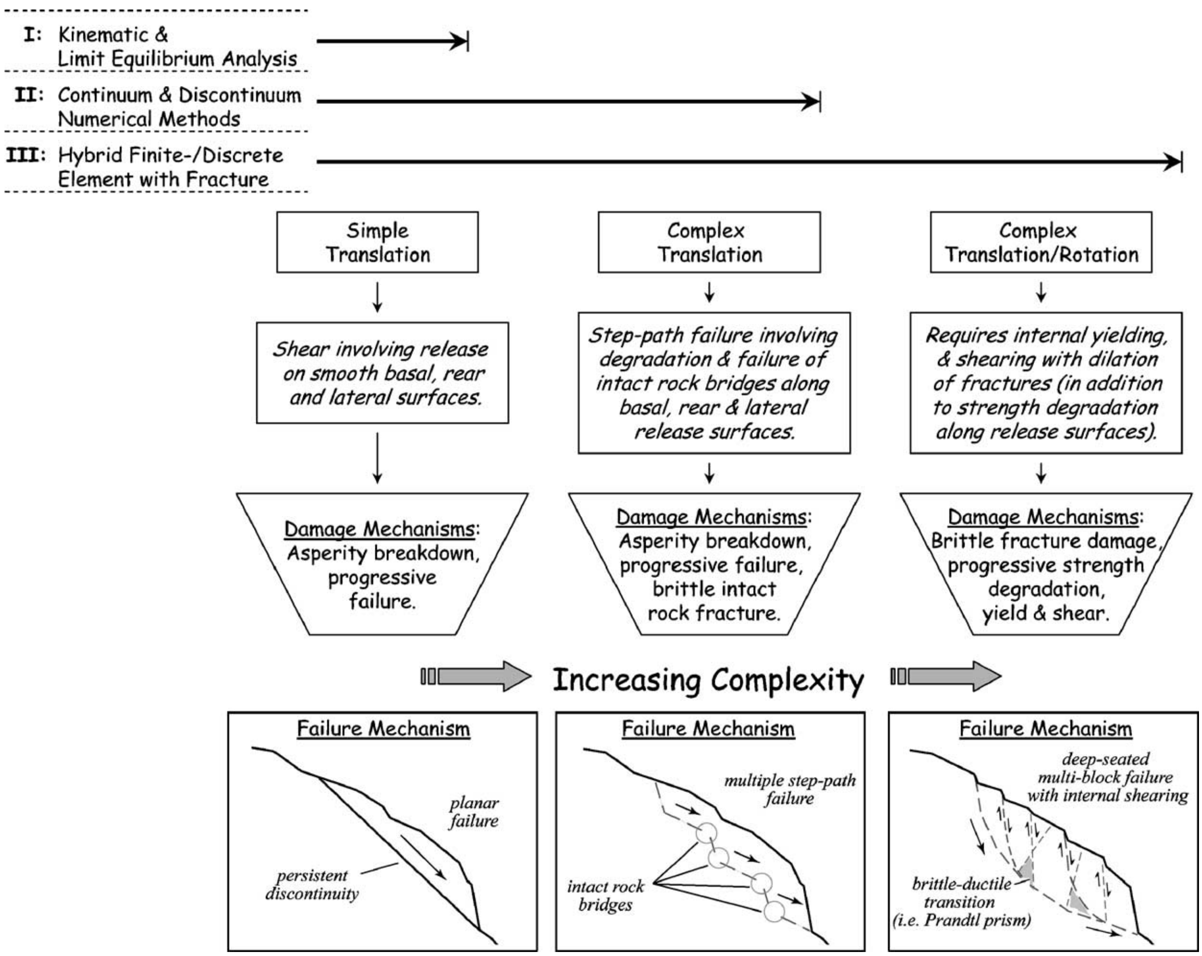

Fig. 2. Flowchart illustrating three levels of landslide analysis and the modes of translational/rotational failure they apply to.

matic and limit equilibrium analysis. These methods are particularly suited to translational failures where basal shear, lateral and rear release all take place on persistent joints, particularly at the residual angle of friction. The authors suggest such conditions are rare in practice.

To account for a non-persistent failure plane, Terzaghi (1962) included an effective cohesion along the shear surface to allow for the increased resistance to shear failure provided by intact rock bridges. Similarly, consideration has also been given to the development of tensile fractures orthogonal to the non-persistent sliding surface where some stepping is required to allow kinematic release. Modification to Level 1 techniques have been attempted by numerous authors to accommodate deformation mechanisms involving step-paths and intact rock fracture (Jennings, 1970; Baczynski, 2000; Kemeny, 2003).

Other authors have viewed the problem of cohesive strength along a potential failure surface in the context of progressive failure (Bjerrum, 1967; Chowdhury and Grivas, 1982). In the case of the latter, the authors considered progressive failure in terms of a probabilistic model of gradual shear strength reduction along the failure surface. To date, limit equilibrium techniques have found little application toward simulation of progressive failure. This is generally due to the complexity involved in the time-dependent geometrical development of even the most simple shear surfaces.

This highlights a major limitation to the Level I techniques, as only processes active along the developing translational shear plane are considered. Where geometrical constraints on the slide mass are present this may necessitate internal deformation, yielding and/or shearing of the rock mass. Mencl (1966) proposed the development of a Prandtl wedge (i.e., damage zone) to explain failure along bi-planar slide surfaces with regard to the 1963 Vaiont slide. Kvapil and Clews (1979) further developed this concept to simulate the transmission of stresses within a rock slope from an active to passive block along a primary system of plane transversal shear surfaces and a secondary system of log spirals (Fig. 3). In a variation of this approach to account for the importance of internal shear, Sarma (1979) formulated a limit equilibrium method using inclined slices. Martin and Kaiser (1984) showed that internal shear and the consequent dilation of the rock mass are a necessary prerequisite to accommodate motion along a basal slip surface.

In summary, the primary kinematic controls on massive rock slope failure can be viewed as both strength degradation in the form of shear plane development (i.e., progressive failure) and strength degradation manifested through internal slide mass deformation (i.e., brittle–ductile yielding and/or shearing). The latter component is most dominant in situations where the failure surface is non-planar (i.e., transition from Level I to Level II analyses).

3. Numerical analysis of rock slopes

Level II analysis techniques (Fig. 2) include numerical modelling methods that provide approximate solutions to problems incorporating intact rock deformation (strain) during rock slope failure. Many of these techniques address complexities relating to geometry, material anisotropy, non-linear behaviour, in situ stresses and the presence of several coupled processes (e.g., pore pressures, seismic loading, etc.). Advances in computing power and the availability of relatively inexpensive commercial numerical modelling codes means that the simulation of potential rock slope failure mechanisms could, and in many cases should, form a standard component of a rock slope investigation.

3.1. Conventional applications

Conventional numerical modelling approaches may be conveniently subdivided into:

- continuum methods (e.g., finite element, finite difference, etc.);
- discontinuum methods (e.g., distinct element; discontinuous deformation analysis, etc.).

Table 2, modified after Coggan et al. (1998), shows the characteristics, applications and limitations of numerical methods currently used in the analysis of rock and mixed rock/soil slopes.

Early numerical analyses of rock slopes were predominantly undertaken using continuum finite-element codes. Kalkani and Piteau (1976), for example, used this method to analyze toppling of rock slopes at Hells Gate in British Columbia, Canada and Krahn and Morgenstern (1976) undertook preliminary finite-element modelling of the Frank Slide in Alberta, Canada. Radbruch-Hall et al. (1976) similarly used a finite-element analysis to simulate the stress distributions in rock slopes in order to investigate mechanisms of high mountain deformation and “sackung” formation. More recently, Stacey et al. (2003) used finite-element analysis in an innovative analysis of extensile strain distributions associated with deep open pit mines. The use of finite-difference codes has predominantly involved the use of the FLAC 2-D and 3-D codes (Itasca, 2004). Board et al. (1996) clearly showed the potential for using continuum 2-D finite-difference codes in conjunction with discontinuum methods in the analysis of high rock slopes at the Chuquicamata open-pit mine in

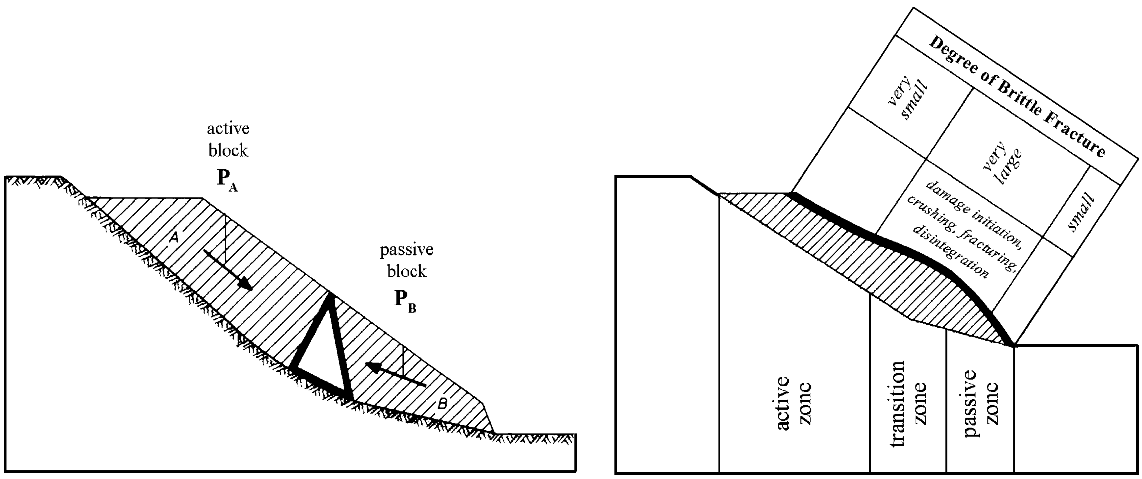

Fig. 3. Schematic representation of a Prandtl’s prism transition zone the corresponding locations of zones with different degrees of fracturing (modified after Kvapil and Clews, 1979).

Table 2
Numerical methods of analysis (modified after Coggan et al., 1998)

Analysis method	Critical input parameters	Advantages	Limitations
Continuum modelling (e.g., finite element, finite difference)	Representative slope geometry; constitutive criteria (e.g., elastic, elasto-plastic, creep, etc.); groundwater characteristics; shear strength of surfaces; *in situ* stress state.	Allows for material deformation and failure, including complex behaviour and mechanisms, in 2-D and 3-D with coupled modelling of groundwater. Can assess effects of critical parameter variations on instability mechanisms. Can incorporate creep deformation and dynamic analysis. Some programs use imbedded language (e.g., FISH) to allow user to define own functions and subroutines.	Users should be well trained, experienced, observe good modelling practice and be aware of model/software limitations. Input data generally limited and some required inputs are not routinely measured. Sensitivity analyses limited due to run time constraints, but this is rapidly improving.
Discontinuum modelling (e.g., distinct element, DDA)	Slope and discontinuity geometry; intact constitutive criteria (elastic, elasto-plastic, etc.); discontinuity stiffness and shear strength; groundwater and *in situ stress* conditions.	Allows for block deformation and movement of blocks relative to each other. Can model complex behaviour and mechanisms (combined material and discontinuity behaviour, coupled with hydro-mechanical and dynamic analysis). Able to assess effects of parameter variations on instability. Some programs use imbedded language (e.g., FISH) to allow user to define own functions and subroutines.	As above, experienced users needed. General limitations similar to those listed above. Need to simulate representative discontinuity geometry (spacing, persistence, etc.). Limited data on joint properties available (e.g., joint stiffness, jk_n and jk_s).

Chile. Coggan et al. (2000) successfully demonstrated the use of both two- and three-dimensional finite-difference analyses in the back analysis of highly kaolinised china clay slopes. Guadagno et al. (2003) adopted a finite difference approach to analyse the influence of cut-and-fill works on slopes in Campania, southern Italy; their models considering both dry and steady state flow seepage conditions. Kinakin and Stead (2005) recently extended the finite-element analyses of Radbruch-Hall et al. (1976) to investigate sackung formation using an elasto-plastic finite-difference model of varied rock slope ridge geometries associated with "sackung" features in British Columbia, Canada. Both finite-element and finite-difference models remain in routine use in engineering landslide investigations and are most appropriate in the analysis of slopes involving weak rock/soils or rock masses where failure is controlled by the deformation of the intact material (i.e., continuum) or through a restricted number of discrete discontinuities such as a bedding plane or fault.

Where the stability of the rock slope is controlled by movement of joint-bounded blocks and/or intact rock deformation then the use of discontinuum discrete-element codes should be considered. Discrete-element codes have found increasing use in the analysis of rock slopes in recent years and are now in routine use in civil and mining engineering. Two principal methods are in use, distinct element (Hart, 1993) and discontinuous deformation analysis (DDA; Shi and Goodman, 1989), the former becoming more common in engineering practice. Example applications of DDA include the analysis of the Vaiont rockslide (Sitar and Maclaughlin, 1997), and of a major rockfall in Japan by Chen and Ohnishi (1999). The predominant discontinuum method in use however, is the distinct-element code 'UDEC' (Itasca, 2004). This code has been used to investigate a wide variety of rock slope failure mechanisms including those ranging from simple planar mechanisms (Costa et al., 1999), to complex deep-seated toppling instability (Board et al., 1996; Benko and Stead, 1999; Hutchison et al., 2000; Nichol et al., 2002) and buckling (Stead and Eberhardt, 1997). These authors illustrate the need to consider both intact rock and joint-controlled displacements in the analysis of complex rock slope instabilities. The use of 3-D distinct element techniques has been more limited to date. Adachi et al. (1991) in an early application of the 3DEC code (Itasca, 2004) analysed toppling slopes along a highway in Japan. Zhu et al. (1996) and Valdivia and Lorig (2000) both present applications of 3DEC in the analysis of open pit mine slope with reinforcement. Corkum and Martin (2002) provide a recent successful and interesting application of the 3DEC code to investigate the stabilising effect of a toe berm on slope deformations at the 731 block adjacent to the Revelstoke Dam, British Columbia, Canada. This stabilised rock slide provided an opportunity to constrain simulated 3DEC movements against monitored slope displacements.

3.2. Further developments in conventional numerical techniques

Recent developments in continuum and discontinuum numerical methods allow simulation of complex rock slope failure processes, including:

- hydro-mechanical coupling;
- dynamic analysis;
- advanced and user-defined constitutive criteria (strain softening, creep, damage, etc.);
- support interaction;
- improved integration with empirical rock mass classification schemes.

Continuum-based analyses of these sort applied to rock slopes are less frequently encountered. One example is the incorporation of groundwater effects by Agliardi et al. (2001) in a finite-difference analysis of sackung-type deep-seated rock slope deformation kinematics in the Rhaetian Alps, Italy. Using a discontinuum approach, Bonzanigo et al. (2001) demonstrate the successful application of coupled hydro-mechanical distinct-element modelling in assessing the effectiveness of deep drainage in stabilizing a massive creeping rock slide in the southern Swiss Alps. Eberhardt and Stead (1998) provide an example of dynamic distinct-element analysis of a natural rock slope in Western Canada (Fig. 4). In this case, the dynamic input was introduced along the bottom boundary of the model as a harmonic function (sine wave) of a specified amplitude (i.e., stress), frequency and time period (Fig. 4a). The model shows an initially stable slope subjected to an earthquake, resulting in yielding and tensile failure of intact rock at the slope's toe (Fig. 4b). Toe failures of this type may then lead to planar failure of the upper slope (Fig. 4c). In addition to material yielding, the oscillating nature of the dynamic load results in rotational type movements, which in turn could induce falls of loose rock. Bhasin and Kaynia (2004) illustrate the application of a dynamic distinct-element analysis of a 700 m high Norwegian rock slope to estimate potential rock volumes associated with catastrophic rock failure.

Strain-softening criterion may be used to simulate internal strength degradation and damage contributing to massive rock slope failure. Eberhardt et al. (2002) show the application of numerical methods in simulating the development of a Prandtl yield zone within a deforming crystalline rock slope. Fig. 5 shows that a zone of yield due to shear damage develops near the base of the eventual slide surface and transforms into a tensile failure zone as straining occurs. This zone of tensile damage continues upwards through the intact slide mass dividing the rockslide into two distinct blocks, approximating the contact between the first and second failure events of a massive rock slide in southern Switzerland. The tensile nature of the failure indicated in this model also supports the strain-dependent frictional strength development concept introduced by Hajiabdolmajid and Kaiser (2002), who applied it to an analysis of the Frank slide. Such studies suggest that the episodic nature of massive rock slides can be explained through brittle tensile fracturing and time-

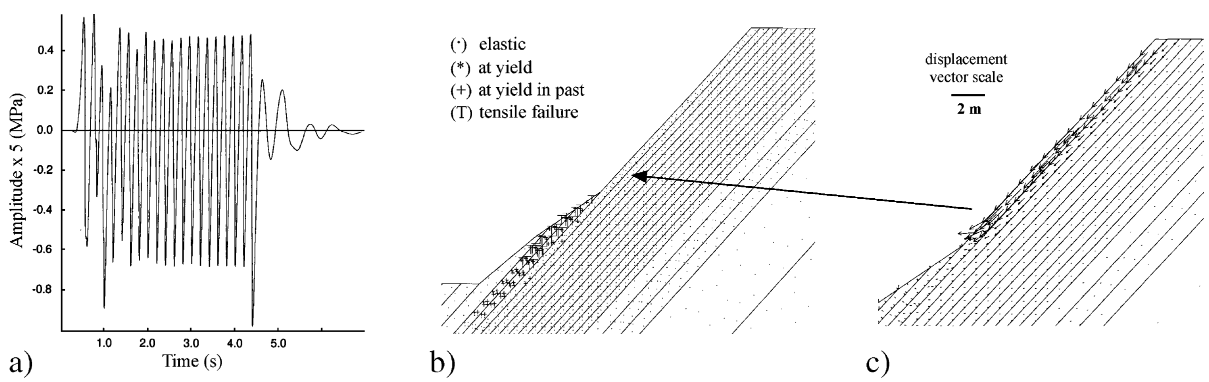

Fig. 4. Dynamic modelling of a natural rock slope using the distinct-element method, showing: (a) modelled seismic wave; (b) subsequent yielding of the slope's toe material; and (c) resulting displacement vectors indicating planar sliding failure (after Eberhardt and Stead, 1998).

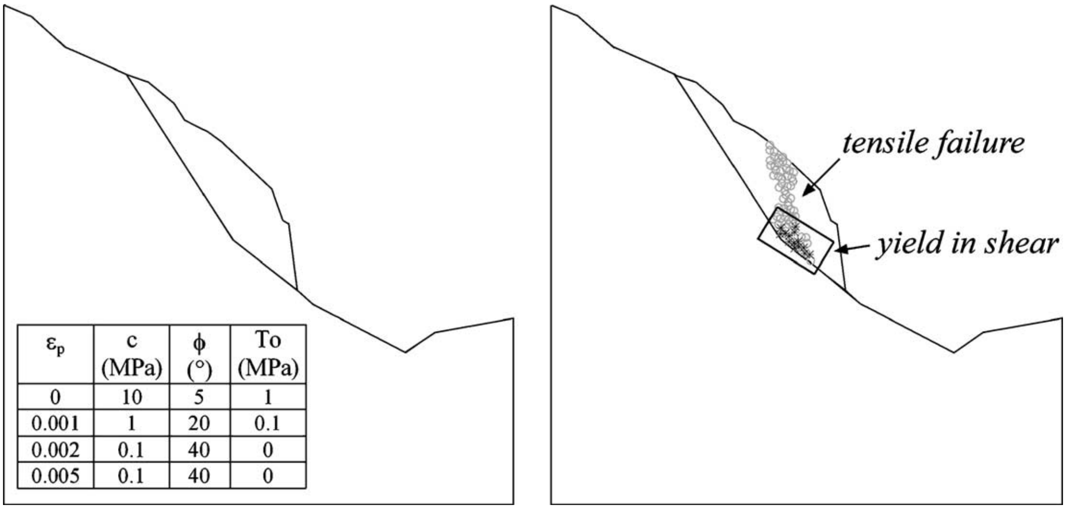

Fig. 5. Distinct-element strain-softening model showing development of Prandtl-type yield zone at base of slide surface and propagation of tensile damage upwards through intact slide mass.

dependent strength degradation. Jin et al. (2003) present an extension of the distinct element method to simulate creep behaviour of jointed rock slopes due to unloading. Viscous deformation of rock joints was modelled using the Kelvin model. Application to the Three Gorges shiplock in China showed close agreement with field instrumentation data.

3.3. Advanced application of numerical techniques to complex rock slope deformation and failure

Continuum and discontinuum codes as described above often fail to realistically simulate the progressive failure of rock slopes, particularly the dynamics of kinematic release accompanying complex internal distortion, dilation and fracture. The importance of developing kinematic release through fracturing in selected mechanisms is a key issue in rock slope analysis that is not addressed by conventional numerical models. Stead et al. (2004) emphasize the need to consider rock slope failures using the principles of fracture mechanics with appropriate consideration of damage, energy, fatigue and time dependency.

Early work to address the influence of fracturing in rock engineering was undertaken by Williams et al. (1985) and Mustoe (1989) who developed a discrete-element code that incorporated fracturing through a Mohr–Coulomb criterion with a tension cut-off. Through their algorithms, an element could fracture through its centroid or along an element edge. Cundall and Strack (1979) adopted a variation of the discrete-element method to simulate particulate material behaviour. This led to the development of the Particle Flow Code, PFC (Itasca, 2004), in which clusters of particles can be bonded together to form joint-bounded blocks. This code is capable of simulating fracture of the intact rock blocks through the stress-induced breaking of bonds between the particles (Table 3). This is a significant development as it allows the influence of internal slope deformation to be investigated both due to yield and intact rock fracture of jointed rock. Wang et al. (2003) demonstrate the application of PFC in the analysis of heavily jointed rock slopes (Fig. 6).

It is evident from both rock slope observations and intensive fragmentation occurring during the failure process that intact brittle fracture mechanisms are an important component of many failures. A major shortcoming of the previously described methodologies is that they only imitate intact rock fracture; they do not follow basic principles related to brittle fracture mechanics. Hybrid finite-/discrete-element codes (Table 3) combine the advantages of both continuum and discontinuum techniques to model intact behaviour, interactions along existing discontinuities and, when incorporating fracture mechanics principles, the initiation and development of new fractures (i.e., the transition from a continuum to a discontinuum). ELFEN (Rockfield, 2004), one example of a hybrid finite-/discrete-element code, enables the modelling of brittle fracture initiation and propagation through adaptive remeshing techniques coupled with contact search algorithms. The program uses a finite-element mesh to represent the intact joint bounded blocks and discrete elements to model joint behaviour. The simulation of fracturing, damage and associated softening within a rock slope prior to and during failure is accomplished

Table 3
Advanced/hybrid numerical methods of analysis

Analysis method	Critical input parameters	Advantages	Limitations
Particle flow code (e.g., PFC, ELFEN)	Problem geometry, particle shape, size and size distribution; particle density, bond stiffness and strength (normal and shear); bonding type and tightness of packing configuration.	Ideal for simulating particle flow, but can also simulate behaviour of solid material (e.g., intact or jointed rock) through bonded assemblage of particles, most notably the fracturing and disintegration of the bonded assemblage. Dynamic analysis possible, as well as 2-D and 3-D simulations. Some programs use imbedded language (e.g., FISH) to allow user to define own functions and subroutines.	Input parameters are based on micromechanical properties, requiring calibration through simulation of laboratory testing configurations (i.e., to correlate particle bonding properties to Young's modulus, compressive strength, etc.). Particles are rigid and often cylindrical (2-D) or spherical (3-D). Simulation of brittle fracture not based on physical laws/principles of fracture mechanics.
Hybrid finite-/discrete-element codes (e.g., ELFEN)	Combination of input parameters listed in Table 2 for both continuum and discontinuum stand alone models (e.g., elastic, elasto-plastic, etc., for continuum; stiffness, shear strength, etc., for discontinuities); damping factors; tensile strength and fracture energy release rate for fracture simulation.	Combines advantages of both continuum and discontinuum methods. Coupled finite-/discrete-element models able to simulate intact fracture propagation and fragmentation of jointed and bedded media. Incorporates efficient automatic adaptive re-meshing routines. Dynamic, 2-D and 3-D analyses possible using wide variety of constitutive models (plastic, visco-plastic, etc.).	Complex problems require high memory capacity. Comparatively little practical experience in use. Requires ongoing calibration and constraints. Yet to be coupled with groundwater.

using a fracture energy approach (Fig. 7). The fracture energy release rate in tension is assumed to control only the post-peak process after the tensile strength limit has been reached (Munjiza et al., 1995). Various constitutive models are available within ELFEN although those used in conjunction with fracture generation are the elasto-plastic Rankine (with an option for a rotating crack model) and Mohr–Coulomb models (Yu, 1999; Klerck, 2000).

Step-path failure has been undertaken in the past using a combination of limit equilibrium and fracture mechanics theory. The use of a hybrid finite-/discrete-element code with fracture propagation, however, is particularly well suited to the simulation of step-path geometries. Fig. 8 illustrates an ELFEN model of a translational failure requiring intact fracture between two joint sets to allow kinematic release. Excess shearing stresses along a discontinuity result in the propa-

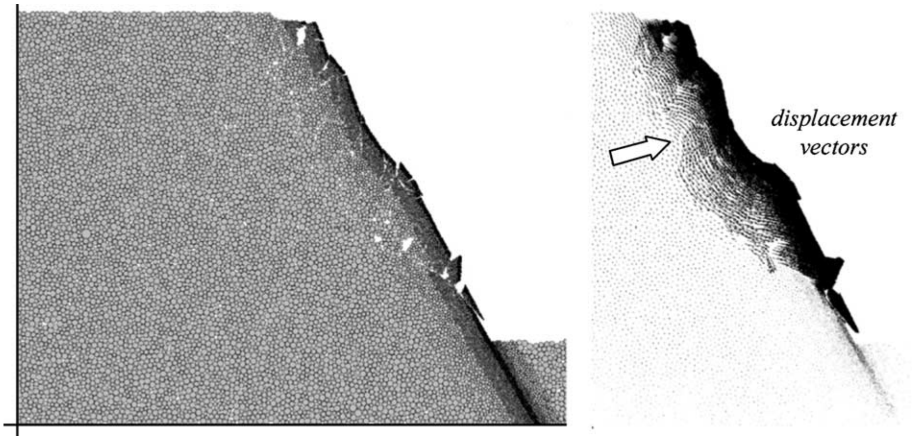

Fig. 6. Simulation of a rock slope failure through an assemblage of bonded particles using a particle flow code (after Wang et al., 2003).

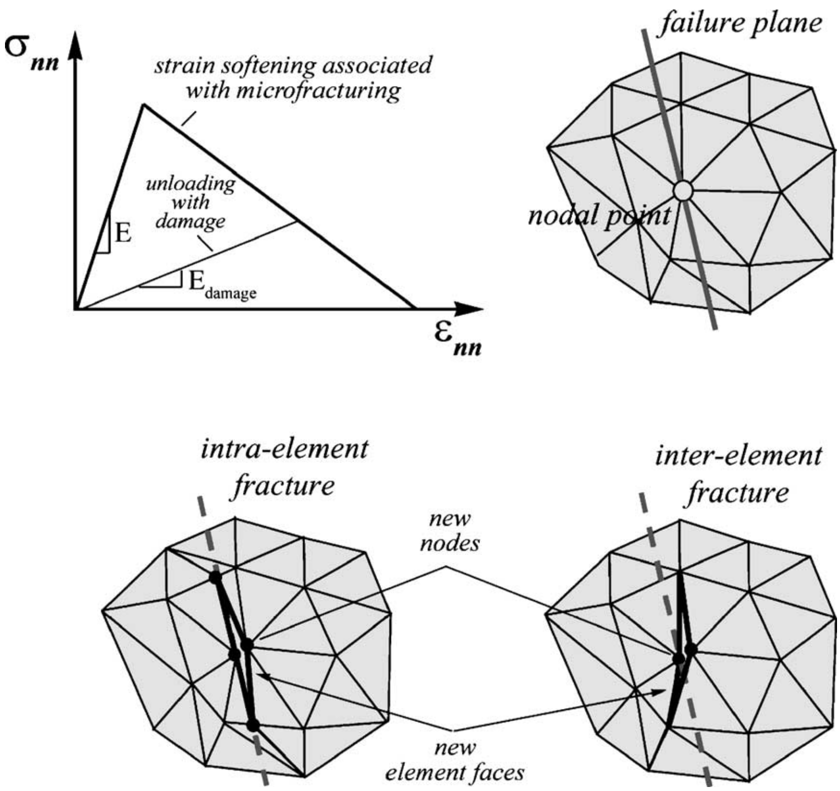

Fig. 7. Stress–strain response to damage within a finite element, and crack insertion procedure used in ELFEN showing crack development either through an element or along an element boundary (modified after Yu, 1999).

gation of orthogonal tensile fractures until eventually cross-over fractures provide a continuous, but stepped, failure surface and overall failure occurs. Preliminary results indicate that the critical factors controlling the step-path generation include a combination of joint persistence, spacing, and frictional strength in addition to the rock mass tensile strength. The location of the potential translational joints within the slope with respect to geometrically induced compressive and tensile stress concentrations may play a key role in failure initiation. Stead et al. (2004) suggest that step-path processes may be active from the micro to macro level depending on time-dependent aspects of deformation. Furthermore, step-path or cross-over features are not restricted to basal shear surface development but are also important in the development of lateral and rear release surfaces. Although a 3-D global step-path failure has yet to be considered by practitioners, the process is undoubtedly extremely important in localized failure throughout massive deforming rock slope masses. Time-dependent degradation of the cohesive and tensional strength of both discontinuities and the intact rock mass are of significant importance.

Bi-planar rock slope failures are common in a variety of tectonic and engineering environments. Early Level I analyses of this failure mechanism were pro-

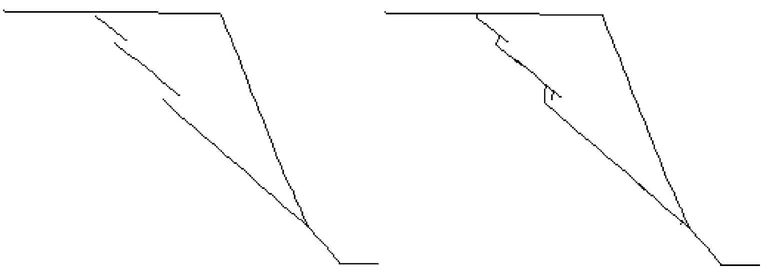

Fig. 8. ELFEN simulation of a stepped-path failure through a 100 m high rock slope.

posed using an active-passive wedge approach (Mencl, 1966; Kvapil and Clews, 1979). An interface was assumed between an upper active driving block and a lower passive or resisting block in order to allow a solution that satisfied the kinematics of the failure geometry. This form of analysis was initially applied to embankment dams with the upper wedge failure surface being the sloping core of a dam and the lower passive failure surface being the foundation of the dam (Seed and Sultan, 1967). The active–passive wedge approach or non-circular methods of slices have been applied to rock slope geometries where the upper failure surface may for example be a high angle fault and the lower failure surface along weak bedding planes or intra-formational shears. Stead and Eberhardt (1997) illustrated the application of discontinuum modelling techniques to the analysis of active–passive failures in surface coal mine footwalls. Major rockslides, such as the Vaiont slide (Mencl, 1966), have exhibited a similar active–passive wedge or chair-shaped failure surface geometry in which internal yielding and fracturing within the rock slope must occur for kinematic release. Evidence of this internal distortion and fracturing can be observed as surface faults and graben features within the post failure topography. Fig. 9 illustrates a hybrid finite-/discrete-element model of a bi-planar failure geometry showing the stages in the development of brittle internal fracturing that accompanies rock slope failure and kinematic release. The development of tensile factures above and below the intersection of the upper and lower failure surfaces is evident. These are followed at a later stage by the development of a wedge interface fracture. Soe Moe et al. (2003) have proposed a modification to the active–passive wedge analysis in which the interface is inserted in the upper wedge rather than at the upper/lower bi-planar surface intersection; this appears to be in closer agreement with the hybrid models than the conventional approach. The authors suggest that only hybrid models containing elements of a discontinuum and intact fracture can realistically simulate the complex processes that occur in such massive bi-planar rock slopes failures or indeed wherever the failure surface has rapid changes in curvature.

Conventional, Level II continuum and discontinuum models although able to simulate certain aspects of progressive shear plane development are not suited to the modelling of progressive failure though brittle fracture initiation and propagation. Fig. 10 shows the use of the ELFEN code in modelling the 1991 Randa rockslide, Switzerland (Eberhardt et al., 2004b). Adoption of a Mohr–Coulomb constitutive criterion with a Rankine crack tensile cutoff closely reproduces the recorded dimensions of the 1991 Randa slide. The

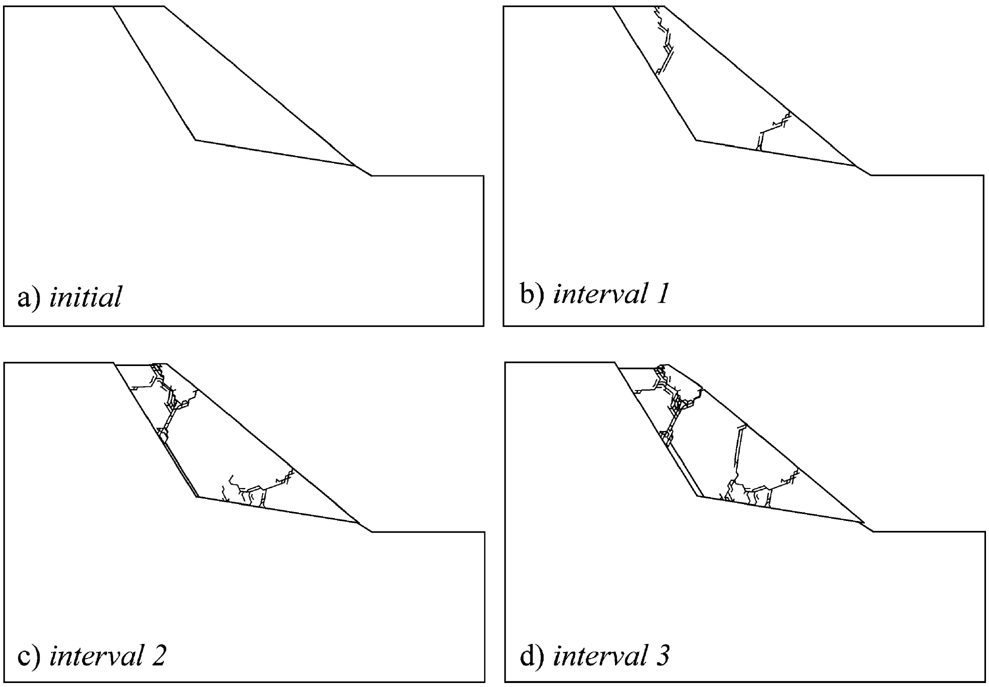

Fig. 9. ELFEN simulation of a 50 m high rock slope with a bi-planar (active–passive) failure surface, showing: (a) the initial problem geometry; and (b–d) three stages of fracture development leading to kinematic release and failure.

progressive development of the failure surface results by the formation of numerous sub-vertical tension/extension cracks (i.e., normal to the direction of down slope strains). These fractures align to enable the formation of a shear plane sub-perpendicular to the tensile fractures. The ELFEN model is also seen to closely reproduce the two observed stages in the rock slope failure as well as a present-day area of movement beyond the failure crest (Fig. 10; Eberhardt et al., 2004b).

Fig. 10 also shows the extension strains for the 1991 Randa rockslide analysis based on a finite-element continuum analysis with a Mohr–Coulomb elasto-plastic yield model, but without the hybrid discrete-element coupling enabling fracture initiation. The model shows that large extensional strains exceeding 0.002 have developed along a path that roughly coincides with the 1991 Randa rockslide failure surface. These strains are well in excess of those reported by Stacey (1981) as being the critical levels for brittle fracture initiation and propagation. A preliminary analysis undertaken using a hybrid discrete-element code with Rankine rotating crack fracture model also indicates the importance of extensile strains during the failure of a simple planar block in a 50 m high slope (Fig. 11). The stages of sub-vertical fracturing associated with movement of the block are clearly shown.

By using hybrid finite-/discrete-element techniques with brittle fracture propagation, the authors feel that the first steps may be taken in what they term a "total slope failure analysis". This is in marked contrast to traditional rock engineering approaches where the analysis of rock slopes has emphasized either the initiation mechanism or the transport/deposition stage. Fig. 12, modified after Couture et al. (1998), shows the varied stages of the total slope failure analysis with the appropriate Level I, II and III analysis codes. Conventional Level I approaches have been used to characterize the

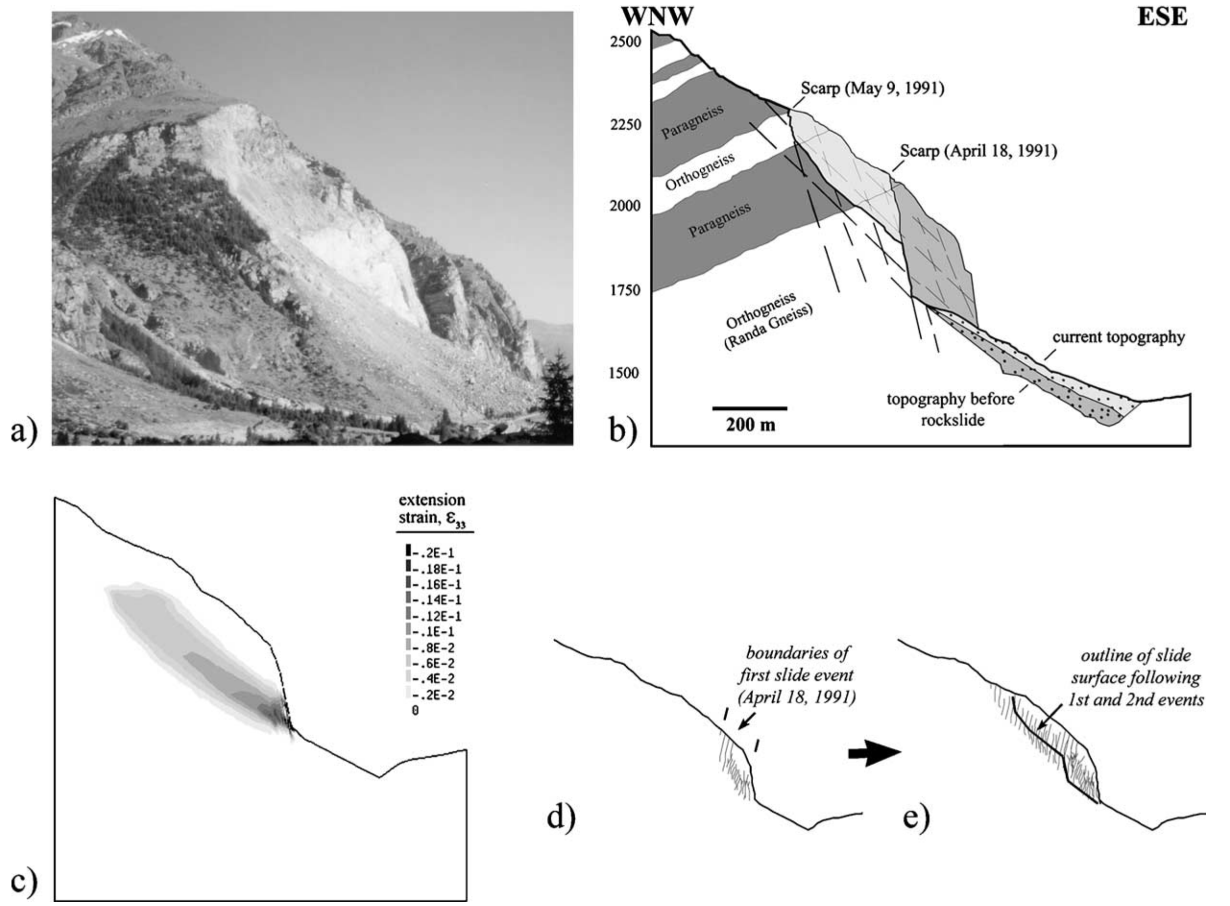

Fig. 10. Analysis of the 1991 Randa rockslide in Switzerland, showing: (a–b) photo and cross-section of the Randa rockslide; (c) modelled extensional strains based on the pre-failure geometry; (d–e) progressive failure analysis using ELFEN in which the staged nature of the actual failure is replicated through extensional strain-driven brittle fracturing (after Eberhardt et al., 2004b).

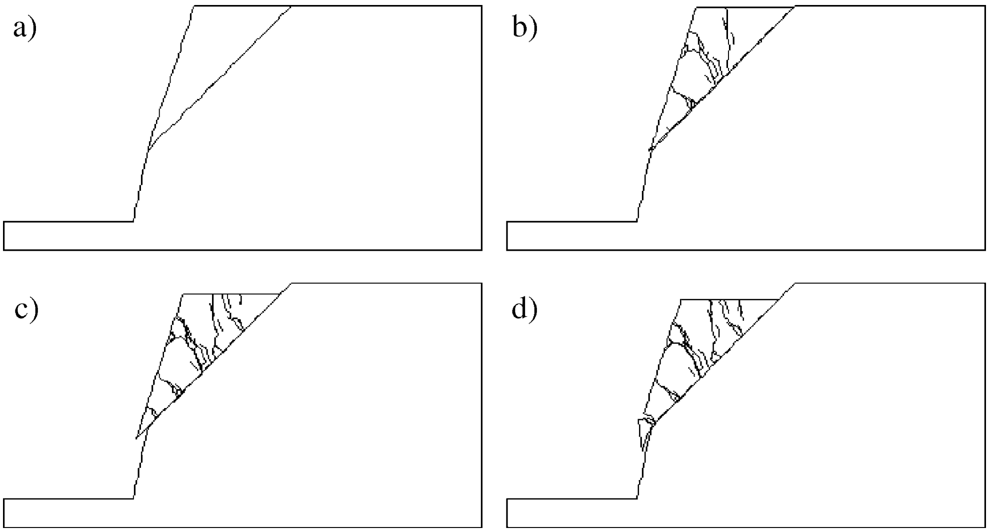

Fig. 11. ELFEN simulation of a 50 m high weak rock slope, showing: (a) the initial problem geometry; and (b–d) three stages of intact rock fracturing driven by extensional strains during planar failure.

hazard presented by the failure initiation using 2-D/3-D deterministic limit equilibrium techniques. The authors suggest, however, that if the true risk is to be ascertained then the characteristics of the deformation as a precursor to failure and the post failure movement must be linked to the initiation analysis. This requires a detailed rock mass characterization in order to allow a combination of continuum, discontinuum and hybrid approaches. The transportation and comminution stage follows the initiation of failure. Comminution may be visualized as occurring both during initial rock failure and during the gravity driven rock debris transport. Both particle flow codes (Calvetti et al., 2000) and hybrid finite-discrete element codes with fracture propagation (Stead and Coggan, in press) have been shown to be suitable for the modelling of rock debris transport. Work by Locat et al. (2003) shows that the study of the comminution process during rock slope is extremely informative with regard to associated energy requirements. The present authors suggest that Level III codes, in conjunction with existing rheological codes (e.g., DAN — Hungr, 1995) offer immense potential in furthering our knowledge of the mechanisms and processes at work during the breakdown of the slope mass continuum through failure initiation, transportation and deposition, that is, the

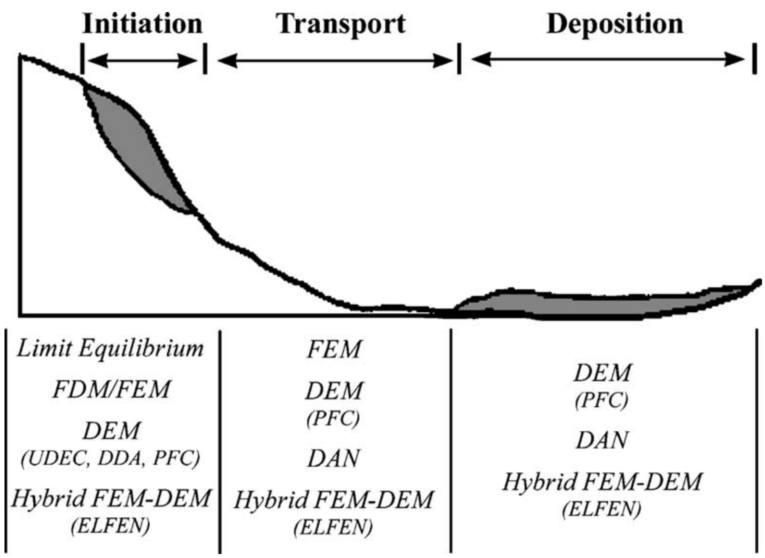

Fig. 12. Varied stages of a "total slope failure analysis" with the appropriate Level I, II and III analysis codes (modified after Couture et al., 1998).

"total slope failure". This is essential if we are to develop robust recommendations with regard to rockslide runouts and the associated temporal and spatial risks due to massive rock slope failure. Fig. 13 shows an ELFEN analysis of the 1881 Elm rockslide in the Swiss Alps, which resulted in the loss of 115 lives. A geological section through the slide (after Heim, 1932) shows that the rockslide occurred within highly deformed slate possessing a closely spaced cleavage that dipped into the slope at 30°. The failure was directly linked to quarrying at the slope toe with failure developing along a complex surface approximately orthogonal to cleavage. Heim (1932) refers to the failure as an example of a "rubble stream" phenomenon in rock avalanche run outs. In this preliminary ELFEN model (Fig. 13), the failure surface according to Heim is assumed. Selected stages in fracturing due to undercutting of the slope are then simulated leading to the failure and subsequent run out of a dry fragmented rock mass.

A second example of a total slope failure analysis is illustrated in Fig. 14. This failure again occurred in highly foliated rocks dipping into the slope, as described by Coggan and Pine (1996). These authors presented a Level II distinct-element analysis of the rock slope failure at the Delabole Slate quarry in Devon, UK, incorporating pore water pressures. They also used slope deformation measurements (electronic distance meters and tension crack monitoring) to constrain the simulated failure mechanism. The pre- and post-failure geometry of the failure is shown in Fig. 14. Coggan and Pine (1996) described the complex failure mechanism as involving the downward movement of chisel shaped blocks, which in turn promoted a combination of toppling and sliding of the lower rock slope blocks beneath them. Selected stages in the hybrid finite-/discrete-element simulation of the Delabole rock slope failure are presented in Fig. 14, which illustrate the fracturing within the middle of the rock slope associated with initiation of toppling and sliding movements followed by fragmentation of the slatey rock mass during post failure transport.

4. Changing data requirements for advanced rock slope analysis

The advent of increasingly sophisticated software necessitates an appropriate change with respect to data collection methodologies and interpretations. The authors suggest that mapping/investigation efforts (or intensity) should match the complexity of the analysis techniques to be utilized. Simple kinematic and limit equilibrium analyses require a Level I mapping intensity level. In these cases, the site investigation should include standard discontinuity survey mapping. The authors emphasize that the recording of orientation alone is insufficient even for preliminary kinematic analysis. It is essential to record geometric data on joint sets (e.g., persistence and spacing) and shear strength (roughness, infill, water, etc.) in order that kinematic analyses can be used in realistic initial hazard assessments. Current kinematic analysis codes such as DIPS (Rocscience, 2004) offer comprehensive discon-

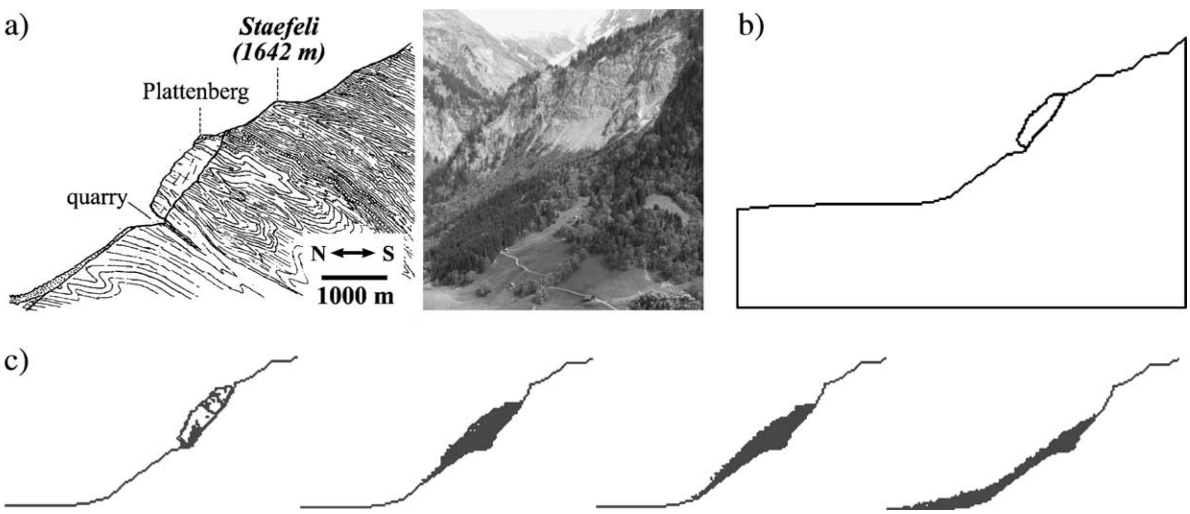

Fig. 13. Preliminary ELFEN simulation of "total slope failure analysis" of the 1881 Elm rockslide in Switzerland, showing: (a) cross-section and present-day photo of the slide scarp; (b) the initial problem geometry; and (c) four stages of failure, passing from failure development through to runout.

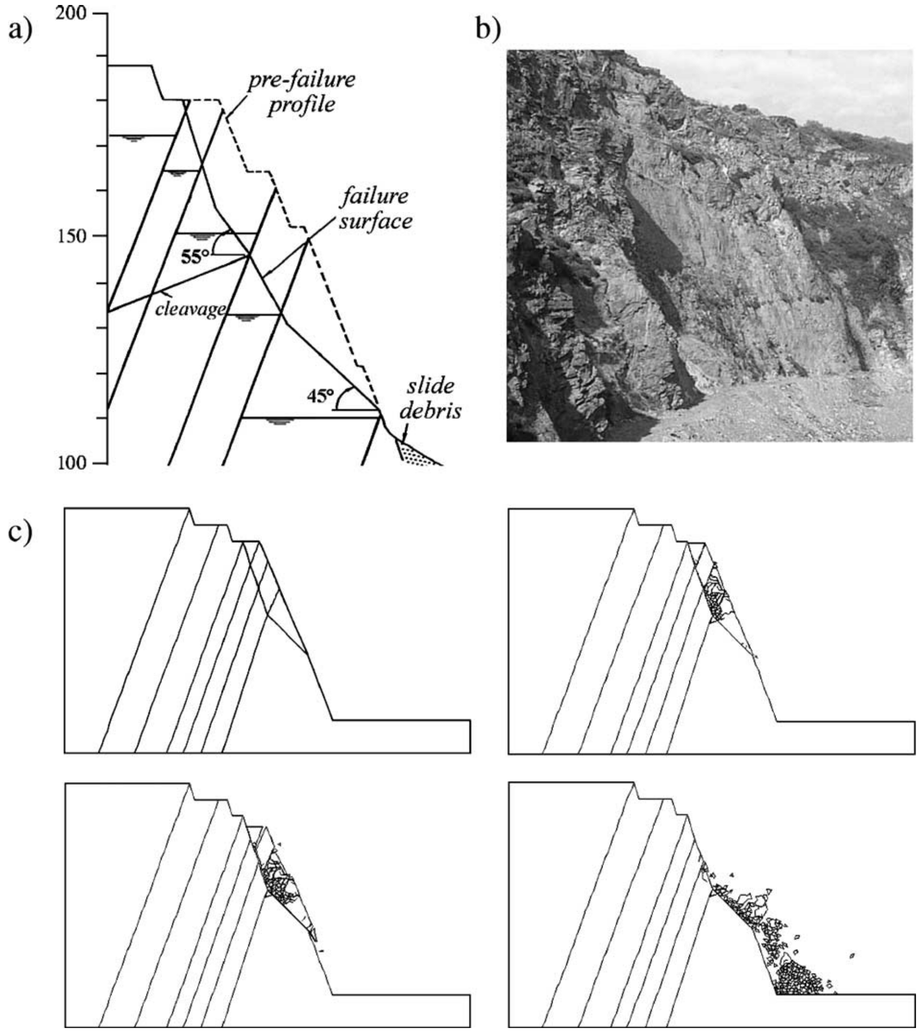

Fig. 14. ELFEN "total slope failure analysis" simulation of the 1967 Delabole slate quarry failure in the U.K., showing: (a–b) cross-section and photo of the rockslide scarp; (c) the initial problem geometry and three stages of failure, passing from failure development through to runout.

tinuity data analysis capabilities for the interpretation of joint properties.

Level II field mapping intensity should involve sufficient data to allow for the use of numerical methods. If continuum codes are to be used to perform a rock slope analysis, then not only should field observations justify their use but data should be collected in order to allow characterization of the rock mass, for example according to the Geological Strength Index, GSI (Hoek et al., 1995; Cai et al., 2004). Such systems can combine field and laboratory data to assess scaled rock mass properties for numerical model input (e.g., using for codes such as RocLab; Rocscience, 2004). If discontinuum codes are being utilized, then it is essential that a rigorous characterization of the rock mass is undertaken, particularly emphasising the discontinuity geometry with respect to the rock slope. Block size variation, joint persistence and joint spacing should be ascertained in order to constrain the discontinuum model input. This requires a further level of sophistication rarely carried out in most field mapping campaigns.

Hybrid codes with fracture propagation require the highest field mapping intensity (Level III), if they are to be used to characterize the "total rock slope failure". Specifically, additional data is required to constrain the degree of intact fracturing either associated with failure in a back-analysis scenario or the probability for release in a hazard assessment investigation. Field data should be gathered throughout the initiation, transportation and deposition zones in order to constrain the degree of comminution associated with failure, and similar controls on debris characteristics (e.g., block size, jointing, tensile strength). It should be stressed that the use of codes involving either fracture mechanics or particle flow theory in rock slope analysis is still very much at the research stage. It is essential that these codes be constrained through analyses in which they are used in conjunction with conventional continuum/discontinuum approaches in order to establish an "experience database" for varied failure mechanisms.

If rock slope analyses are to fully utilize recent (and continuing) developments in numerical methods it is essential to characterize the rock mass both spatial and temporally. Geostatistical approaches to rock mass variation and their application toward parameter uncertainty have been the subject of limited research to date (Pascoe et al., 1998). It is essential that if risk assessments are to be undertaken that the spatial variation of properties within a rock mass is adequately treated. The authors suggest that the increased use of geophysical techniques (e.g., radar, active and passive seismic, etc.) in the characterization of rock slopes is essential. Dynamic changes in rock mass characteristics/properties during rock slope deformation should also be undertaken to assess associated stress-induced brittle damage. Work carried out at the Randa In Situ Rockslide Laboratory (Willenberg et al., 2004; Eberhardt et al., 2004a) has moved in this direction, providing an excellent example of the multi-disciplinary studies in rock slope characterization required to properly constrain advanced numerical modelling and improve hazard assessment. The increasing availability of remote sensing imagery should also be maximized in application to rock slide hazard assessment and model constraint. Developments in LiDAR (Light Detection and Ranging) and InSAR (Interferometric Synthetic Aperture Radar) are particularly relevant in monitoring both deformation associated with the initiation zone and characteristics of the rock debris within the transportation and deposition zones.

The requirements for data presentation and interpretation are increasingly important as the sophistication of both rock slope characterization and modelling increases. Improved linkages between Geographic Information Systems (GIS) and numerical modelling techniques are an important research area if modelling is to be adequately constrained. Current hybrid finite-/discrete-element codes with fracture propagation offer Virtual Reality Imaging (VRI) of rock slope deformation and failure processes. This has the potential to allow unique interpretations of 3-D failure mechanisms. These capabilities, in associated with currently available virtual reality caves or domes, have yet to be utilized in rock slope stability and hazard assessment, despite successful applications in the petroleum and mining industries (Henning et al., 2002).

5. Conclusions

The authors have attempted to illustrate the wide range of tools available to the engineer and geoscientist for the analysis of rock slopes with particular emphasis on emerging powerful numerical modelling codes that allow realistic simulation of rock slope failure. The importance of brittle behaviour and internal deformation within deforming rock slopes due to a combination of yield and fracturing has been emphasized. Developments in discrete-element and hybrid finite-/discrete-element codes have demonstrated the significant potential in the analysis of total slope failure analysis failure, from initiation through transportation/comminution to deposition. Further progress in the application of these techniques requires additional constraints with improved rock slope characterization both in terms of input properties and deformation instrumentation.

Several key issues require attention if rock slope failure mechanisms are to be understood and hazard assessment improved. These include: further research on time dependent or progressive rock slope failure and the role of cumulative internal rock slope damage; the characterization of the influence of groundwater pore pressures and flow on the deformation of massive rock slopes represents, an important "missing-link" if numerical models of rock slopes are to be adequately constrained (the assumption of tenuous water-tables in fractured rock slopes is an area of considerable model uncertainty); and scale effects, both in relation to rock mass strength properties and the influence of groundwater.

Advanced numerical models have, in recent years, received a much wider acceptance both in research and in routine engineering practice. It is essential that the rapid development in sophisticated rock slope analysis codes be balanced by a concomitant increase in the quantity and quality of engineering geological field data collected. Although it is still often possible to use these codes to supplement our understanding of failure mechanisms in data limited situations, good geological and geotechnical data is a prerequisite for most numerical analyses. If the use of current rock slope numerical models is to be maximised the engineer and geologist must work in tandem to ensure that the appropriate field, laboratory and slope monitoring data are collected during engineering rock slope investigations.

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