Discrimination of the flow law for subglacial sediment using in situ measurements and an interpretation model

Jeffrey L. Kavanaugh and Garry K. C. Clarke
Department of Earth and Ocean Sciences, University of British Columbia, Vancouver, British Columbia, Canada

Received 3 June 2005; revised 10 September 2005; accepted 15 September 2005; published 21 January 2006.

[1] Subglacial hydrological and mechanical processes play a critical role in determining the flow characteristics and stability of glaciers and ice sheets. To study these processes, we have measured simultaneously basal water pressure, pore water pressure, sediment deformation, glacier sliding, and sediment strength beneath Trapridge Glacier, Yukon Territory, Canada. To interpret these data, we have developed a simple hydromechanical model of processes beneath a soft-bedded alpine glacier. The glacier bed is divided into soft-bedded regions that are hydraulically connected to the subglacial drainage system, soft-bedded but hydraulically unconnected regions, and hard-bedded regions. Each region is represented as a one-dimensional column. The columns are coupled by a simple ice dynamics model that accounts for water-pressure-driven changes in basal shear stress distribution. Synthetic responses for subglacial instruments are calculated from the modeled basal conditions, providing a framework for improving interpretation of field records. The model is used to determine which of several till flow laws best represents conditions beneath Trapridge Glacier. Investigated are linear-viscous, nonlinear-viscous, nonlinear-Bingham, and Coulomb-plastic tills. Pore water pressures, sediment deformation profiles, and sliding rates are calculated for each flow law. Comparison of synthetic and field instrument responses suggests that till behavior is best represented as Coulomb-plastic. Model results also suggest that the ploughmeter is the most diagnostic in situ indicator of till behavior currently available and that using long-term observations of sediment deformation profiles in regions of varying pore water pressure can result in an underestimation of flow law nonlinearity.


1. Introduction

[2] Sediment deformation and basal sliding can contribute greatly to the motion of ice masses that are underlain by water-saturated sediments. Basal motion is the dominant flow mechanism of some alpine glaciers [e.g., Raymond, 1971; Engelhardt et al., 1978; Boulton and Hindmarsh, 1987; Truffer et al., 2001] and at least one arctic glacier [Copland et al., 2003]. Furthermore, basal motion provides the key to understanding fast flow of ice streams [e.g., Blankenship et al., 1986; Alley et al., 1987a, 1987b; Tulaczyk et al., 2000a; Andrews and MacLean, 2003; Truffer and Echelmeyer, 2003], tidewater glaciers [e.g., Meier and Post, 1987; Glasser and Hambrey, 2001], and surging glaciers [e.g., Meier and Post, 1969; Kamb et al., 1985; Fowler, 1987; Kamb, 1987; Raymond, 1987]. Much emphasis has been placed on the flow dynamics of Antarctic ice streams, which play an important role in the mass balance of the potentially unstable West Antarctic Ice Sheet [Mercer, 1978] and provide modern analogues to fast flowing regions of the Laurentide and Cordilleran Ice Sheets inferred from geologic evidence [e.g., Morner and Dreimanis, 1973; Clayton and Moran, 1982; Brown et al., 1987; Hooyer and Iverson, 2002]. More recently, much work has been focused on improving the understanding of how changes in subglacial drainage system characteristics influence basal motion, both in general terms [e.g., Iken and Bindschadler, 1986; Harbor et al., 1997; Iken and Truffer, 1997; Boulton et al., 2001; Mair et al., 2002; Clarke, 2005] and in the context of such phenomena as spring speedup events [e.g., Iken et al., 1983; Röthlisberger and Lang, 1987; Kavanaugh and Clarke, 2001; Mair et al., 2003; Anderson et al., 2004]. Despite the fundamental role basal motion plays in glacier dynamics, our understanding of the underlying processes remains incomplete.

[3] Studies of ice dynamics have used a wide range of flow laws for describing the behavior of subglacial sediments, from linear viscous [e.g., Alley et al., 1987b; MacAyeal, 1989] to rate-independent [e.g., Tulaczyk et al.,]
2000a], while some recent work suggests that subglacial sediment can be treated as viscous material even though it is probably not [Hindmarsh, 1997; Fowler, 2002].

[4] Subglacial sediments with different rheological behaviors would respond differently to a given set of subglacial forcings. In situ measurements made by subglacial instruments provide a means of observing these responses. During the summer of 1996, instruments installed at the bed of Trapridge Glacier, Yukon Territory, Canada (Figure 1) recorded simultaneous measurements of basal water pressure, pore water pressure, sediment strain rate, basal sliding and sediment strength. These records are sufficiently complete to help determine which of the proposed flow laws best describes the mechanical behavior of subglacial sediments.

[5] To this end, we have developed a simple hydromechanical model of the processes governing basal motion of a soft-bedded alpine glacier. The glacier bed is divided into three general regions, and each basal region is modeled as a single one-dimensional column. The time evolution of pore water pressure, sediment dilatancy, sediment deformation and glacier sliding is calculated in modeled soft-bedded regions; hard-bedded regions are considered rigid and impermeable. Sediment properties are determined by the pore water pressure, porosity and preconsolidation history of the sediment. The columns are coupled by a simple ice dynamics model, allowing investigation of the effects of stress bridging between regions.

[6] Sediment deformation profiles are calculated using four flow laws that have been proposed in the literature: linear-viscous, nonlinear-viscous, nonlinear-Bingham and Coulomb-plastic. Synthetic instrument responses are calculated from the modeled pore water pressure, sediment deformation, sliding and sediment strength conditions. The magnitude and form of the calculated instrument responses are compared to the field records, allowing us to determine which sediment flow law provides the best qualitative match.

2. Field Instrument Records

[7] Figure 2 shows simultaneous measurements of basal water pressure, pore water pressure, sediment strain rate, basal sliding, and sediment strength recorded by five instruments during the period 20–25 July 1996 at Trapridge Glacier, Yukon Territory, Canada (Figure 1). The responses in Figure 2 exhibit typical magnitudes and phase relations (with respect to diurnal water pressure fluctuations) for each instrument type during the summer season at Trapridge Glacier. The five instruments were installed in three boreholes located within ~7.0 m of each other at the glacier bed. Sensors P1 and PL1 shared one hole; PZ1 and BT1 shared another (Figure 1c). Water level fluctuations in each borehole immediately following drilling indicates that all three boreholes were placed in hydraulically connected regions of the bed, and borehole depths measured upon completion of drilling suggest that the glacier bed is sufficiently flat to assume uniform basal water pressures. The close proximity of the boreholes suggests that basal conditions and forcings were uniform over the region in which they were installed.

[8] Figure 2a shows records for pressure transducers P1 and PZ1, with pressure expressed in units of pressure head $p_{ph} = p/p_w g$. Pressure transducer P1 was suspended within the borehole above the glacier bed. Because Trapridge Glacier is polythermal [Jarvis and Clarke, 1975; Clarke et al., 1984; Clarke and Blake, 2001], boreholes drilled into the glacier typically freeze closed within ~12 hours of drilling. Pressures measured by P1 thus accurately represent
the water pressure in the subglacial drainage system. PZ1 was installed at a depth of 0.15 m within the subglacial sediments. Indicated pressures are taken to represent the pore water pressure within the sediments at this depth. The records for both P1 and PZ1 show strong diurnal pressure fluctuations, indicating continued hydraulic connection with the subglacial drainage system. The record for transducer P1 exhibits peak pressures close to the estimated flotation value of 62 m. The record for PZ1 shows pressure variations that are slightly lower in amplitude than those indicated in the record for P1, and with a diurnal pressure cycle that is slightly lagged in phase in comparison with P1.

Figure 2b shows the strain rate in the subglacial sediments inferred from the tilt record for tilt cell BT1. The tilt cell (Figures 3a and 3b), which measures 0.11 m long and incorporates pressure transducer PZ1 (Figures 2a and 3a), was installed 0.15 m into the sediments. During the 5-day period shown in Figure 2, the tilt sensor rotated approximately 14.7° (assumed down glacier). The tilt angle time series data were smoothed with a 3-hour moving boxcar filter prior to differentiation. Strain rate variations are out-of-phase with water pressure fluctuations.

Figure 2c shows inferred glacier sliding rates from the record for slidometer SL1. The anchor (see Figures 3c and 3d) was inserted 0.12 m into the subglacial sediments. Prior to differentiation, raw sliding displacement data were smoothed with a 3-hour moving boxcar filter. Because there is no mechanism for “rewinding” stripped line, the slightly negative sliding rates indicated in the record are clearly nonphysical and are likely artifacts of the instrument design. Strong correlation between sliding rate and subglacial water pressure is observed during days 202–204; sliding rates during days 205–206 are not as clearly related to pressure variations.

Figure 2d. Field instrument records during 20–25 July (days 202–207) 1996. (a) Records for PZ1 (solid line) and P1 (gray line). (b) Strain rate in till calculated from the tilt record for BT1 (solid line). (c) Sliding rate determined from displacement record for SL1. (d) Force record for PL1 (solid line). The pressure record for P1 is shown as gray lines in Figures 2b–2d.

3. Flow Laws for Subglacial Sediments

The rheological nature of subglacial sediments remains as one of the more pressing questions in glaciology. Studies to date have variously concluded that the behavior of subglacial sediments is only slightly nonlinear [e.g., Boulton and Hindmarsh, 1987], highly nonlinear [e.g., Kamb, 1991; Hooke et al., 1997] or rate-independent (typically described as “Coulomb-plastic”; [e.g., Iverson et al., 1997; Iverson et al., 1998; Tulaczyk et al., 2000a]. Models of ice stream dynamics have generally assumed subglacial sediments to behave as a linear-viscous [Alley et al., 1987b; MacAyeal, 1989; Thorsteinsson and Raymond, 2000] or mildly nonlinear [Alley, 1989] fluid, although recent models have also considered Coulomb-plastic tills [e.g., Tulaczyk et al., 2000b]. (Here and for the remainder of this manuscript, the word “till” is used to describe any un lithified subglacial sediments, not just those that are basally derived.). Because an ice mass underlain by a highly nonlinear till is more prone to unstable behavior than is one resting on a linear-viscous till [Alley, 1990; Kamb, 1991], the flow characteristics of till play a critical role in the stability of ice sheets and glaciers.
In this study, we will investigate four commonly used flow laws for subglacial sediments. The first assumes sediments to deform as a Newtonian viscous material (Figure 4a):

\[ \dot{\varepsilon} = \frac{1}{2\eta_0} \tau. \]  

Here the strain rate \( \dot{\varepsilon} \) is linearly related to the shear stress \( \tau \) and the till viscosity \( \eta_0 \) is independent of pore water pressure. This relation has been used to model the flow of ice streams [e.g., Alley et al., 1987b; Alley et al., 1989; MacAyeal, 1989] and the subglacial properties of alpine glaciers [e.g., Fischer and Clarke, 1994].

The second and third flow laws investigated were proposed by Boulton and Hindmarsh [1987]. These relations were derived from observations made in near-margin sediments beneath Breidamerkurjökull, Iceland. The first of these relations considers sediments a nonlinear-viscous material (Figure 4b):

\[ \dot{\varepsilon} = B_1 \tau^a (\rho')^{-b}. \]  

As pore water pressures increase the effective pressure \( (\rho' = \rho_I - p) \), where \( \rho_I \) is the ice overburden pressure and \( p \) the pore water pressure) decreases, weakening the till. The second relation treats till as a Bingham material (Figure 4c), in which the strain rate depends on the amount by which the shear stress exceeds the yield strength \( \sigma_Y \):

\[ \dot{\varepsilon} = B_2 (\tau - \sigma_Y)^a (\rho')^{-b}. \]  

The yield strength \( \sigma_Y \) is determined by the Mohr-Coulomb failure criterion,

\[ \sigma_Y = c_0 + \rho' \tan \phi, \]  

in which \( c_0 \) is the cohesion and \( \phi \) the friction angle. At stress values below \( \sigma_Y \) the till behaves elastically; at values above the yield strength the till deforms viscously. The terms \( B_1, B_2, a, \) and \( b \) in (2) and (3) are positive-valued tuning parameters.

The fourth flow law investigated assumes that till behaves as a Coulomb-plastic material (Figure 4d). Assuming that the till has been sufficiently sheared to reach a residual state [see Iverson et al., 1998], the strength of such a material is linearly related to the effective pressure but independent of the strain rate. Laboratory studies by Iverson et al. [1998] and Tulaczyk et al. [2000a] suggest that till behavior can be approximated as Coulomb-plastic, and this behavior has been used to model basal motion of the Puget Lobe of the Cordilleran Ice Sheet [Brown et al., 1987] and of

Figure 3. Instruments used at Trapridge Glacier. (a) Tilt cell. (b) Installation of tilt cell. (c) Slidometer. (d) Installation of slidometer. (e) Ploughmeter. (f) Installation of ploughmeter.

Figure 4. Stress-strain rate relations for modeled till flow laws. (a) Linear-viscous till. (b) Nonlinear-viscous till, with \( a = 1.33 \) and \( b = 1.80 \). (c) Nonlinear-Bingham till, with \( a = 0.625 \) and \( b = 1.25 \). Shaded region represents transition zone between elastic (\( \tau < \sigma_Y \)) and viscous (\( \tau > \sigma_{\text{min}} \)) behaviors. (d) Coulomb-plastic till (solid line) and hyperbolic tangent approximation of Coulomb-plastic behavior used in model (dashed line).
both alpine glaciers [e.g., Iverson, 1999; Truffer et al., 2001] and ice streams [e.g., Tulaczyk et al., 2000b]. Plastic behavior is also suggested by analysis of till from the base of Whillans Ice Stream (formerly Ice Stream B) by Kamb [1991].

At stresses below the yield strength \( \sigma_Y \) determined by the Mohr-Coulomb failure criterion (4), the behavior is elastic and no permanent deformation occurs; flow at stresses greater than the yield strength is instantaneous:

\[
\dot{\varepsilon} = \begin{cases} 
0, & \text{if } \tau < \sigma_Y \\
> 0, & \text{if } \tau \geq \sigma_Y
\end{cases}
\]  

(5)

No direct relationship between shear stress and deformation rate exists for values of \( \tau \geq \sigma_Y \) in the absence of additional constraints (provided, for example, by neighboring sticky spots or marginal drag), deformation would occur at the rate necessary to prevent the applied shear stress from exceeding \( \sigma_Y \). The abrupt transition between nondeforming and deforming states for Coulomb-plastic till creates a high degree of numerical stiffness. To reduce this stiffness, we model Coulomb-plastic behavior as

\[
\dot{\varepsilon} = \frac{\dot{\varepsilon}_0}{2 \left[ 1 + \tanh \left( 2\pi \frac{\tau - \sigma_Y}{\Delta \tau} \right) \right]}.
\]  

(6)

Assigning a sufficiently large value to the reference strain rate \( \dot{\varepsilon}_0 \) (i.e., one that is much larger than modeled strain rates) and a small value to the failure range \( \Delta \tau \) allows close approximation to the Coulomb-plastic behavior of (5) (Figure 4d, dashed line). The hyperbolic tangent \( \tanh(x) \) is an exponential, rather than circular, function. As such, the factor \( 2\pi \) has the effect of largely confining the transition between nondeforming and deforming states to the interval \( \sigma_Y \pm \Delta \tau/2 \). Without this factor, which plays a similar role in (13), the transition would occur over a wider range of shear stress values.

### 4. Model

#### 4.1. Basal Representation and Model Geometry

Spatial variations in drainage system morphology and sediment properties lead to considerable nonuniformity in mechanical and hydraulic characteristics of the bed. While much complexity could be introduced to account for these variations and may eventually prove necessary to explain the broad range of subglacial phenomena observed in instrument records, our goal is to explore general subglacial behaviors under typical conditions. We thus take a simplified view of the subglacial environment, in which the glacier bed is divided into three categories: (1) soft-bedded regions that are hydraulically connected to the subglacial drainage system, (2) soft-bedded but poorly connected regions and (3) hard-bedded regions. Only general assumptions are made about the distribution of bed types (discussed below), and each region is assumed to cover an areal fraction \( \alpha_i \) of the bed such that \( \alpha_1 + \alpha_2 + \alpha_3 = 1 \).

#### 4.2. Pore Water Pressure Evolution

Subglacial sediment in the two soft-bedded columns is considered to be a fully saturated two-component mixture of water and solid particle matrix. The expression used to calculate the time evolution of pore water pressure within the till layer was developed assuming that (1) the equations governing till porosity and dilatancy equations follow those of Clarke [1987], (2) hydraulic permeability is determined by the Kozeny-Carman relation [Carman, 1961], and (3) water flux within the till columns obeys Darcy’s law.

Typically, modeling of water flow through a porous medium is accomplished by writing the equations of state in terms of an Eulerian, or spatially fixed, coordinate system. While this approach is appropriate for cases in which deformation of the medium is negligible, a dilatant till can undergo significant volume changes. We therefore use a Lagrangian approach, in which the equations of state are expressed in terms of the initial configuration of particles. In such a coordinate system, \( Z \) represents the initial vertical position of a particle in the till column; at some later time \( t \) its position is \( z(Z, t) \). The Lagrangian equation for the time evolution of pore water pressure within the till is

\[
\frac{\partial p(Z, t)}{\partial t} = \left\{ \frac{1}{\mu} \left[ \frac{1}{2} \frac{\partial D}{\partial p} + 3 \kappa_\beta(p) \right] \left[ \frac{1}{J(t)} \frac{\partial p}{\partial Z} + \rho e \right] \frac{\partial p}{\partial Z} \right. \\
\left. - \frac{\kappa(p)}{J(t)} \left[ \frac{1}{2} \frac{\partial D}{\partial p} \left( \frac{\partial p}{\partial Z} \right)^2 + \kappa(p) \frac{1}{J(t)} \frac{\partial^2 p}{\partial Z^2} \right] \right. \\
\times \left\{ (\alpha(p) + n(Z, t) \beta J(t))^{-1} \right. \\
\left. \right. \\
\right.
\]  

(7)

Here \( p \) is the pore water pressure and \( \rho, \mu, \kappa, \beta, \) and \( \alpha \) are the density, viscosity, and compressibility of water. The terms \( n, \kappa, \alpha \) represent the porosity, hydraulic permeability, and compressibility of the solid matrix. The acceleration due to gravity is \( g \), and \( J \) is the Jacobian of the transformation from \( Z \) to \( z \). See Appendix A for thorough discussions of the Lagrangian representation and the development of this equation.

Dilatation of subglacial sediments results in changes in porosity \( n \) and therefore the hydraulic permeability \( \kappa \). The time evolution of the porosity is related to the rate of change in pore water pressure, the local strain rate within the till layer, and the consolidation state by

\[
\frac{\partial n}{\partial t} = \begin{cases} 
\frac{1}{(1 + e)^2} \left( B_0 \left( \rho \dot{e} + \rho \dot{e} \dot{e} \right) - D_0 \dot{e} (e - e_C) \right) & \text{left of NCL} \\
\frac{1}{(1 + e)^2} \left( B_1 \left( \rho \dot{e} + \rho \dot{e} \dot{e} \right) - D_0 \dot{e} (e - e_C) \right) & \text{on NCL}.
\end{cases}
\]  

(8)

Here \( e \) is the void ratio at a given depth within the till column and \( e_C \) is the critical state void ratio. The term \( B_i \) is a swelling index and \( B_0 \) is a compression index. The rate of change of the pore water pressure is given by \( \dot{p} \) and \( D_0 \) is a scaling factor. The term \( \dot{e} \) prevents the void ratio from diverging as the effective pressure value approaches zero. The “normal consolidation line”, or NCL, relates the void ratio to the effective pressure for a virgin soil [see Clarke, 1987]. Equation (8) is developed in Appendix A.

#### 4.3. Deformation Profile

Assuming simple shear in the till, the strain rate is given by

\[
\dot{\varepsilon} = \frac{1}{2 \left( \frac{\partial \varepsilon}{\partial Z} \right) \frac{\partial Z}{\partial t}}
\]  

(9)
Solving (9) for $\partial v_z/\partial Z$ and integrating up from the base of the till layer yields the velocity profile $v_z(Z,t)$:

$$v_z(Z,t) = 2 \int_0^Z J(t) \varepsilon(Z',t) dZ'. \quad (10)$$

We assume that no slip occurs between the till base and the underlying substrate, and neglect any motion in lower layers of till (as has been inferred beneath Black Rapids Glacier by [Truffer et al., 2000]). Depending on the flow law chosen, the strain rate $\varepsilon(Z,t)$ is determined by equation (1), (2), (3) or (6). Till displacement is calculated by integrating the velocity field with respect to time:

$$\frac{\partial}{\partial t} s(Z,t) = v_z(Z,t). \quad (11)$$

### 4.4. Glacier Sliding

[22] In addition to facilitating the deformation of subglacial sediments, increased subglacial water pressure also enables basal sliding by submerging roughness features and decoupling the ice from the bed. Several models have been developed to relate glacier sliding to basal shear stress, effective pressure and bed roughness [e.g., Weertman, 1957, 1964; Liboutry, 1968, 1987; Kamb, 1970]. While these models assume the glacier bed to be rigid and impermeable, the regions of interest in the model developed here are underlain by soft, deforming sediments. Here we use an alternative sliding model, in which the sliding velocity is related to the driving stress $\tau(t)$ acting upon the region by

$$v_{SL}(t) = C_{SL}(p) \frac{h_{SL}}{\mu_{SL}} \tau(t). \quad (12)$$

The sliding coefficient $C_{SL}$ is assumed to be related to the water pressure at the ice-bed interface:

$$C_{SL}(p) = \frac{1}{2} \left[ 1 + \tanh \left( 2\pi \frac{p(h(0),t) - p_{SL}}{\Delta p_{SL}} \right) \right]. \quad (13)$$

Here $h(0)$ is the vertical position of the ice-bed interface. The value of $C_{SL}$ varies between zero and unity depending on the water pressure at the ice-bed interface. At water pressures much greater than the reference pressure $p_{SL}$ in (13), $C_{SL} = 1$ and the sliding velocity is $v_{SL} = (h_{SL}/\mu_{SL})\tau$, equal to the rate at which a layer of lubricating material of thickness $h_{SL}$ and viscosity $\mu_{SL}$ would deform when subjected to shear traction $\tau$. At such pressures, roughness features of the bed are assumed to be completely submerged and the shear stress is transferred to the bed by viscous coupling through the layer of lubricating slurry. At water pressures significantly lower than the reference pressure, sliding is assumed to be negligible because of the strong mechanical coupling between the ice and bed provided by roughness features. The transition between nonsliding and sliding modes is assumed to occur over the pressure range $p_{SL} \pm \Delta p_{SL}/2$. In this transition zone, partial submergence of roughness features reduces their effectiveness and allows increased sliding, and shear traction is assumed to be transferred to the bed by a combination of mechanical resistance and viscous coupling. The sliding velocity $v_{SL}$ is given by the rate of change of the sliding distance $s_{SL}(t)$,

$$\frac{\partial}{\partial t} s_{SL}(t) = v_{SL}(t) = C_{SL}(p(h(0),t)) \frac{h_{SL}}{\mu_{SL}} \tau(t). \quad (14)$$

The total ice velocity and displacement in each column are simply the sum of the deformational and sliding components:

$$v(t) = v(h(0),t) + v_{SL}(t) \quad (15)$$

$$s(t) = s(h(0),t) + s_{SL}(t). \quad (16)$$

Although the assumption that the lubricating slurry behaves as a viscous fluid is questionable, this sliding law formulation incorporates several important details: (1) rapid sliding is allowed at high drainage system pressures, (2) strong ice-bed coupling is provided at low system pressures, and (3) the full shear stress $\tau(t)$ acting upon a column is transferred to the bed at all times, as required by continuum mechanics.

### 4.5. Ice Dynamics Model and Basal Shear Stress

[23] For a glacier with thickness $H$ and surface slope $\theta$, the driving stress acting on the bed is $\tau_0 = \rho g H \sin \theta$. The local shear stress can vary from this nominal value if basal conditions favor the transfer of stress to or from neighboring regions. Because subglacial water pressure strongly influences both sediment strength and basal sliding, diurnal pressure variations can drive a cyclic transfer of shear stress between regions of the bed. Although the transfer of shear stress between basal regions can result in unequal stress distribution, the area-averaged shear stress is equal to the nominal value. In our three-column representation, this is expressed

$$\alpha_1 \tau_1 + \alpha_2 \tau_2 + \alpha_3 \tau_3 = \tau_0, \quad (17)$$

where $\alpha_1$, $\alpha_2$ and $\alpha_3$ are assumed fixed. It follows that

$$\alpha_1 \frac{\partial}{\partial t} \tau_1 + \alpha_2 \frac{\partial}{\partial t} \tau_2 + \alpha_3 \frac{\partial}{\partial t} \tau_3 = 0. \quad (18)$$

[24] A rigorous treatment of the transfer of stresses between regions would require computation of the full three-dimensional stress and velocity fields within the ice. Although such algorithms have been developed [e.g., Blatter, 1995], they add greatly to the computational complexity and require knowledge of the distribution of basal stresses. We instead develop a simple ice dynamics model based on the work of Fischer and Clarke [1997] that allows investigation of the effects of stress bridging while retaining computational simplicity. We assume that the columns representing the different basal regions are located across slope (i.e., aligned perpendicularly to the flow direction). Soft-bedded regions are assumed to be separated from each other by a characteristic distance $L_S$; hard-bedded and soft-bedded regions are separated by distance $L_H$. This geometry allows us to calculate stress...
bridging in terms of simple shear of the ice between columns. Ice is treated as a nonlinear Maxwell material:

\[ \dot{\varepsilon} = \frac{1}{2G_f} \tau + A_0 \tau^N. \]  

(19)

The short-term elastic response is determined by the rigidity \( G_f \) and, for \( N = 3 \), the material deforms over long timescales according to Glen’s law [Glen, 1958]. If we designate the shear stress transferred from column \( b \) to column \( a \) as \( \tau_{b \rightarrow a} \), rearranging (19) yields

\[ \dot{\tau}_{b \rightarrow a} = 2G_f (\dot{\varepsilon}_{b \rightarrow a} - A_0 \tau_{b \rightarrow a}^N). \]  

(20)

Assuming uniform simple shear of the ice between columns gives

\[ \dot{\varepsilon}_{b \rightarrow a} = \dot{\varepsilon}_{xy} = \frac{1}{2} \frac{\partial \varepsilon_x}{\partial y} \frac{\Delta V}{2L}. \]  

(21)

where \( \Delta V = V_b - V_a \) is the difference in ice velocity between columns and \( L \) is the characteristic distance separating the columns. Depending on which pair of columns are being considered, \( L \) is replaced by either \( L_S \) or \( L_{HT} \). Substitution of (21) into (20) yields

\[ \dot{\tau}_{b \rightarrow a} = 2G_f \left( \frac{\Delta V}{2L} - A_0 \frac{\tau_{b \rightarrow a}^N}{2} \right). \]  

(22)

[25] The preceding discussion considers stress bridging between one pair of columns. Full accounting of stress bridging takes into account the transfers between each pair of columns and the relative areal coverage of each column in a manner that satisfies equation (18):

\[ \frac{\partial}{\partial t} \tau_1 = \dot{\tau}_{2 \rightarrow 1} + \dot{\tau}_{3 \rightarrow 1}, \]

\[ \frac{\partial}{\partial t} \tau_2 = -\frac{\alpha_1}{\alpha_2} \tau_{2 \rightarrow 1} + \dot{\tau}_{3 \rightarrow 2}, \]

\[ \frac{\partial}{\partial t} \tau_3 = -\frac{\alpha_1}{\alpha_3} \tau_{3 \rightarrow 1} - \frac{\alpha_2}{\alpha_3} \tau_{3 \rightarrow 2}. \]  

(23)

Values for \( \dot{\tau}_{2 \rightarrow 1}, \dot{\tau}_{3 \rightarrow 1} \) and \( \dot{\tau}_{3 \rightarrow 2} \) are calculated by substituting appropriate values for velocity contrast \( \Delta V \), separation distance \( L \) and transferred stress \( \tau_{b \rightarrow a} \) for each pair of columns into (22). The shear stress acting upon the till in a given region is assumed constant throughout the layer (i.e., no \( z \) variation; Alley [1989] has called this the “thin till approximation”). For simplicity we further neglect any vertical velocity gradient in the ice, and flow mechanisms such as enhanced creep of the basal ice, regelation, and ploughing are ignored.

[26] This ice dynamics model assumes uniform simple shear of ice between basal regions, and neglects any changes in the vertical velocity profiles within the ice column. Blatter et al. [1998] modeled the stress and velocity conditions in the vicinity of an isolated sliding spot in an otherwise homogeneous nonsliding slab. Their modeling results suggest that the surface velocity distribution would vary smoothly over length scales equivalent to several ice thicknesses. The assumption of uniform simple shear between columns is therefore likely a reasonable simplification.

4.6. Synthetic Instrument Responses

[27] Instrument responses are calculated from the modeled pore water pressure, till deformation, glacier sliding, and till strength conditions. Because all of the records shown in Figure 2 were recorded by instruments installed in a hydraulically connected region of the bed, and because no suitable records exist for the unconnected region, comparison between synthetic and field instrument responses will be limited to the connected region of the bed. Modeled instruments include pressure transducers, ploughmeters, bed tilt sensors, and slidometers. In the field, these instruments are usually installed ~0.05–0.25 m into the subglacial till depending on instrument type and local bed properties. The true depth of installation is uncertain, however, as (1) hot water drilling disturbs the sediments at the base of the borehole and (2) it is possible for soft subglacial sediments to ooze up into the borehole. Because variations in installation depth influence the instrument response, we calculate instrument responses for a range of installation depths.

4.6.1. Bed Tilt

[28] Assuming simple shear of the sediments, the average strain rate in the depth range occupied by a tilt cell is given by

\[ \dot{\varepsilon} = \frac{1}{2} \frac{\partial \tan \theta_{BR}}{\partial z} \]  

(24)

We assume that the tilt cell is advected passively with the deforming till. The bottom of the instrument is thus located at \([x_b(t), z_b(t)] = [s(Z_0, t), Z_0] \), where \( Z_0 \) is the installation depth (i.e., the vertical position of the tip of the tilt cell within the till column). Because the tilt cell has a fixed length \( l_{BT} \), the position of the tilt cell top is determined by finding the point on the calculated deformation profile \( s(Z, t) \) with the straight line distance \( l_{BT} \) from \([x_b(t), z_b(t)] \) (Figure 3b). The tilt angle is given by

\[ \theta_{BR} = \tan^{-1} \left( \frac{z_t - z_b}{x_t - x_b} \right). \]  

(25)

Because the Eulerian coordinates \( z_b \) and \( x_t \) used in determining the tilt angle are calculated from the Lagrangian representation of the till matrix, rotation of the device due to dilatant expansion and contraction of the till is incorporated into (25).

4.6.2. Pressure Transducer

[29] A small pressure transducer can be incorporated into the tip of the tilt cell housing, allowing simultaneous measurement of basal deformation and pore water pressure. The pore water pressure is thus

\[ p_{pZ}(t) = p(Z_0, t), \]  

(26)

where \( Z_0 \) is the installation depth of the tilt cell.

4.6.3. Slidometer

[30] Ideally, the displacement measured by a slidometer would result solely from glacier sliding and would be unaffected by basal deformation. However, because it is
necessary to firmly affix the line to the bed, the anchor is placed some distance into the till. The displacement signal is therefore contaminated by any sediment deformation that occurs above the anchor placement. Assuming that the connecting string does not cut through the till and no slippage of the anchor occurs, the total displacement indicated by the sidrometer is

\[
x_{SL}(t) = s_{SL}(t) + \left\{ \int_{Z_0}^{h(0)} J(t) \left[ 1 + \left( \frac{\partial x(Z, t)}{\partial Z} \right)^2 \right]^{\frac{1}{2}} dZ \right\} - [h(0) - Z_0],
\]

where \(Z_0\) is the installation depth of the anchor. The true sliding distance is \(s_{SL}(t)\) and the bracketed term on the right-hand side of is the component contributed by sediment deformation.

4.6.4. Ploughmeter

[31] Following Fischer and Clarke [1994], the bending moment measured by the strain gauges is calculated using one of two relations. For flow laws that have linear or nearly linear dependencies on \(\tau\), the force per unit length is calculated as

\[
F' = \frac{4\pi\eta(Z, t)\Delta V(Z, t)}{\ln(2\bar{a}/r_{PL})} + 0.5
\]

[Batchelor, 1970; Cox, 1970; Tillett, 1970]. Here \(r_{PL} = 0.015\) m is the ploughmeter radius and \(\bar{a}\) is the ploughmeter tip-to-gauge center distance. The force acting upon the ploughmeter by a viscous fluid is proportional to both the velocity contrast \(\Delta V(Z, t) = V(t) - v(Z, t)\) between the plough meter and sediments at depth \(Z\) and the effective viscosity \(\eta\) at that depth, given by

\[
\eta(Z, t) = \frac{1}{2\bar{a}(Z, t)}\tau(t).
\]

This term incorporates any stress- and pressure-dependent nonlinearities in flow; for linear-viscous tills \(\eta(Z, t) = \eta_0\).

[32] The total force per unit length acting on a ploughmeter moving through a Coulomb-plastic till is

\[
F'(Z, t) = 4r_{PL}(2 + \pi)\sigma Y(Z, t)
\]

[Humphrey et al., 1993; Fischer and Clarke, 1994], where the yield stress \(\sigma_Y\) is given by the Mohr-Coulomb failure criterion (4). Ignoring changes in pore water pressure resulting from motion of the ploughmeter through the sediments, the yield strength of Coulomb-plastic till is independent of the strain rate. The velocity contrast \(\Delta V(Z, t)\) between the ploughmeter and till therefore does not appear in (30). (The “excess pressure effect” [Iverson, 1999], which has been observed in ploughmeter records at Unteraragletcher [Rousselot and Fischer, 2005], is likely small compared to the observed diurnal pressure variations of Figure 2a and is therefore ignored.)

[33] Because ploughmeter calibrations are typically performed by hanging objects of known weight from the tip, calibrations (and subsequent field records) are commonly expressed in units of force (kN) rather than of bending moment (Nm). To allow direct comparison with field records, we divide the calculated bending moment \(M\) by the tip-to-gauge center distance \(l_g\):

\[
F = \frac{M}{l_g} = \int_{Z_0}^{h(0)} \left[ h_g - Z \right] F'(Z) dZ.
\]

where \(F'(Z)\) is the force per unit length applied between \(Z\) and \(Z + dZ\).

[34] The total force acting upon a ploughmeter due to motion through a till is given by substituting (28) for linear-viscous or nonlinear-viscous tills or (30) for Coulomb-plastic tills into (31). For nonlinear-Bingham tills, this substitution depends on the value of the shear stress. At stresses below \(\sigma_Y\) elastic behavior dominates, and we assume that the force acting on the plough meter is given by equation (30). At high basal shear stresses, visco-elastic behavior dominates, and (28) is used. At intermediate shear stress values, till behavior is neither truly elastic nor truly viscous. For given values of \(a\) and \(\sigma_Y\) in (3), the minimum effective viscosity occurs at \(\sigma_{min} = \sigma_Y(1 - a)\). In the range of values \(\sigma_Y < \tau < \sigma_{min}\) (Figure 4c, shaded region), the bending moment is therefore taken to be the greater of the values calculated using (28) and (30).

5. Model Parameters

[35] In this section we will discuss the choice of values for the physical constants, boundary conditions, and initial conditions required by the model. Values chosen for the model are based, where possible, on field or laboratory measurements of the physical properties of ice, water, and subglacial sediment and represent our best guess at the conditions present in the subglacial environment. Although the number of parameters in the model is large, we consider only a small subset of these parameters to be “tunable” (see below). The general characteristics of the modeling results presented in the following sections are not closely tied to any particular combination of model parameter values; sensitivity tests of the model [Kavanaugh, 2000] show that they are retained through a wide range of reasonable parameter values.

5.1. Physical Constants and Till Flow Law Parameters

[36] Table 1 shows values for physical constants used in the model. Values used for parameters \(G_I\) and \(A_0\) are appropriate for glacier ice at \(-5^\circ\)C, which represents an approximate depth-averaged temperature value for Trapridge Glacier. In the ice dynamics model developed above, it is the quantity \(A_D L\), where \(L\) is the characteristic length scale between columns, that determines ice behavior. We will thus consider \(A_D L\) fixed and take \(L\) as the adjustable parameter.

[37] The geotechnical properties of Trapridge Glacier till are not well known. Size distribution analysis of a sample of recently exposed basal till from Trapridge Glacier showed that fine-grained solids (silt- and clay-sized particles) account for roughly 40% of the solid volume [Clarke, 1987]. Other quantities, such as the in situ porosity, compressibility, permeability and shear strength, are poorly constrained. We take parameter values pertaining to these
properties from previous instrument studies at Trarbridge Glacier or from geotechnical tests of tills obtained from other locations.

[38] Assumed till property parameter values used in the model are shown in Table 1. The value chosen for the mineral density \( \rho_s = 2800 \text{ kg m}^{-3} \) is representative of the mix of metamorphic and igneous materials found beneath Trarbridge Glacier. Reference values chosen for the normal consolidation void ratios \( e_0 \) and \( e_1 \) and for the critical state reference void ratio \( e_{CS} \) are slightly lower than those used by Clarke [1987] and are likely more representative of Trarbridge till, yielding initial porosities \( \phi_0 \) and \( \phi_1 \) (corresponding to a clay-sized particle with grain diameter 1.2 \( \mu \text{m} \)) results in realistic values for till. The rate of dilatant till expansion or contraction is determined by the scaling factor \( D_0 = 0.20 \); this value results in a time constant of 1/5s.

[39] Table 2 shows the assumed parameter values for the four modeled till flow laws. The viscosity chosen for the linear-viscous and nonlinear-Bingham flow laws, we adopt the values for \( B_2 \), \( a \) and \( b \) in (2) and (3) proposed by Boulton and Hindmarsh [1987], but use a value for \( B_1 \) in (2) that is approximately 30 times greater than that proposed by Boulton and Hindmarsh [1987] in order to produce deformation rates closer to those measured at Trarbridge Glacier.

Both nonlinear-Bingham and Coulomb-plastic tills deform only at shear stresses greater than a yield stress determined by the Mohr-Coulomb failure criterion. To date, no shear strength measurements have been performed on Trarbridge Glacier till, and thus we use values for \( c_0 \) and \( \phi \) measured for a clay-rich till by Iverson et al. [1998].

Shear failure of an ideal plastic material would be localized to an infinitely thin layer. However, because till is composed of particles with sizes ranging from clays (diameter <1 \( \mu \text{m} \)) to boulders (diameter >1 m), till deformation will be distributed over a layer with finite thickness. The values we have chosen for the reference strain rate \( \dot{\varepsilon}_0 = 1 \times 10^{-4} \text{s}^{-1} \) and failure range \( \Delta \tau = 5 \text{ kPa} \) in (6) result in deformation that is distributed over a layer up to 0.24 m in thickness, comparable in scale to the diameter of the largest commonly occurring clasts.

5.2. Boundary Conditions and Initial Conditions

Table 2 lists values prescribed to boundary and initial conditions for the model results presented in the following sections. The main tuning parameters in the model include the water pressure at the base of the till layer \( p_{B} \), till layer thickness \( h \), length scales \( L_H \) and \( L_S \), areal fractions \( \alpha_t \) and glacier sliding reference pressure \( p_{GL} \). The effects of varying these key parameters are discussed by Kavannaugh [2000].
Values for ice thickness $H = 70.0 \text{ m}$ and surface slope $\theta = 7.0^\circ$ were chosen to represent the geometry of Trapridge Glacier, and the thickness of the deformable till layer is assumed to be $1.0 \text{ m}$ ($\alpha_1$). Appropriate values for areal fractions $\alpha_1$, $\alpha_2$ and $\alpha_3$ are not well established. In the study area, attempts to install instruments in the bed are usually successful, suggesting that the study area is largely soft bedded. Crevasse patterns in the vicinity of the study area suggest that resistance to flow could be provided mainly by marginal drag. We make the rough assumptions that (1) hydraulic convection occurs in 25% of soft-bedded regions and (2) hard-bedded and hydraulically connected regions cover approximately equal portions of the bed. These assumptions give nominal values $\alpha_1 = 0.20$, $\alpha_2 = 0.60$ and $\alpha_3 = 0.20$.

Water pressure values are prescribed at the ice-bed interface in the two soft-bedded columns. In order to model general summer behaviors at Trapridge Glacier, we choose pressure forcing functions for columns 1 and 2 that mimic typical summer mode pressure records in connected and unconnected regions, respectively. Hydraulic connection in column 1 is simulated by assigning a sinusoidally varying pressure function to the top of the till column:

$$p_1(h, t) = p_{T1} + \Delta p_{T1} \cos (\omega t).$$

The pressure $p_1(h, t)$ thus represents the drainage system pressure. Taking $p_{T1} = 490.0 \text{ kPa}$, $\Delta p_{T1} = 98.0 \text{ kPa}$ and $\omega = 2\pi \text{ d}^{-1}$ yields diurnally varying pressures ranging between 40.0 and 60.0 m of head, similar to the record for P1 (Figure 2a). Unconnected regions generally exhibit steady, near-flotation pressure values, and so the water pressure at the top of column 2 is $p_2(h, t) = p_{T2} = 588.0 \text{ kPa}$ (equivalent to 60.0 m of head).

The water pressure at the base of the till layer is poorly constrained. If the region beneath the till layer is relatively impermeable, $p_B$ would reflect a long-term average of pressures at the ice-bed interface. If the till layer is underlain by a highly permeable aquifer, however, $p_B$ may differ from this value depending upon the aquifer pressure. Studies by Stone [1993] suggest that Trapridge Glacier is underlain by an aquifer, and modeling work by Flowers and Clarke [2002] suggests that the aquifer is effective at removing water from the glacier bed. For the modeling scenarios presented here, we assign a nominal basal pore water pressure of $p_B = 294.0 \text{ kPa}$ (30.0 m of head) to both till columns. The initial pore water pressure profile within the till layer for each of the two soft-bedded columns is assumed to vary linearly with depth between the values prescribed at the top and bottom of the till layer:

$$p(Z, 0) = p_B + (p_T - p_B)(h - Z)/h.$$  

Void ratio (and therefore porosity) values for each column are initialized to the critical state value [see Clarke, 1987] given the assumed pressure profiles.

In the majority of the modeling scenarios presented here, we assume that glacier sliding occurs only in the hydraulically connected regions of the bed, even though modeled water pressures in the hydraulically unconnected region are near flotation. This assumption, which is valid if the volume of free water in unconnected regions is insufficient to submerge roughness elements of the bed, has been made primarily to prevent pervasive ice-bed decoupling. For a discussion of the consequences of relaxing this assumption, see Kavanaugh [2000].

The sliding parameters in (13) assigned to column 1, at $\Delta p_{SL} = 629.7 \text{ kPa}$ ($p_{SL}$) and $p_{SL} = 700 \text{ kPa}$ result in 20% decoupling at flotation pressure ($p = p_f$) and $\approx7\%$ decoupling at $p = 0.90 p_f$. This value of $p_{SL}$ was chosen to yield sliding velocities similar to those recorded by SL1 (Figure 2). We assume that the lubricating slurry has viscosity $\eta_{SL} = 2.0 \times 10^7 \text{ Pa s}$ (0.1% of the linear till viscosity) and a layer thickness of 0.01 m.

In nature, glacier motion over hard-bedded regions is due to some combination of creep flow, ice fracture and basal sliding. These mechanisms are ignored in the simple ice dynamics model employed here; ice above column 3 is assumed static and provides necessary resistance to glacier flow. The value $L_1 = 1500 \text{ m}$ chosen for the length scale of variation between hard- and soft-bedded regions yields flow rates comparable to those observed at Trapridge Glacier during the summer season ($\approx0.07-0.12 \text{ m d}^{-1}$). This value is unrealistic (as it exceeds the glacier width at the elevation of the study area), but accounts for the contribution to ice motion made by the mechanisms of enhanced creep, regelation and crevassing not directly incorporated into the ice dynamics model. Making the rough assumption that $L_2 \approx L_1/10$ yields $L_2 = 150.0 \text{ m}$.

Clearly, model results are strongly influenced by the values chosen for the model geometry, till flow law parameters, and boundary conditions. It is important to stress, however, that the modeling results presented in this study are representative of those obtained by using reasonable values for the model variables and are not the product of a high degree of tuning. Varying model parameters within a reasonable range (e.g., within the range of values suggested by the field or laboratory study from which a given value was drawn) yields responses that are similar in magnitude and phase (with respect to the modeled drainage system pressure) to the results presented in the following section. For a full discussion of the effects of varying model parameters, we refer the reader to Kavanaugh [2000]. This model has also been successfully applied to an investigation of subglacial conditions beneath Haut Glacier d’Arolla by U. H. Fischer et al. (Ice bed coupling during a glacier speedup event, submitted to Journal of Geophysical Research, 2005) with only minor adjustments made to the tuning parameters discussed above; the modeled glacier geometry and drainage system pressure functions were also changed to reflect conditions at Haut Glacier d’Arolla.

6. Modeling Results

6.1. Pore Water and Deformation Profiles
Figure 5. Modeled pore water pressure and deformation profiles. Modeled pore water pressure profiles in the connected region at (a) $t = 0.00$ d (peak system pressure), (b) $t = 0.25$ d, (c) $t = 0.50$ d (minimum system pressure), and (d) $t = 0.75$ d. Pressure value in shaded region above $Z = 1.0$ m represents flotation pressure. (e) Modeled pressure profile in unconnected region. Figures 5f–5y show modeled sediment deformation profiles. Profiles for connected region are shown at times corresponding to pressure profiles (Figures 5a–5d). Profiles in unconnected region are shown for $t = 0.00$ d (solid line) and $t = 0.50$ d (dashed line). Total ice velocity is shown in shaded region above $Z = 1.0$ m. (f–i) Connected region deformation profiles for linear-viscous till. (j) Deformation profiles in unconnected region at times of maximum (solid line) and minimum (dashed line) system pressure modeled for linear-viscous till. (k–o) Deformation profiles for nonlinear-viscous till. (p–t) Deformation profiles for nonlinear-Bingham till. (u–y) Deformation profiles for Coulomb-plastic till.
modeled drainage system pressure”. Figures 5a–5d show modeled pore water pressure profiles in the connected region at 0.25 d intervals; Figure 5e shows the pressure profile for the unconnected region. Pore water pressures at the time of maximum system pressure are shown in Figure 5a. Pressure values during decreasing system pressure are shown in Figure 5b; Figure 5d shows pressures increasing from the daily minimum of 40.0 m (shown in Figure 5c). The diurnal pressure signal diffuses into the till layer to a depth of \(0.5\) m. This depth is a function of (1) the permeability of the sediments, (2) the magnitude of pressure variation, and (3) the period of variation; a pressure variation with a larger magnitude or longer period would penetrate to a greater depth in a till with a similar permeability structure. Because the strength of nonlinear-viscous, nonlinear-Bingham, and Coulomb-plastic tills depends on the pore water pressure, the magnitude and period of pressure variations also control the depth to which deformation occurs for these flow laws.

Modeled till deformation profiles are shown in Figures 5f–5y. Linear-viscous (Figures 5f–5j) and nonlinear-viscous (Figures 5k–5o) tills both exhibit deformation over the full till layer thickness. Because the strength of linear-viscous till is independent of pore water pressure, uniform deformation is seen with depth at all pressures. In contrast, nonlinear-viscous till (Figures 5k–5o) shows increased deformation rates near the top of the till layer during times of high modeled system pressure.

Nonlinear-Bingham till (Figures 5p–5t) shows deformation that is limited to the upper portion of the till layer. Because pore water pressures generally decrease with depth in the till layer, the resulting high Mohr-Coulomb yield strengths prohibit deformation at depth. As a result, deformation is confined to the top \(0.4\) m of the till layer in the connected region and to the upper \(0.15\) m in the unconnected region.

Deformation of Coulomb-plastic till (Figures 5u–5y) occurs only where pore water pressures are greatest in the till column (or, more accurately, where effective pressures are lowest). This behavior results in a diurnal migration of the depth of active deformation. At the time of minimum system pressure, the deforming region is centered at a depth of \(0.27\) m (Figure 5w), corresponding to the depth of maximum pore water pressure (Figure 5c). Later, when the maximum pore water pressure is again found at the ice-bed interface (Figure 5x), deformation is confined to the top of the till layer.

Figure 6 shows total daily sediment deformation profiles in both the connected and unconnected regions for each flow law. Examination of the deformation profiles indicates that sediment deformation is increasingly concentrated toward the top of the till layer as the flow law nonlinearity increases. These profiles also provide an important cautionary message: If we compare the total daily deformation profile with the instantaneous deformation profiles for Coulomb-plastic till (Figures 5u–5x, respectively), we see that the strong strain concentration apparent Figures 5u–5x are obscured in Figure 6d. This suggests that long-term observations of sediment deformation profiles in regions where pore water pressures vary can result in an underestimation of flow nonlinearity. Similar viscous-flow-like deformation profiles have been modeled by Iverson and Iverson [2001] for Coulomb-plastic tills in the absence of pore water pressure variations.

6.2. Modeled Instrument Responses

6.2.1. Pressure Transducer

Records for field and modeled pressure transducers are shown in Figure 7. The pressure records for P1 (gray line) and PZ1 (solid line) are shown in Figure 7a; modeled responses shown in Figure 7b. The gray line in Figure 7b represents the prescribed pressure function at the ice-bed interface.
6.2.2. Tilt Cell

Too low. Suggest that modeled hydraulic permeabilities are slightly greater lags in the synthetic pore water pressure records reasonable agreement. Marginally lower amplitudes and estimated installation depth for PZ1 (0.15 m) show reasonably in the drainage system. Modeled and field records at the amplitude and lagged in phase in comparison with those in the till layer exhibiting variations that are smaller in similar to that between P1 and PZ1, with pressures within the till layer exhibiting variations that are smaller in amplitude and lagged in phase in comparison with those in the drainage system. Modeled and field records at the estimated installation depth for PZ1 (0.15 m) show reasonable agreement. Marginally lower amplitudes and greater lags in the synthetic pore water pressure records suggest that modeled hydraulic permeabilities are slightly too low.

6.2.2. Tilt Cell

Tilt cells [Blake et al., 1992] measure deformation of the subglacial till (Figures 3a and 3b). Upon installation the cell is oriented approximately vertically, and any deformation of till in the range of depth occupied by the sensor results in rotation of the device. Figure 8 shows till strain rates calculated from field (Figure 8a) and modeled (Figures 8b–8e) tilt cell records. Modeled responses are calculated for installation depths of 0.15 (solid line), 0.25 (short-dashed line) and 0.35 m (long-dashed line). All synthetic records indicate minimum deformation rates at times of high system pressure due to ice-bed decoupling during these times; this behavior is also seen in the field record for BT1 (Figure 8a). Linear-viscous till (Figure 8b) exhibits strain rates that are uniform over the full till depth, yielding identical responses for all three modeled installation depths. Although the form of the response for linear-viscous till is very similar to that of BT1, the indicated strain rates are an order of magnitude lower. Deformation rates for nonlinear-viscous till (Figure 8b) vary with installation depth, with higher strain rates seen nearer the top of the till layer. Till stiffening at low system pressures results in a slight reduction in deformation rate during these times. Indicated peak deformation rates are 6–14% those indicated by BT1. [57] No deformation is observed at depth during times of high system pressures for nonlinear-Bingham and Coulomb-plastic tills (Figures 8d and 8e). The stiffness of the Bingham till dictates that sliding dominates at these times; for Coulomb-plastic till, deformation is confined to a thin region above the till cell. The calculated strain rates for both records show strong dependence on installation depth, with peak strain rates occurring later with deeper installation. Peak deformation rates for Bingham till are ~4–10% of those indicated by BT1. Coulomb-plastic till exhibits higher deformation rates than the other flow laws, with maximum values that are 24–55% of those exhibited by BT1. The twin strain rate peaks at installation depths of 0.15 and 0.25 m arise from cyclic migration of the deforming region with system pressure variations. Slightly negative strain rates indicated in the responses for Bingham and Coulomb-plastic tills at peak system pressure is due to dilatant expansion of the till.

6.2.3. Slidometer

Slidometers [Blake et al., 1994] consist of a thin line, a spool (onto which the line is wound and that turns a small potentiometer as the glacier slides over the bed), and an anchor (which attaches the line to the bed; see Figures 3c and 3d). Figure 9 shows glacier sliding rates determined from field (Figure 9a) and modeled (Figures 9b–9e) slidometer records. Responses are calculated for installation depths of 0.10 (short-dashed line), 0.20 (long-dashed line) and 0.30 m (long- and short-dashed line), and the true modeled sliding rate is also shown (solid line). Modeled records show sliding rates and phase relations similar to those indicated during days 202.5–204.5 in the record for SL1 (Figure 9a). Low sediment deformation rates in the region above the anchor for linear-viscous, nonlinear-viscous and Bingham tills (Figures 9b–9d) yield indicated sliding rates that are close to the true sliding rates for all installation depths; deformation in the region above the anchor adds ~10% to the true value for these flow laws. Coulomb-plastic till exhibits significant deformation in the uppermost portion of the till layer at times of rising system pressure. As a result, indicated sliding rates are approximately double the true sliding rate (Figure 9e) and peak pressure head (m)

<table>
<thead>
<tr>
<th>Pressure head (m)</th>
<th>Day number 1996</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>202</td>
<td>60</td>
</tr>
<tr>
<td>203</td>
<td>60</td>
</tr>
<tr>
<td>204</td>
<td>60</td>
</tr>
<tr>
<td>205</td>
<td>60</td>
</tr>
<tr>
<td>206</td>
<td>60</td>
</tr>
<tr>
<td>207</td>
<td>60</td>
</tr>
</tbody>
</table>

Figure 7. Comparison of field and modeled pore-water pressure records. (a) Records for P1 (gray line) and PZ1 (solid line) during 20–25 July (days 202–207) 1996. Transducer PZ1 was installed at an estimated depth of 0.15 m. (b) Five-day pore water pressures modeled for linear-viscous till at depths of 0.15 (solid line), 0.25 (short-dashed line) and 0.35 m (long-dashed line). The gray line represents the drainage system pressure. Pore water pressures for other till flow laws are similar.
indicated sliding rates occur at times of increasing drainage system pressure. While this behavior is not seen in the record for SL1, it has been observed in other sliding records for Trapridge Glacier [Blake et al., 1994; Fischer and Clarke, 1997].

6.2.4. Ploughmeter

A ploughmeter [Fischer and Clarke, 1994] consists of a 1.54 m steel rod that is instrumented with strain gauges and is installed so that the tip is embedded ~0.1–0.2 m into the subglacial sediments (Figures 3e and 3f). Relative motion between the sediments and the ploughmeter imparts a bending moment on the steel rod that is measured by the strain gauges. Modeled ploughmeter records are shown in Figures 10b–10e, with the record for PL1 shown for comparison (Figure 10a). Ploughmeter responses are calculated for installation depths of 0.10 m (solid line), 0.20 m (short-dashed line) and 0.30 m (long-dashed line), with greater installation depths resulting in higher calculated force values. Figure 10b shows the modeled plough meter response for linear-viscous till. Force values are seen to be in-phase with system pressure (gray line). Because till properties do not vary with time, variations in the calculated force result solely from increased sliding velocities at times of high system pressure. Although forces in the record for PL1 (Figure 10a, solid line) are of similar magnitude to those shown in Figure 10b, peak modeled force values exhibit an in-phase relation with water pressure (gray line) as opposed to the out-of-phase relation seen in the records for PL1 and P1.

Maximum plough meter force values for nonlinear viscous till (Figure 10c) occur at times of rising system pressure. At this time, increasing sliding rates drag the ploughmeter through till that is relatively stiff. Diffusion of high pressures down into the till layer results in till softening and reduced plough meter force values, with softening occurring at later times for greater installation depths. Stiffening of the till following peak system pressure gives rise to the small increase noted in the 0.10 m response (solid line). Maximum force values range between ~2 and ~6 times those indicated by PL1. Ploughmeter responses for Bingham till (Figure 10d) exhibit similar characteristics to those for nonlinear viscous till. Neither till flow law reproduces the observed phase relationship between ploughmeter force values PL1 and PL1 (Figure 10a). Maximum force values calculated for Bingham till are ~2–4 times those seen in the record of PL1.

Figure 8. Comparison of till strain rates calculated from field and modeled tilt cell records. (a) Strain rate record for BT1 (solid line) during 20–25 July (days 202–207) 1996. The pressure record for P1 (gray line) is shown for comparison. Tilt cell BT1 was installed at an estimated depth of 0.15 m. Figures 8b–8e show modeled 5-day strain rates for installation depths of 0.15 (solid line), 0.25 (short-dashed line), and 0.35 m (long-dashed line). Modeled drainage system pressure (gray line) shown for comparison. Modeled strain rates for (b) linear-viscous, (c) nonlinear-viscous, (d) nonlinear-Bingham, and (e) Coulomb-plastic tills.
Ploughmeter responses for Coulomb-plastic till (Figure 10e) are similar in both magnitude and phase to records for field instruments PL1 and P1 (Figure 10a). Maximum force values occur immediately following the time of minimum system pressure, and minimum force values lag peak pressure by $\frac{1}{24} - 1$ hours due to the low permeability of the till.

7. Discussion

In the following discussion we compare modeled instrument responses with field records in an effort to determine the till flow law that provides the best qualitative match to observed variations in hydrological and mechanical conditions.

Modeled strain rates for linear-viscous and nonlinear-viscous tills are an order of magnitude lower than those indicated by the field record for BT1 (Figures 8a–8c). This is a result of the fact that deformation of these tills occurs over the full till layer thickness (Figures 5f–5o). Thus, while the total deformation rates for these flow laws tills are relatively high, the strain rate at any given depth is low. For nonlinear-Bingham till, deformation is limited to the upper portion of the till layer (Figures 5p–5t), but the inherent stiffness of this flow law results in peak modeled strain rates that are just 4–10% of those indicated by the record for BT1 (Figure 8d). Peak modeled strain rates for Coulomb-plastic till are significantly greater than rates calculated for the other flow laws, measuring 24–55% of those indicated by BT1.

The record for BT1 indicates rotation throughout the diurnal pressure cycle. In this regard, the signal for linear-viscous till (Figure 8b) most closely matches the field record, though at strain rates an order of magnitude lower. Modeled tilt cell rotation for Coulomb-plastic till occurs only during times of low system pressure due to strain localization. A more realistic treatment of strain distribution for Coulomb-plastic till, taking into account grain-grain interactions that would distribute failure over a length scale related to the grain size distribution, could produce a deformation signal closer in character to that for a linear-viscous till (though at reduced strain rates). Strain rates indicated in the record for BT1 (Figure 8a) are too high to be representative of rates over a significant layer thickness.

Figure 9. Comparison of indicated sliding rates calculated from field and modeled slidometer records. (a) Indicated sliding rates for SL1 (solid line) during 20–25 July (days 202–207) 1996. Pressures reported by P1 (gray line) is shown for comparison. The anchor for slidometer SL1 was installed at an estimated depth of 0.12 m. Figures 9b–9e show modeled 5-day sliding rates for installation depths of 0.10 (short-dashed line), 0.20 (long-dashed line) and 0.30 m (short- and long-dashed line). The true sliding rate is shown as the solid line, with the modeled drainage system pressure (gray line) included for comparison. Indicated strain rates for (b) linear-viscous, (c) nonlinear-viscous, (d) nonlinear-Bingham, and (e) Coulomb-plastic tills.

Ploughmeter responses for Coulomb-plastic till (Figure 10e) are similar in both magnitude and phase to records for field instruments PL1 and P1 (Figure 10a). Maximum force values occur immediately following the time of minimum system pressure, and minimum force values lag peak pressure by $\sim 1$–4 hours due to the low permeability of the till.
and are likely indicative of localized deformation. It is important to note that none of the till flow law formulations used in this study incorporates any elastic response to changes in shear stress (that is, the sediment rigidity is considered infinite). Elastic behavior has been observed in till beneath Storglaciären \[Iverson, 1999\], and its inclusion in the model could change the character of tilt cell responses. The most significant changes would likely be observed for nonlinear-Bingham and Coulomb-plastic tills during times when stresses are below the yield stress $\tau_Y$; at modeled, no deformation occurs at these times.\[65\] Modeled slidometer records for all flow relations show similar character to that for SL1 (Figure 9) and are thus not diagnostic. In contrast, significant differences between flow laws are seen in the phase relationship between modeled ploughmeter force values and drainage system pressures. Because of these differences, comparison of field and modeled ploughmeter responses provides perhaps the clearest means of identifying the flow law that best describes till behavior. The magnitude of computed force values for linear-viscous till is in reasonable agreement with those for PL1 (Figures 10a and 10b), but the phase relation with system pressure is opposite to that for PL1. Synthetic ploughmeter responses for nonlinear-viscous and Bingham tills (Figures 10c and 10d) are approximately an order of magnitude larger than those for PL1. The ploughmeter force/system pressure phase characteristics also differ, with peak force values occurring at times of rising system pressure. Modeled ploughmeter responses for Coulomb-plastic till show responses that are similar to that exhibited by PL1, both in magnitude and phase with respect to the subglacial drainage system pressure.\[66\] These comparisons of field and modeled responses for tilt cells, slidometers and ploughmeters indicate that of the four flow laws investigated, till behavior is best represented by Coulomb-plastic failure. Of the instruments used in this determination, the ploughmeter is perhaps the most diagnostic indicator of till behavior. Only Coulomb-plastic till reproduces the out-of-phase relationship with drainage system pressure commonly observed in field records from Trapridge Glacier. Over the range of model parameter and boundary condition values that we have investigated, no combination has produced the observed phase relationship for any of the three other flow laws. In contrast, this phase relationship is a robust feature of Coulomb-plastic till, resulting directly from the rate independence of such materials.

Figure 10. Comparison of field and modeled ploughmeter records. (a) Force record for PL1 (solid line) during 20–25 July (days 202–207) 1996. The pressure record for P1 (gray line) shown for comparison. Ploughmeter PL1 was installed at an estimated depth of 0.14 m. Figures 10b–10e show modeled 5-day ploughmeter records for installation depths of 0.10 (solid line), 0.20 (short-dashed line) and 0.30 m (long-dashed line). Modeled drainage system pressure (gray line) shown for comparison. Modeled force records for (b) linear-viscous, (c) nonlinear-viscous, (d) nonlinear-Bingham, and (e) Coulomb-plastic tills.
Features in the records for PL1 and SL1 (Figures 2c and 2d) further suggest rate-independent behavior. The record for SL1 (Figure 2a) shows two distinct behaviors: during days 202–204 the sliding rate shows a strongly in-phase response to the diurnal drainage system pressure variations, while during days 205–207 the sliding rate varies on an approximately semidiurnal timescale. The record for PL1, which was installed <1 m away, does not reflect the change in sliding rate behavior. Similar independence between ploughmeter force values and glacier flow rate has been observed at Storglacièreen [Hooke et al., 1997]. In addition, antiphase relationships between ploughmeter force records and subglacial drainage system pressures have been previously observed at Trapridge Glacier [Fischer and Clarke, 1994], Storglaciereen [Fischer et al., 1998], and Unterargaletcher [Fischer et al., 2001]. These characteristics strongly suggest Coulomb-plastic behavior.

It is likely that the simple ice dynamics model used in these studies greatly oversimplifies the nature of the interactions between various regions of the glacier bed. The across-glacier arrangement of the columns assumed in this model approximates the situation in which boundaries between the basal regions are aligned parallel to the direction of glacier flow. The similarities between field instrument records and modeled instrument responses suggest that this approximation is acceptable for this region of Trapridge Glacier.

8. Conclusions

We have developed a hydromechanical model of the processes that govern basal motion of a soft-bedded alpine glacier. The simple three-column representation of the glacier bed has allowed us to model the dynamic response of a glacier to diurnal variations in subglacial drainage system pressure. Calculation of pore water pressure, sediment deformation and sediment stiffness profiles, along with glacier sliding, permits us to model the responses of several commonly used subglacial instruments. Comparison of modeled responses to field instrument records from Trapridge Glacier allows us to determine which flow law best describes typical instrument responses.

Of the four tested rheological models, modeled instrument responses for Coulomb-plastic till yield the best qualitative match to field instrument records from Trapridge Glacier. Synthetic slidometer and ploughmeter responses show good agreement in both magnitude and phase with field responses, and the modeled tilt cell responses are closest in magnitude to the field records. Modeled responses for linear-viscous, nonlinear-viscous and nonlinear-Bingham tills fail to produce the observed phase relationship between system water pressure and ploughmeter force values, and till deformation rates for these laws are an order magnitude lower than those indicated by the field record.

For the chosen values of till thickness and basal pore water pressure, Coulomb-plastic deformation occurs to a depth of 0.35 m. This value shows good agreement with the deformation depth of ~0.3 m estimated at Trapridge Glacier by Blake [1992]. Furthermore, the close agreement between field and modeled ploughmeter responses for Coulomb-plastic till supports the assumption that till in the actively deforming layer is in a residual state. Although at any one time deformation of Coulomb-plastic till can occur over a narrow band, migration of the actively deforming zone with cyclic pressure variations leads to time-integrated deformation profiles that imply nearly linear flow behavior. This demonstrates that caution must be exercised when determining flow law parameters from long-term deformation profiles. Similarly, deformation in the uppermost portion of the till can lead to overestimation of the contribution of sliding to basal motion. This is especially true for Coulomb-plastic tills, which exhibit strong strain localization.

Appendix A: Development of Pore Water Pressure Evolution Equation

A1. Till Porosity and Dilatancy

Development of the till porosity and dilatancy relations used in the model directly follows that of Clarke [1987]. In particular, we direct the reader’s attention to the discussions of sediment compressibility and shear deformation and to equations (29)–(45) of that paper.

In addition to the pore water pressure-driven changes in void ratio described by Clarke [1987], we wish to consider void ratio changes due to the reorganization of sediment grains upon shear deformation (or “dilatancy”); We assume the rate of expansion or contraction to be proportional to both the strain rate in the soil and the difference between actual and critical state void ratios (as described by Clarke [1987]):

$$\dot{e}_D(z) = -D_0 \dot{e}(e - e_{CS}). \quad (A1)$$

Here $\dot{e}_D$ is the rate of change in void ratio due to dilatant reorganization and $D_0$ is a scaling factor. Differentiating equations (38) and (41) of Clarke [1987] and adding the above relation (A1) (noting that $\partial p/\partial t = -\partial p/\partial t$ for constant values of $p_l$) yields

$$\frac{\partial \dot{e}}{\partial t} = \left\{ \begin{array}{ll}
\frac{1}{B_l(p' + p'_l)} p - D_0 \dot{e}(e - e_{CS}) & \text{left of NCL} \\
\frac{1}{B_l(p' + p'_l)} p - D_0 \dot{e}(e - e_{CS}) & \text{on NCL} 
\end{array} \right. \quad (A2)$$

(with the Normal Consolidation Line, or NCL described by equation (37) of Clarke [1987]). Equation (8) follows from (A2) if we note that

$$n = \frac{e}{1 + e} \quad (A3)$$

and

$$\frac{\partial n}{\partial t} = \frac{1}{(1 + e)^2} \frac{\partial e}{\partial t} \quad (A4)$$

A2. Hydraulic Permeability

The porosity $n$ of a soil gives an averaged measure of the packing of grains in the solid matrix. Because tight packing of the grains results in highly restricted water passageways, low-porosity soil will exhibit a low hydraulic permeability $k$. Much effort has been made to determine the
relationship between the porosity and permeability of a soil. Perhaps best known is the Kozeny-Carman relation [Carman, 1961]:

\[ \kappa = \frac{n^3}{5(1-n)^2 S_0} \]  

(A5)

The term \( S_0 \) is the solid surface area per unit volume; the factor of 1/5 is that suggested by Carman. This relation has yielded reasonable estimates of porosity for clean sands [Bourbié et al., 1987], but its applicability to tills remains unclear.

### A3. Lagrangian Representation of Water Transport

The usual approach to modeling water flux through a porous medium is to cast in the Lagrangian, or spatial, description. This approach is appropriate for elastic aquifers that experience only small strains, in which case the distinction between unstrained and strained states can be neglected. For materials that can undergo large volume change, such as dilatant till, this simplification is inappropriate, and thus we use a Lagrangian representation of till. With this approach, spatial and temporal variability are expressed in terms of the initial conditions, so that \( p(X, t) \) describes the spatial and temporal distribution of pressure.

We assume that properties of the bed are uniform for some distance both up and down glacier from the location of the till column. Thus as deformation within the column moves sediments down glacier and therefore away from the one-dimensional column, those sediments are immediately replaced by sediments with identical properties. With this assumption, we can express \( p \) and other physical properties as functions of their vertical position \((Z, t)\) within the till column, where \( Z \) is the initial position of a particle in the column. The vertical position of such a particle changes with time in response to dilatation and compression, and thus a solid particle having initial position \( Z \) will at some later time be located at \( z \), where

\[ z(Z, t) = \int_0^Z \left( \frac{1-n(Z', 0)}{1-n(Z', t)} \right) dZ' \]  

(A6)

From (A6) the Jacobian of the transformation from \( Z \) to \( z \) is

\[ J(t) = \frac{\partial z}{\partial Z} = \frac{1-n(Z, t)}{1-n(Z, 0)} \]  

(A7)

If we make the usual assumption that solids are incompressible, the solid mass \( m_s \) per unit area of bed is given in the material description by

\[ m_s = \rho_s \int_0^{h(0)} (1-n(Z, t)) J(t) dZ. \]  

(A8)

The condition for conservation of solid mass is \( \frac{dm_s}{dt} = 0 \), which leads to the local form expression

\[ \frac{\partial}{\partial t} \left[ (1-n(Z, t)) J(t) \right] = 0. \]  

(A9)

Equation (A9) is an identity: solid mass is automatically conserved in the Z-t coordinate system. Thus in the Lagrangian representation the solid mass balance condition is automatically satisfied and the water balance condition is the one that leads to field equations. In the Lagrangian representation the water mass (per unit area of bed) is

\[ m_w = \int_0^{h(0)} \rho_w(Z, t) n(Z, t) J(t) dZ. \]  

(A10)

Within the region \( 0 \leq Z \leq h(0) \) water mass varies because of dilatation or compression of the till layer. This change in water mass is a consequence of the fact that water moves independently of the solid matrix. As the pore volume in a given region increases or decreases in response to dilatation or contraction of the solid matrix, water flows in or out of the region to accommodate the changes in pore volume. The change in total water mass for column of till is given by the difference in water mass flux \( \rho_w q_w \) between the upper and lower boundaries of the column \( Z = 0 \) and \( Z = h \); here \( q_w \) denotes the volume flux of water. Thus

\[ \frac{dm_w}{dt} = -\rho_w(h, t) q_w(h, t) + \rho_w(0, t) q_w(0, t) \]

\[ = - \int_0^{h(0)} \frac{\partial}{\partial Z} \left[ \rho_w(Z, t) q_w(Z, t) \right] dZ. \]  

(A11)

Equations (A10) and (A11) lead to the local form expression of water balance

\[ \frac{\partial}{\partial Z} \left[ \rho_w(Z, t) n(Z, t) J(t) \right] = - \frac{\partial}{\partial Z} \left[ \rho_w(Z, t) q_w(Z, t) \right]. \]  

(A12)

From (A12) it is clear that changes in pore volume due to dilatation or compression result in nonzero vertical water flux values. If we assume that water obeys the simple equation of state

\[ \rho_w(p) = \rho_w(p_0) \exp[\beta (p - p_0)], \]  

(A13)

we get

\[ \frac{\partial \rho_w}{\partial t} = \beta \rho_w \frac{\partial p}{\partial t}, \]  

(A14)

\[ \frac{\partial \rho_w}{\partial Z} = \beta \rho_w \frac{\partial p}{\partial Z}. \]  

(A15)

We assume that transport of water in the cross- or down-glacier directions is negligible compared to vertical flow driven by diurnal pressure variations. Water flux \( q_w \) in the column is assumed to be governed by Darcy’s law, which in the material description is given by

\[ q_w(Z, t) = \frac{\kappa(Z, t)}{\eta_w} \left[ \frac{1}{J(t)} \frac{\partial (p(Z, t))}{\partial Z} + \rho_w g \right]. \]  

(A16)

Expanding the left-hand side of (A12) and using relations (A7), (A14), and (34) of Clarke [1987] leads to

\[ \frac{\partial}{\partial t} \left[ \rho_w(Z, t) n(Z, t) J(t) \right] = \rho_w(Z, t) \left[ \alpha(p) + n(Z, t) \beta |J(t)| \frac{\partial}{\partial t} p(Z, t) \right]. \]  

(A17)
Expanding the right-hand side of (A12) and incorporating (A15) gives
\[
\frac{\partial}{\partial t} (\rho(Z,t)q(Z,t)) = -\rho(Z,t) \frac{\partial q(Z,t)}{\partial Z} \frac{\partial}{\partial t} (p(Z,t)) + \frac{\partial}{\partial Z} \rho(Z,t) q(Z,t) \cdot (A18)
\]
Equating (A17) and (A18), substituting the Lagrangian expression of Darcy’s law (A16), and rearranging yields the pressure evolution equation (7).


Liboutry, L. (1968), General theory of subglacial cavitation and sliding of temperate glaciers, J. Glaciol., 7(49), 21–58.


Mair, D., I. Willis, U. H. Fischer, B. Hubbard, P. Nienow, and A. Hubbard (2003), Hydrological controls on patterns of surface, internal and basal motion during three “spring events”: Haut Glacier d’Arolla, Switzerland, J. Glaciol., 49(167), 555–567.


G. K. C. Clarke, Department of Earth and Ocean Sciences, University of British Columbia, 6339 Stores Road, Vancouver, B. C., Canada V6T 1Z4. (clarke@eos.ubc.ca)

J. L. Kavanaugh, Department of Earth and Atmospheric Sciences, University of Alberta, 1-26 Earth Sciences Building, Edmonton, AB, Canada T6G 2E3. (jeff.kavanaugh@ualberta.ca)