Pergamon

Engineering Fracture Mechanics Vol. 59, No. 2, pp. 103–115, 1998

Printed in Great Britain
PII: S0013-7944(97)00095-7
0013-7944/98 $19.00 + 0.00

THE EFFECT OF NEIGHBOURING CRACKS ON ELLIPTICAL CRACK INITIATION AND PROPAGATION IN UNIAXIAL AND TRIAXIAL STRESS FIELDS

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Abstract—The study of brittle fracture and its relationship to strength is a fundamental part of rock mechanics and a number of other engineering disciplines. The initiation, propagation and coalescence of these fractures results in the degradation of material strength and eventually leads to failure. Results show that the interaction of stresses between neighbouring tips of elliptical cracks aligned parallel to the direction of loading can have a significant influence on one another in terms of crack initiation and propagation. Depending on the distance separating each crack and the loading conditions imposed on the medium surrounding them, the addition of neighbouring cracks can act to either suppress or promote crack growth. © 1998 Elsevier Science Ltd

Keywords—crack initiation, elliptical cracks, uniaxial loading, triaxial loading, stress shadows, crack interaction.

1. INTRODUCTION

THE EFFECTS of stress-induced brittle fracturing on the progressive degradation of intact rock strength is a major concern in assessing the degree of damage that occurs around an underground excavation and, hence, its long term stability. The analysis of brittle fracture in solid materials loaded in compression has been based largely on the concept of a crack acting as a stress concentrator through which the fracture process is initiated. Griffith [1] postulated that in the case of a linear elastic material, brittle fracture is initiated through tensile stress concentrations at the tips of small, thin cracks randomly distributed within an otherwise isotropic material. These cracks were used by Griffith to explain the discrepancy between the observed tensile strength of materials and the theoretical tensile strength based on the concept of molecular cohesion.

The propagation of cracks in compression has been the subject of numerous studies involving a variety of materials including glass [2, 3], hard plastics [4, 5], plaster [6], ice [7] and rock [8–10]. These studies have shown that crack growth occurs in the direction of the major principal stress (σ_1). For cracks not aligned with σ_1, the cracks grow along a curved path to align themselves with σ_1. In either case, it is the initiation, propagation and coalescence of these cracks that results in the degradation of material strength and, eventually, failure.

Analysis of crack behaviour in a compressive stress field has progressed from the simple case of a single crack, to en echelon arrays of cracks, to multiple random arrays of cracks. Few studies, however, have examined how these cracks may interact in either promoting or inhibiting the growth of neighbouring cracks. The work presented in this paper uses the boundary element method to model the mutual influences neighbouring cracks have on crack initiation and propagation.

2. NUMERICAL MODELLING OF CRACK INTERACTION

Numerical modelling to simulate crack initiation and propagation in rock has been used by a number of researchers. Ingraffea [11], Kemeny and Cook [12] and Dyskin *et al.* [13] focused on the use of linear elastic fracture mechanics (LEFM) to model crack stability and propagation trajectories by incorporating a stress intensity factor into the numerical formulation to dictate whether crack propagation would occur or not. Additional studies have concentrated on the concept of a process zone at the crack tip to model non-linear effects[14, 15]. To a lesser extent the effects of crack interaction have been studied. Studies have been conducted on the modelling of crack coalescence[16, 17], however, little work has been done on how the stresses surrounding these coalescing cracks interact in terms of promoting or inhibiting crack propagation. A study was therefore conducted in which a series of two-dimensional boundary element models was used to examine these effects. The results are reported in this paper.

2.1. *Methodology*

Crack initiation and propagation were modelled following a similar process to that outlined by Ingraffea *et al.*[18]. The sequence of events begins by computing the stress intensity factors for a given crack length and determining crack stability under a prespecified load. If the crack is unstable, the crack length is increased, and if the crack is stable, the load is increased. This process is repeated, thereby producing a relationship between stable crack length and applied stress. One of the limitations in this methodology is that the problem geometry requires remeshing for each crack length increment and reanalysis for each load increment. In cases where a large number of model runs are required, the boundary element method can be an efficient tool since only the boundary of the problem geometry requires discretization. To model the effects of crack interaction, Dyskin *et al.*[19] note that a high number of crack models must be run, thus making the use of the boundary element method more attractive than the finite element method. For these reasons, a boundary element approach was chosen, as it allowed for the quick analysis of numerous crack models.

The nature of the boundary element approach chosen, however, required a number of assumptions and modifications to be made to the LEFM approach summarized by Ingraffea

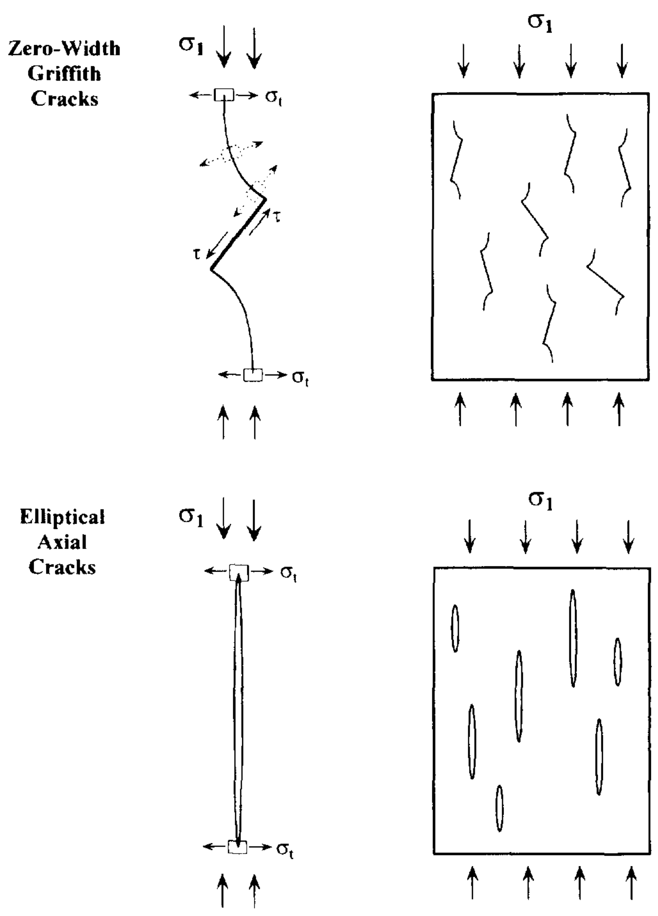

Fig. 1. Examples of "zero-width Griffith" and "elliptical axial" cracks

et al. [18]. For example, crack geometries were modelled as axial cracks represented by ellipses of finite width. Most LEFM approaches, however, assume a zero-width Griffith type crack (Fig. 1). In cases where the crack is aligned parallel to the principal stress direction, the zero-width crack is unaffected by the applied compressive stress field and, therefore, cannot be propagated. To propagate, the zero-width crack must be inclined to the principal stress direction. Dzik and Lajtai [20] note that, theoretically, an axial crack can only be propagated in the axial direction by removing the zero-width simplification or by allowing a finite deformation in the lateral direction.

Another modification to Ingraffea *et al.*'s [18] approach was to replace the stress intensity formulation to model crack tip failure with an empirical fracture criterion. The Uniaxial Strength Ratio (USR) failure criterion calculates a safety factor in terms of the ratio of material strength to the induced stresses surrounding the crack tip. Described in detail by Dzik and Lajtai [20], the USR criterion is derived through the Rocker function [21, 22] which describes material strength as a function of its compressive, C_o, and tensile, T_o, strength:

$$\sigma_{1f} = C_o\left(1 - \frac{\sigma_3}{T_o}\right)^R \tag{1}$$

where R is a fitting constant with the most likely value of 0.5. The Rocker function represents an equivalent strength curve passing through the stress point (σ_3, σ_1) as shown in Fig. 2. Assuming an initial crack length, crack initiation is defined as the stress level required to produce a factor of safety, SF, below 1.0 (i.e. tensile failure of the crack tip material):

$$\mathrm{SF} = \frac{\sigma_{1f}}{\sigma_1}. \tag{2}$$

The use of the boundary element formulation also required the incorporation of an averaging distance to properly portray the critical stress concentrations required at the crack tip to initiate crack propagation. Since elements are located only along the periphery of the crack ellipse, high stress concentrations are calculated that do not take into account the redistribution of stresses around the crack tip due to material yield. In other words, the largest tensile stress concentration must coincide with the crack tip boundary, with a steep stress gradient away from the crack tip. A very small averaging radius is required to accurately represent the redistribution of stresses in this region. This procedure is somewhat analogous to incorporating a process zone into the analysis. Dzik and Lajtai [20] found that in order to obtain results that match relevant experimental data, the averaging distance is approximately two to three times the minor axis width of the elliptical crack.

Modelling proceeded such that, if fracture was indicated near the crack tip, the crack was extended parallel to the applied load. This process of crack extension was continued until the crack length became stable. By following the routine of incrementally increasing stress levels and crack length until stability was reached, a stable crack length curve can be constructed.

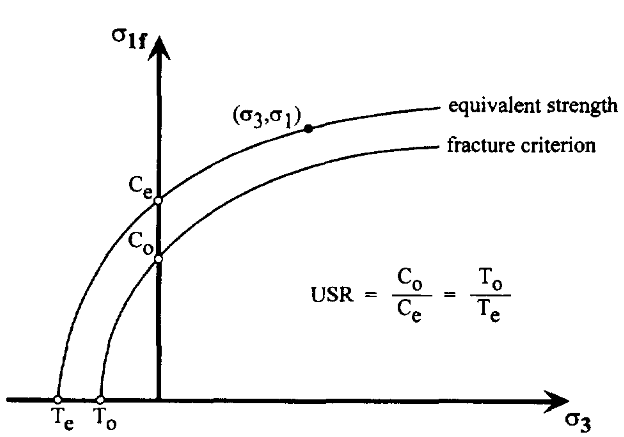

Fig. 2. USR fracture criterion (after ref. [20]).

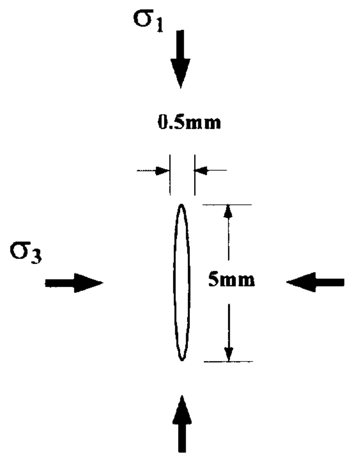

Fig. 3. Problem geometry and material properties used for single crack models.

Using the single crack geometry shown in Fig. 3, simulation of uniaxial loading produced a stable crack length vs applied axial stress curve with a decreasing slope (Fig. 4) with increasing stress. This indicates that, as the axial load is increased, the crack length required to stabilize crack propagation for the same stress increment increases. Material properties for these models were chosen to represent Lac du Bonnet granite from Manitoba, Canada (Table 1).

3. STRESS SHADOW EFFECTS IN A UNIAXIAL STRESS FIELD

Two axial cracks were added to the single crack geometry to examine the influence of their respective stress shadows on the initiation and growth of the central crack (Fig. 5). With the addition of these cracks, crack initiation and propagation differ from the case of a single crack in that crack initiation occurs at a higher stress level for the single crack case (Fig. 6). This conflicts with observations presented by Hoek and Bieniawski[3] who found that crack initiation in glass plates occurred at lower stresses when multiple cracks were present. Du and Aydin [23] found that crack interaction depends both on the distance between cracks and the relative position of the cracks, with the strongest interaction occurring when cracks are offset such as in an

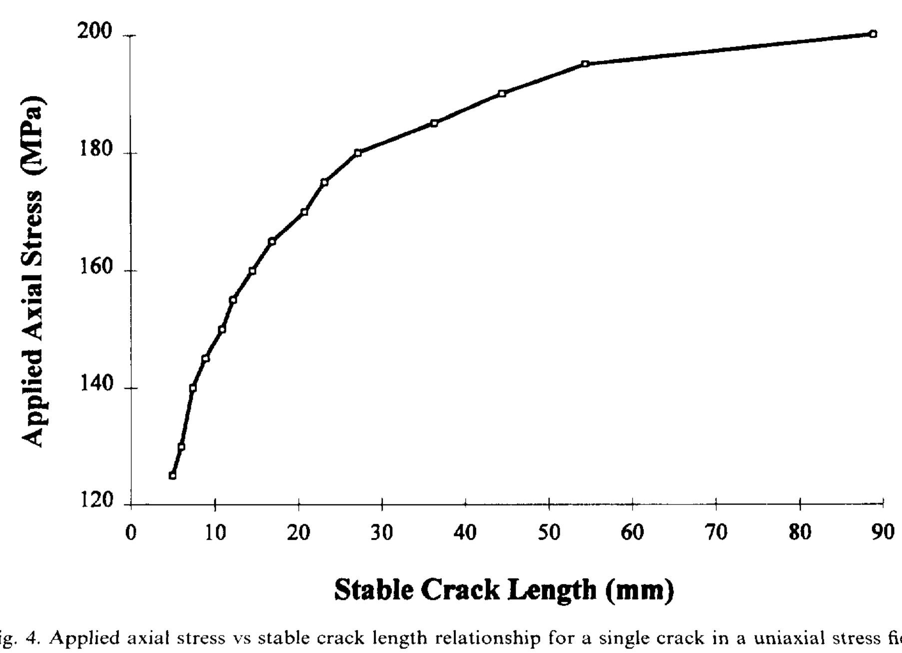

Fig. 4. Applied axial stress vs stable crack length relationship for a single crack in a uniaxial stress field.

Table 1. Material properties for Lac du Bonnet granite used in modelling study

Material property	Value
Young's modulus	70 GPa
Poisson ratio	0.2
Uniaxial compressive strength	225 MPa
Tensile strength	10 MPa
Rocker exponent	0.5

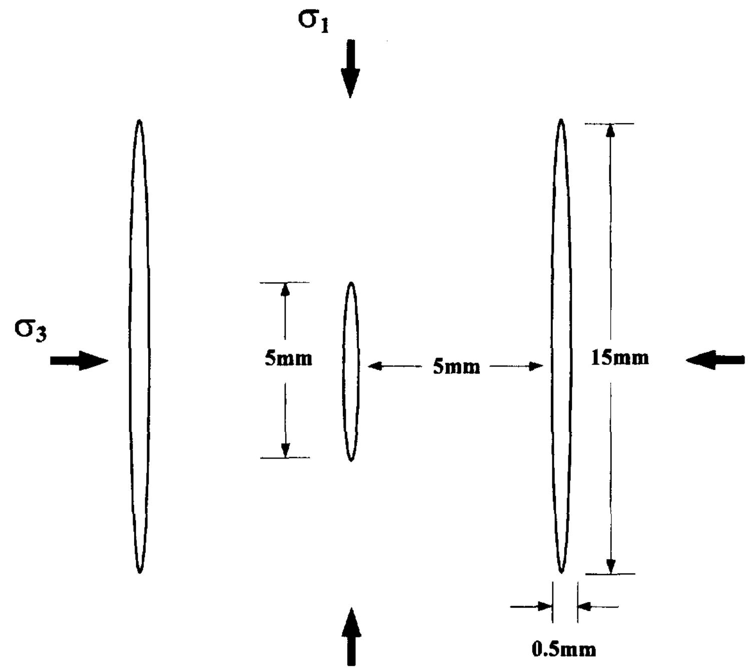

Fig. 5. Problem geometry and material properties used in multiple crack array models for determination of stress shadow effects.

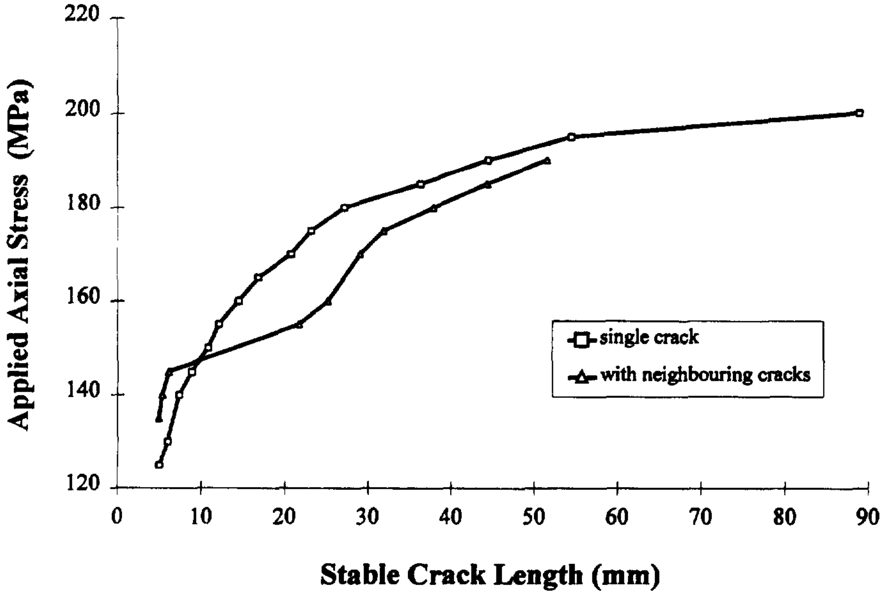

Fig. 6. Applied axial stress vs stable crack length relationships for both single and multiple crack arrays in a uniaxial stress field.

en echelon array. Depending on the geometry of the array, this interaction may result in stress conditions that either inhibit or promote crack initiation. It is apparent that for the particular crack arrangement used in this study, crack initiation is inhibited by the presence of the two neighbouring cracks.

A second effect of the addition of neighbouring cracks is in terms of crack propagation. Initially, induced stresses retard crack growth, but at about 150 MPa they appear to greatly enhance crack growth (Fig. 6). This agrees well with modelling results provided by Kachanov and Laures [24] who noted that shielding or amplification of stresses can occur in a uniaxial stress field when multiple cracks are used. They found that a major crack produces shadows normal to its major axis which may shield nearby microcracks. Crack growth then continues at approximately the same rate as without the stress shadows, but at a lower stress for a given crack length, indicating that the stress field continues to influence propagation (Fig. 6). Due to the nature of the elastic solution, as the central crack lengthens and moves farther away from the two neighbouring cracks' zone of influence, the two curves will eventually converge.

These differences can be best explained by noting that in a uniaxial stress field a tensile stress zone exists around each of the three crack tips if the cracks are aligned approximately parallel to the applied compressive load (Fig. 7). When the propagating crack is small its zone of influence is not within the zone of influence of the larger neighbouring cracks. Therefore, although crack growth is promoted, the influence of the neighbouring cracks is small. As the crack grows due to increased loading, its tensile stress shadow gradually approaches the tensile stress shadows produced by the other cracks and this development accelerates crack propagation. After the crack extends past the zone of influence of the two peripheral cracks, crack growth decelerates. The zone in which the propagating crack is most influenced by the induced

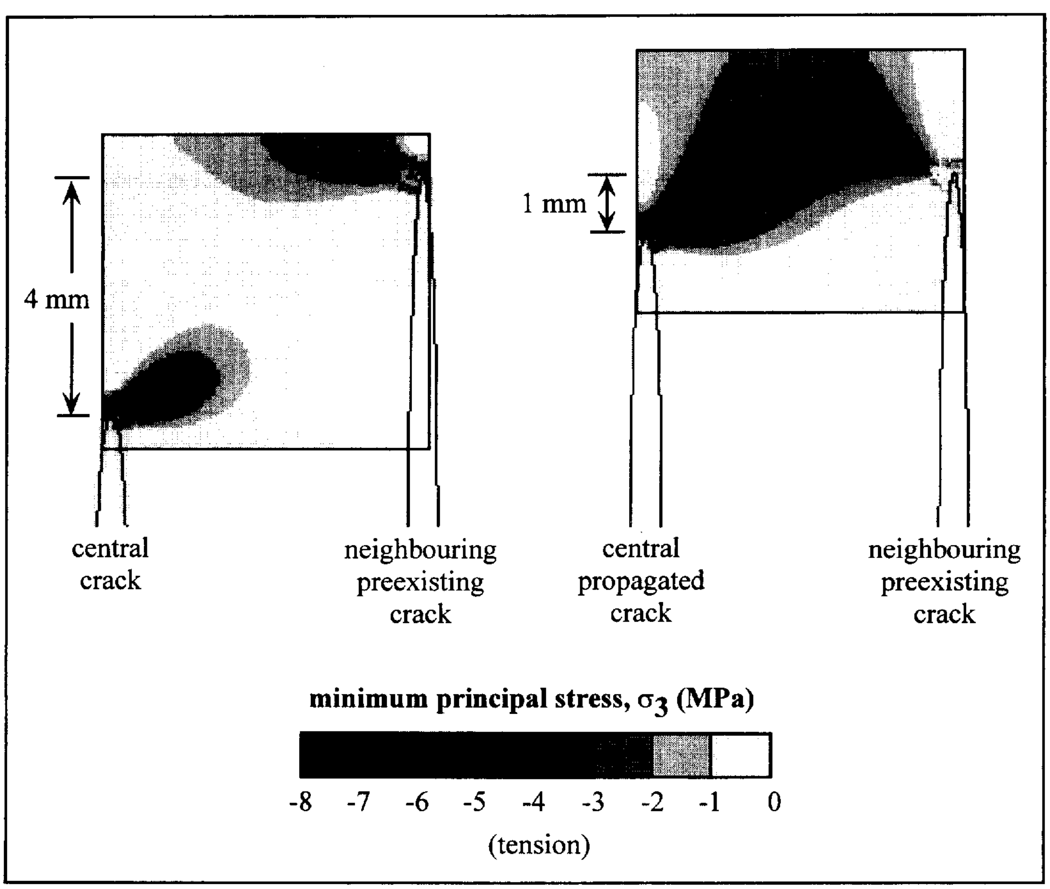

Fig. 7. Minimum principal stress (σ_3) contours surrounding crack tips in multiple crack array under uniaxial loading conditions.

stresses, as shown in Fig. 6, is between a central crack length of 10–20 mm, coinciding with the location of the neighbouring crack tips.

4. STRESS SHADOW EFFECTS IN A TRIAXIAL STRESS FIELD

The addition of confining stress to the single crack model results in a reduction of the tensile stresses near the crack tip. Therefore, crack initiation occurs at much lower stresses for a single crack loaded uniaxially than for one loaded triaxially. Similar results were found by Adams and Sines [25] through the testing of polymethylmethacrylate (PMMA) plates with an embedded crack. Modelling results indicate the opposite effect when peripheral cracks are included in the model. Confinement was added to the multiple crack array model (Fig. 5) by simulating 20 MPa in the σ_2 and out-of-plane directions. Both the applied out-of-plane stress and the intermediate principal stress were identical in magnitude so as to avoid any crack propagation in the out-of-plane direction.

With the addition of triaxial loading to the multiple crack array, results show that the predicted tensile stress zone which forms around the central crack tip is enhanced and appears at a lower σ_1 than in the uniaxial case. The development of larger tensile stresses results in a crack initiation stress 40 MPa lower than in the uniaxial case (Fig. 8). Although crack initiation begins sooner in the triaxial case than in the uniaxial case, crack propagation in the triaxial case is much slower with crack growth occurring on the scale of only a few millimetres over a change in applied stress of 140 MPa (Fig. 8). In comparison, total crack propagation in the uniaxial multiple crack model is approximately 35 mm over only 40 MPa of applied axial stress. This indicates that stress shadows resulting from the addition of peripheral cracks and the addition of a confining load seriously retards crack growth. In the uniaxial case, the stress zones around the tips of the neighbouring cracks are tensile and were shown to promote crack propagation. Examination of the stress zones around the neighbouring crack tips in the triaxial case reveal that not only are these stresses not tensile, but the minimum principal stresses are consistently around 30 MPa compressive (Fig. 9) regardless of the applied axial load. This phenomenon accounts for the extremely slow crack propagation that occurs. Whereas peripheral cracks in a uniaxial stress field enhance crack growth due to the interaction of the tensile stress shadows which form around all three crack tips, the compressive stress shadows that form around the neighbouring crack tips in a triaxial stress field suppress the tensile stress zone around the central crack, effectively restricting crack growth to only a few millimetres. Initially, large tensile

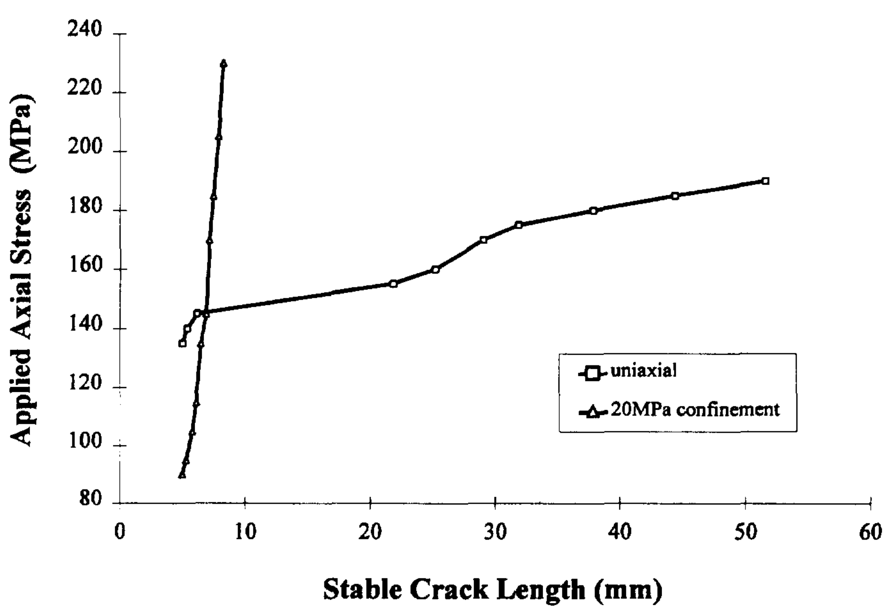

Fig. 8. Applied axial stress vs stable crack length relationships for a multiple crack array with and without an applied confining pressure.

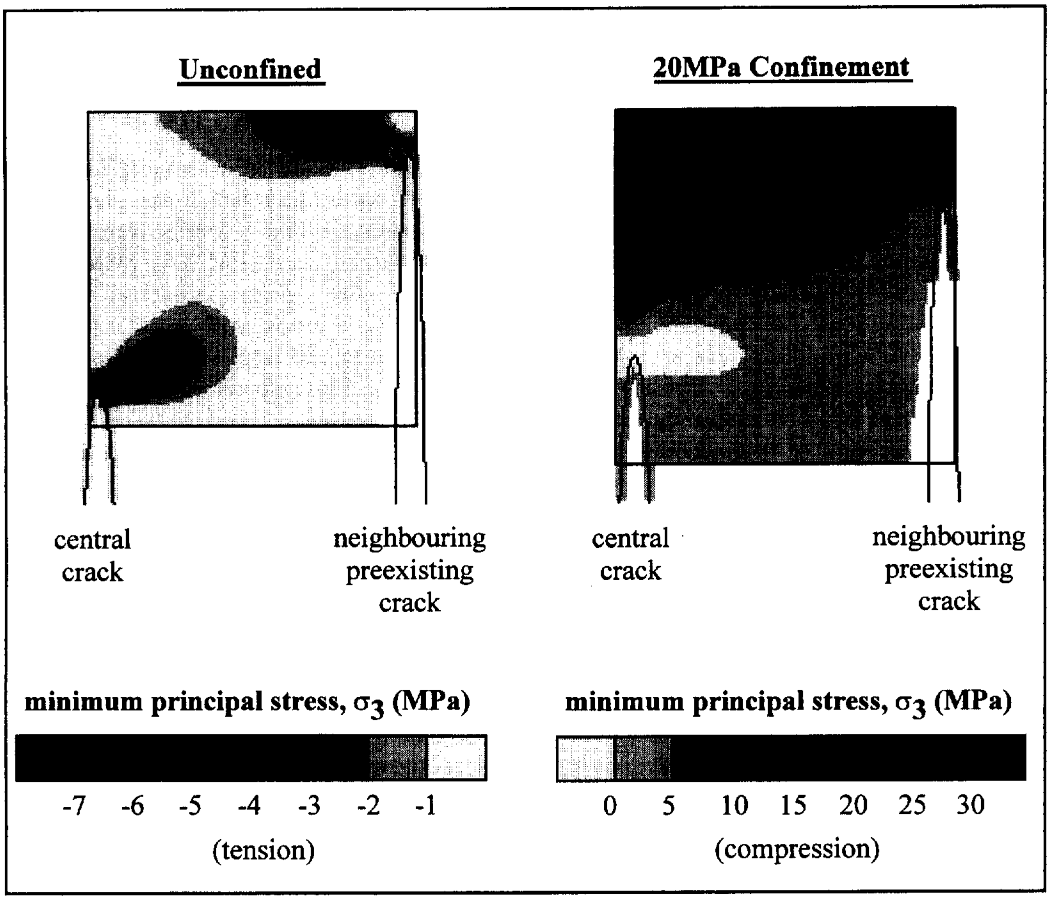

Fig. 9. Minimum principal stress (σ_3) contours surrounding crack tips in multiple crack array under uniaxial and triaxial loading conditions.

stress zones form around the middle crack tip early on so that crack initiation occurs sooner in a triaxial stress field. However, as the crack grows and approaches the compressive stress zones surrounding the tips of the neighbouring cracks, crack growth is essentially halted (Fig. 8).

Additional modelling shows that as the confining stress applied to the multiple crack array is varied, its influence on promoting crack initiation changes. Models of a multiple crack array with varying confining stress show that central crack initiation occurs at decreasing applied axial compressive stresses with increasing confining stress (Fig. 10). Crack initiation stress decreases from 140 MPa for the uniaxial condition to 45 MPa at 30 MPa of confinement. Similar findings were made by Hamajima *et al.* [26] using discrete element modelling. These models also reinforce observations made for the case of 20 MPa confinement regarding crack propagation. With increasing confining pressure, the magnitude of compressive stresses surrounding the peripheral crack tips increases, thereby increasing the restraint on propagation of the central crack. With an increase in confining pressure from 10 to 20 MPa, the compressive stress magnitudes around the tips of the peripheral cracks increase from an approximate range of 10–15 MPa to 20–30 MPa.

4.1. *Application of modelling results to laboratory observations*

The modelling results suggest that with the addition of confining stress the stress level required to achieve crack initiation should be lower than that required under uniaxial loading. It would also appear that, although cracks growing in a triaxial stress field may be smaller in length due to the restraining effect compressive stress shadows have on crack growth, the number of cracks, or crack density, would be greater than that in a uniaxial stress field due to the ease with which they initiate. These findings are supported by crack counting studies using opti-

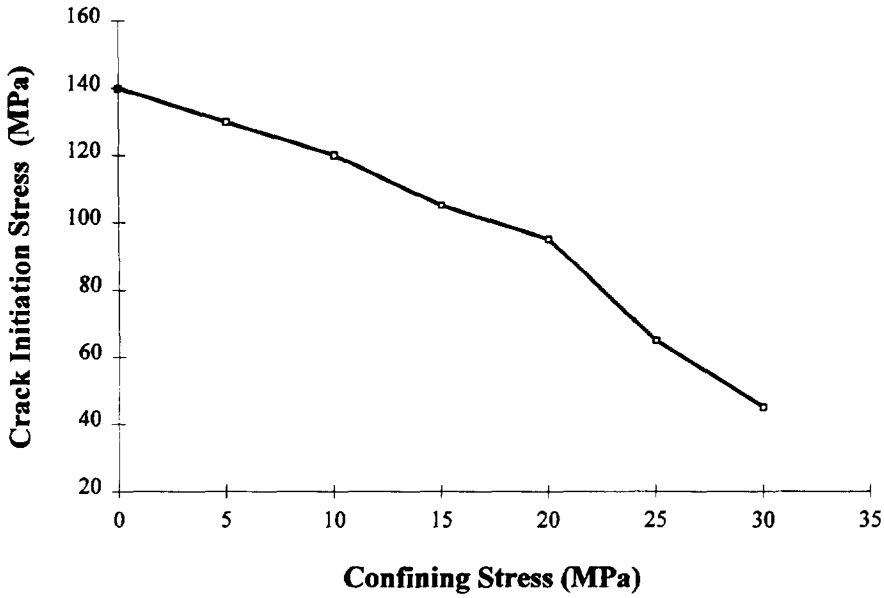

Fig. 10. Effect of confining stress on crack initiation for a multiple crack array.

cal microscopes and scanning electron microscopes (SEM) on thin sections taken from rock samples previously loaded through uniaxial and triaxial laboratory testing. Thin section studies by Kwong [27] and Bezys [28] indicate that crack density is higher in samples tested triaxially than those tested uniaxially. Wawersik and Brace [29] and Kranz [30] both observed an increase in crack density with the addition of confining stress. Hugman and Friedman [31] noted that as the confining pressure is increased the density of microcracks developing before failure also increases.

Studies comparing the lengths of cracks in samples tested uniaxially and triaxially are more limited. Model studies indicate that, due to the adverse conditions created by stress shadows under triaxial conditions, crack lengths will be small relative to those found for the uniaxial case. Similar results were found by Dey and Wang [32] using a two-dimensional stress inhomogeneity model. They noted that, with the addition of confining pressure, axial crack growth was strongly suppressed. Observations made on Indiana limestone by Myer *et al.* [33] also substantiate these results. In their studies, visual inspection revealed that the dominant micromechanical process associated with failure under uniaxial conditions was the growth of long extensile cracks. They found that the addition of confining pressure limited the extent of stable crack growth and limited the amount of crack interaction. Based on these observations, Myer *et al.* [33] concluded that lack of confinement results in lower densities of longer extensile cracks which eventually interact to form macrofractures, while confined compression produces more uniform populations of shorter cracks due to a lack of crack interaction. Results from our study demonstrate that in a multiple crack array under triaxial loading conditions, stresses around a crack tip may be compressive and inhibit other cracks extending into their compressive stress field.

5. ZONE OF INFLUENCE OF PERIPHERAL CRACKS

The zone of influence of peripheral cracks depends upon the relative size of the cracks to the central crack and their relative position (or distance away) from the propagating central crack. Cases with varying crack lengths and crack distances were analysed for both uniaxial and triaxial loading conditions. In terms of uniaxial loading, results show that as peripheral cracks are moved away from the central crack, the tensile stresses surrounding the middle crack tip decrease. This reduction increases the applied axial stress required to initiate cracking (Fig. 11). When the peripheral cracks are between 5 and 10 mm from the central crack, a compressive

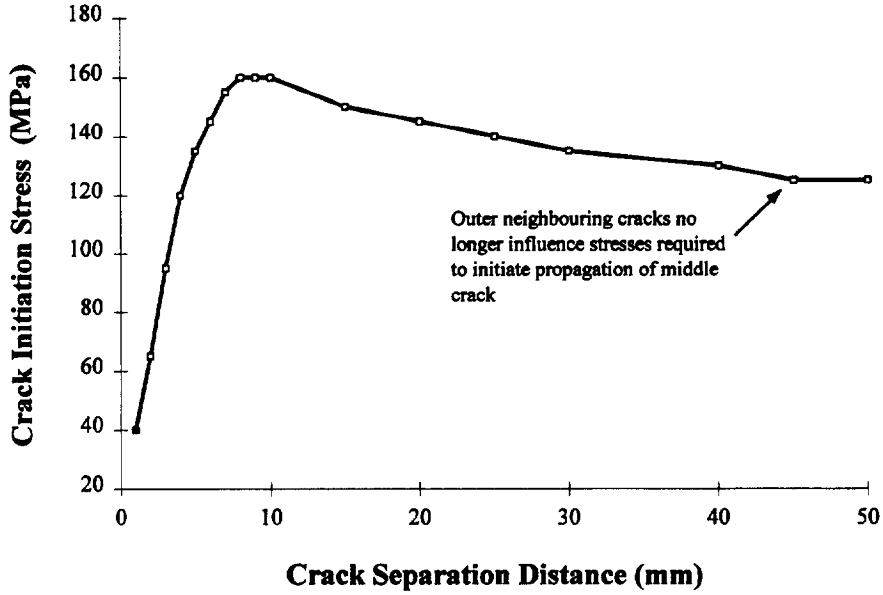

Fig. 11. Zone of influence of peripheral cracks on the middle crack and its crack initiation stress level in a uniaxial stress field. Zone of influence is taken as the horizontal distance separating the middle crack from the two neighbouring cracks.

stress shadow forms between the middle and peripheral cracks and results in a higher crack initiation stress than that for a single crack (i.e. an isolated crack without pre-existing cracks). With no peripheral cracks, the crack initiation stress for a single 5 mm crack under uniaxial loading is approximately 130 MPa (Fig. 6). This uniaxial crack initiation stress is once again achieved under the multiple crack conditions when the peripheral cracks are separated approximately 45 mm from the middle crack. These results indicate that the zone of influence of the stress shadows resulting from the inclusion of two cracks 15 mm in length and 0.5 mm in width is approximately 45 mm on either side of the central crack.

Shortening or lengthening of the peripheral cracks also effects crack initiation. The effect is dependent on the interaction between the stress shadows surrounding the middle crack and peripheral cracks. Under uniaxial loading conditions, tensile stress zones form around both the

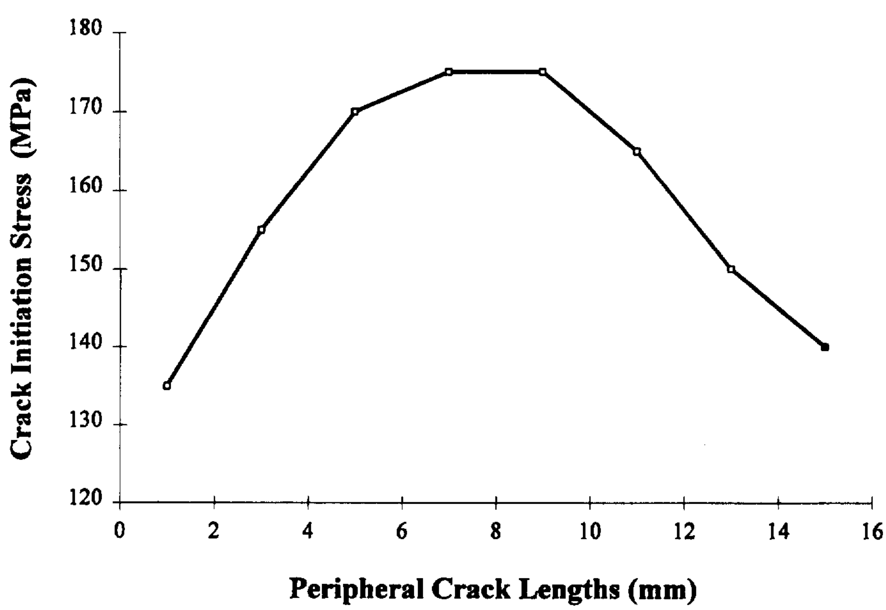

Fig. 12. Influence of peripheral crack length on the middle crack and its crack initiation stress level in a uniaxial stress field.

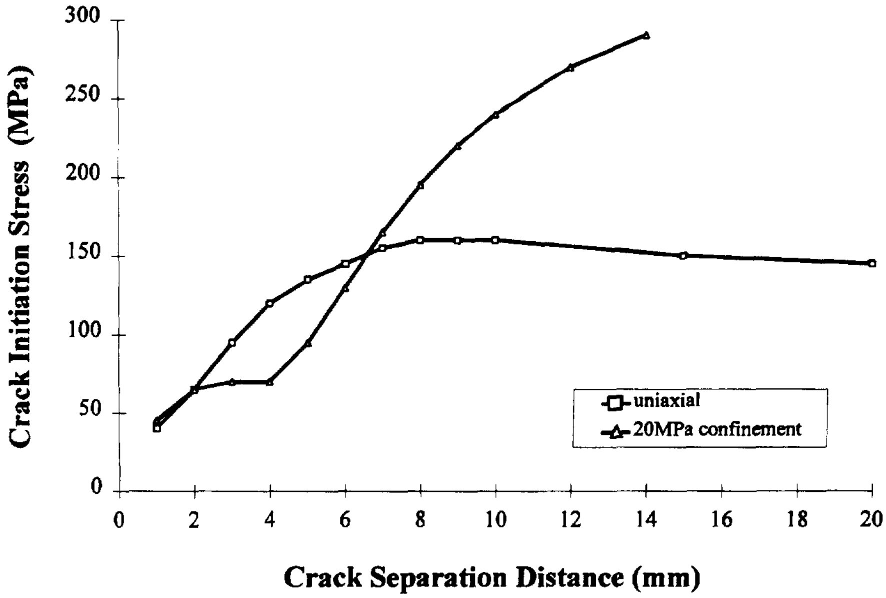

Fig. 13. Zone of influence of peripheral cracks on the middle crack and its crack initiation stress level in a triaxial stress field. Zone of influence is taken as the horizontal distance separating the middle crack from the two neighbouring cracks.

middle and outer crack tips when the peripheral crack lengths are small relative to the middle crack length, resulting in reduced crack initiation stresses (Fig. 12). As the peripheral cracks are lengthened, the stresses around them change from tensile to compressive. The appearance of these compressive stress zones occurs between crack lengths of approximately 7 mm and 9 mm. At crack lengths greater than 10 mm the stress shadows around the outer cracks become tensile again with a resulting drop in the required crack initiation stress.

These findings indicate that the relative position and size of any neighbouring cracks have a significant effect on the crack initiation and propagation of neighbouring cracks. Similar results have been described by other authors. Using photoelastic models, Bombolakis [34] found that the stress required for initial crack growth depends strongly on the crack spacing. Similar to the results shown in Fig. 11, Bombolakis [34] showed that as the crack spacing decreased, the applied stress required to initiate crack growth also decreased. Peng and Oritz[35] found in their studies that the initiation and propagation of individual cracks under compression was predominantly governed by the local configuration of the microstructure. Similarly, Kranz [36] and Dey and Wang [32] noted that significant changes in the tensile stresses near the crack tip occur as a function of both crack separation and relative orientation. In general, cracks can inhibit or promote propagation of adjacent cracks depending on their relative positions, size, and the degree of interaction between the induced crack tip stress concentrations.

The application of confining pressure significantly alters the zone of influence and the behaviour of cracks within it. Initially, as the peripheral cracks move away from the central crack, the higher tensile stress zone surrounding the propagating crack resulting from the stress drop induced in between the two peripheral cracks remains, thus keeping the crack initiation stresses lower than in the uniaxial case (Fig. 13). However, once the pre-existing cracks are separated sufficiently for the pressure drop to disappear, triaxial loading has an adverse effect and requires a higher applied load to initiate cracking since the deviatoric stresses are lower than in the uniaxial case. The farther away the outer cracks are located, the more the triaxial load prevents crack initiation, thereby requiring higher axial loads for crack initiation.

6. CONCLUSION

Crack initiation, growth and crack interaction are complicated processes. Numerical modelling results have shown that stress shadows in a multiple crack model have a significant effect on crack initiation and propagation. The following conclusions are made based on these results.

- For crack propagation, the addition of peripheral cracks can act to either suppress or promote crack growth depending on the distance separating the cracks and the loading conditions.
- A material loaded triaxially should contain a large number of small cracks, whereas a material loaded uniaxially should exhibit fewer but longer cracks.
- The complexities created through the interaction of stress shadows indicate that the process of crack initiation and propagation is far more complex than that revealed in single crack models and single crack fracture criteria. Although analytical models and criteria based on single cracks have proven useful in providing a basis for the study of material fracture and failure, they appear to be quite limited in simulating realistic crack propagation in rock. Further modelling studies are needed to increase understanding of how cracks interact with one another in the progressive degradation of a material before ultimate failure.

Acknowledgements—Parts of this work have been supported by NSERC Operating Grants and Atomic Energy of Canada Limited. The authors wish to thank Ed Dzik for the use of his program EJBEM.

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(*Received* 1 *May* 1997, *in final form* 8 *July* 1997, *accepted* 15 *July* 1997)