Simulation of Vatnajökull ice cap dynamics

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[1] We apply a coupled model of ice sheet dynamics and subglacial hydrology to investigate the dynamics and future evolution of the Vatnajökull ice cap, Iceland. In this paper we describe a new theoretical approach to introducing longitudinal stress coupling in the ice dynamics solution, and we analyze our ability to simulate the main features of Vatnajökull, with and without longitudinal stress effects. Equilibrium ice cap configurations exist for Vatnajökull but under a narrow range of climatic boundary conditions. Equilibrium reconstructions have an average ice thickness greater than what is observed at Vatnajökull, consistent with our inability to capture surge dynamics in Vatnajökull’s outlet glaciers. Hydrological regulation of basal flow, longitudinal stress coupling, and a simple parameterization of the subglacial heat flux from Vatnajökull’s geothermal cauldrons all help to reduce average ice thickness in the equilibrium reconstructions, but cases that reproduce the present-day ice volume have an ice cap area that is 5–10% less than the actual ice cap. Present-day reconstructions that adopt a realistic climate spin-up for the period 1600–1990 provide improved fits to the modern-day ice cap geometry. This indicates that climatic disequilibrium also plays a significant role in dictating Vatnajökull’s morphology. Simulations for the period 1600–2300 illustrate that air temperature is the dominant control on Vatnajökull’s volume and area. Longitudinal stress coupling and hydrological coupling both increase Vatnajökull’s sensitivity to future warming.


1. Introduction

[2] We examine our ability to simulate the dynamics of the Vatnajökull ice cap, Iceland, in a three-dimensional ice sheet model based on the shallow ice approximation. This approximation of glacier dynamics is standard in continental-scale ice sheet modeling, but simplifications to the momentum balance in the shallow ice approximation are of questionable validity for Vatnajökull. Vatnajökull is a mesoscale ice complex (∼8000 km²) characterized by complex bedrock topography and high topographic and climatic gradients, requiring kilometer-scale model resolution to capture the important details of the ice cap’s surface morphology, outlet glaciers, and glaciological structure. Longitudinal stress coupling may be important in some regions of the ice cap at this resolution, as Vatnajökull’s average ice thickness is close to 400 m and longitudinal coupling effects are expected to extend 3–20 ice thicknesses [Budd, 1970; Kamb and Echelmeyer, 1986; Paterson, 1994, p. 264]. We address this by introducing longitudinal stress coupling as a perturbation to the shallow ice flow, using an approach adapted from Kamb and Echelmeyer [1986]. This paper summarizes our theoretical approach to longitudinal stress coupling and assesses our ability to capture the main features of Vatnajökull in steady state and transient simulations.

[3] The ice sheet model is coupled with the University of British Columbia (UBC) subglacial hydrology model [Flowers and Clarke, 2002; Flowers et al., 2003]. For these simulations the hydrological model simulates the exchange, storage, and transport of meltwater in the subglacial and groundwater systems. Basal flow in the ice cap is parameterized as a function of the spatially and temporally evolving subglacial water pressure, while the changing ice cap geometry feeds back on the hydrological simulation. A companion paper describes scenarios for the future evolution of the ice cap and implications for meltwater routing [Flowers et al., 2005].

[4] This paper examines whether we can improve model reconstructions of the present-day dynamics and geometry
of Vatnajökull through the introduction of more complex physical processes, such as longitudinal stress coupling and subglacial hydrological controls on basal flow. In addition to these increases in ice sheet model sophistication, we explore the impact of a simple parameterization of the primary geothermal cauldrons that underlie the Vatnajökull ice cap. Our standard model does not attempt to capture the influence of these features, although they are responsible for several m yr⁻¹ of basal melt and can be expected to influence both ice dynamics and subglacial hydrological conditions [Björnsson, 2002].

[3] We also examine the sensitivity of our Vatnajökull ice cap reconstructions to some of the assumptions and simplifications in our mass balance modeling. Equilibrium simulations are explored for a suite of sea level temperature scenarios, to test the range of temperature conditions that are compatible with the current ice sheet geometry. Precipitation fields over Vatnajökull are complex and a dynamical model of orographic precipitation is probably needed to capture their present-day spatial and seasonal patterns. Simulations presented here are based on observational precipitation patterns, although we explore the application of a multivariate statistical model of precipitation for Vatnajökull, to test the potential feedbacks of changing ice sheet model sophistication, we introduce the theory in a Cartesian reference frame. spherical (Earth) coordinates, but for notational simplicity and latitude

\[ \frac{\partial}{\partial \lambda} \]

several m yr⁻¹ of basal melt and can be expected to influence both ice dynamics and subglacial hydrological conditions [Björnsson, 2002].

[5] Glen’s flow law parameterizes this nonlinear rheologic behavior according to the constitutive relationship

\[ \dot{\varepsilon}_y = B(T) f(\sigma') \sigma'_y, \]  

where \( B(T) \) is a temperature-dependent viscosity coefficient,

\[ B(T) = B_0 \exp \left( -\frac{Q}{RT} \right). \] 

In this expression, \( Q \) is a creep activation energy, \( R \) is the ideal gas law constant, and \( B_0 \) is an empirically derived parameter [Paterson, 1994]. The nonlinearity in the constitutive relationship, equation (3), comes through a complex dependence on \( \sigma' \) [Glen, 1958],

\[ f(\sigma') = \Sigma^{(n-1)/2}, \]

where \( n \) is an empirically derived power law coefficient and \( \Sigma^{1/2} \) is the second invariant of the deviatoric stress tensor,

\[ \Sigma^{1/2} = -\frac{1}{2} \left( \sigma''_{ij} \sigma''_{ij} \right). \]

Collectively, the term \( B(T) \Sigma^{(n-1)/2} \) gives a measure of the ice “deformability,” essentially an inverse effective viscosity (1/\( \mu \) in (2)). The effective viscosity of ice is seen to vary as a function of the stress regime, such that ice is more deformable when it is under greater deviatoric stress. Under low-stress conditions, ice has a high effective viscosity and deformation is reduced. This stress softening leads to enhanced ice deformation in steep outlet valleys of an ice cap, relative to the low-stress region in the vicinity of ice divides. We return to this point in the discussion of longitudinal stress coupling below.

[6] Paterson [1994] discusses choices of \( B_0, n, \) and \( Q \) and gives recommended values for simulating glacier flow. Multiyear field measurements and modeling studies indicate that \( n = 3 \) provides a good description of ice deformation in the Vatnajökull ice cap [Adalgisdo´ttir et al., 2000; Adalgisdo´ttir, 2003]. Adalgisdo´ttir’s studies also provide exceptional constraint on the flow law parameter \( B_0 \) in equation (4). Since Vatnajökull is a temperate ice cap, believed to be at the pressure melting point throughout, \( B(T) \) in (4) becomes a well-constrained constant. Following Adalgisdo´ttir [2003], we adopt a reference value of \( B = 3.0 \times 10^{-5} \) Pa⁻³ yr⁻¹.

2. Ice Dynamics

[7] We examine the ice sheet modeling equations below for a generalized three-dimensional space coordinate \( x_n \), using summation convention to denote vector and tensor fields. In Cartesian space, \( x_n \) has components \((x, y, z)\), while on a spherical grid \( x_n \) has components \((\lambda, \theta, z)\), for longitude \( \lambda \) and latitude \( \theta \). We define the vertical coordinate \( z \) to be positive upward. Our ice dynamics model is couched in spherical (Earth) coordinates, but for notational simplicity we introduce the theory in a Cartesian reference frame. Define the ice surface topography \( h(x, y) \), bed topography \( h_b(x, y) \), and ice thickness \( H(x, y) = h(x, y) - h_b(x, y) \). The ice cap surface slope is calculated from \( \frac{\partial h}{\partial x} \), with \((x, y)\) components \( \partial h_b/\partial x \) and \( \partial h_b/\partial y \). In this section we present the governing equations for the general case of a three-dimensional, polythermal ice mass, where ice effective viscosity is temperature-dependent. Vatnajökull is isothermal, allowing significant simplifications for the specific application to Vatnajökull that follows in sections 3 to 5.

2.1. Ice Deformation

[8] Define the stress tensor \( \sigma_{ij} \), the strain rate tensor \( \dot{\varepsilon}_{ij} \), and the three-dimensional ice velocity field \( v_i \) with velocity components \((u, v, w)\). Strain rates in the ice are defined from

\[ \dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \] 

A linear viscous fluid deforms following

\[ \dot{\varepsilon}_{ij} = \frac{1}{\mu} \sigma_{ij}, \]  

where \( \mu \) is the material viscosity. Glacier ice deforms as a nonlinear or non-Newtonian fluid [Nye, 1952; Glen, 1955], as a function of the deviatoric stress \( \sigma'_y = \sigma_{ij} - \sigma_{kk} \sigma_{yy} \). Lab experiments and field studies both demonstrate that the deviatoric stress field drives ice deformation, with no dependence on the glaciostatic stress \( \sigma_{kk} \).

\[ \sigma'_y = B(T) f(\sigma') \sigma'_y, \]  

where \( B(T) \) is a temperature-dependent viscosity coefficient,

\[ B(T) = B_0 \exp \left( -\frac{Q}{RT} \right). \] 

In this expression, \( Q \) is a creep activation energy, \( R \) is the ideal gas law constant, and \( B_0 \) is an empirically derived parameter [Paterson, 1994]. The nonlinearity in the constitutive relationship, equation (3), comes through a complex dependence on \( \sigma' \) [Glen, 1958],

\[ f(\sigma') = \Sigma^{(n-1)/2}, \]

where \( n \) is an empirically derived power law coefficient and \( \Sigma^{1/2} \) is the second invariant of the deviatoric stress tensor,

\[ \Sigma^{1/2} = -\frac{1}{2} \left( \sigma''_{ij} \sigma''_{ij} \right). \]

Collectively, the term \( B(T) \Sigma^{(n-1)/2} \) gives a measure of the ice “deformability,” essentially an inverse effective viscosity (1/\( \mu \) in (2)). The effective viscosity of ice is seen to vary as a function of the stress regime, such that ice is more deformable when it is under greater deviatoric stress. Under low-stress conditions, ice has a high effective viscosity and deformation is reduced. This stress softening leads to enhanced ice deformation in steep outlet valleys of an ice cap, relative to the low-stress region in the vicinity of ice divides. We return to this point in the discussion of longitudinal stress coupling below.

[9] Glen’s flow law relates strain rates to deviatoric stresses in the ice,

\[ \dot{\varepsilon}_{ij} = B(T) \Sigma^{(n-1)/2} \sigma'_y. \]
With the assumption that vertical gradients in horizontal velocity are much larger than horizontal gradients of vertical velocity, vertical shear strain rates can be calculated from

\[
\frac{\partial u}{\partial z} \approx 2 B(T) \Sigma^2 \gamma(Z)^{\frac{1-\alpha}{2}} \sigma'_{zz},
\]

\[
\frac{\partial v}{\partial z} \approx 2 B(T) \Sigma^2 \gamma(Z)^{\frac{1-\alpha}{2}} \sigma'_{zz}.
\]  

(8)

Given knowledge of the stress field, these expressions can be vertically integrated to give a representation of horizontal ice velocities \(u(z)\) and \(v(z)\).

[12] Ice stresses are estimated from the Stokes’ flow representation of conservation of momentum in an ice mass, \(\partial_t \sigma_{ij} = \rho g_{ij}\), for ice density \(\rho\) and gravitational acceleration vector \(g = (0, 0, -g)\). Under the shallow ice or “zeroth-order” approximation for glacier flow [cf. Hutter, 1983], horizontal stress gradients are assumed to be negligible relative to vertical gradients of horizontal shear stress in ice mass [Nye, 1959, 1965, 1969]. This allows a reduction of the momentum balance equations to simple expressions for vertical shear stress as a function of local gravitational driving stress,

\[
\sigma'_{zz}(z) = -\rho g (h_s - z) \frac{\partial h_s}{\partial x},
\]

\[
\sigma'_{xz}(z) = -\rho g (h_s - z) \frac{\partial h_s}{\partial y}.
\]  

(9)

The assumptions in the shallow ice approximation permit the additional simplification

\[
\Sigma^2 \approx \sigma'_{zz}(z)^2 + \sigma'_{xz}(z)^2 = |\rho g (h_s - z)\left(\left(\frac{\partial h_s}{\partial x}\right)^2 + \left(\frac{\partial h_s}{\partial y}\right)^2\right)|.
\]

(10)

This gives expressions for the horizontal velocities in (7) as a straightforward function of local ice thickness and surface slope. Defining the slope

\[
\alpha_s = \left[\left(\frac{\partial h_s}{\partial x}\right)^2 + \left(\frac{\partial h_s}{\partial y}\right)^2\right]^{1/2},
\]

(11)

horizontal velocities can be calculated from a vertical integration of (7),

\[
u(z) = v_b + \frac{2B}{n+1} \rho g \alpha_s^{n-1} \left(\frac{\partial h_s}{\partial x}\right) \left[(h_s - z)^{n+1} - H^{n+1}\right],
\]

\[
u(z) = v_b + \frac{2B}{n+1} \rho g \alpha_s^{n-1} \left(\frac{\partial h_s}{\partial y}\right) \left[(h_s - z)^{n+1} - H^{n+1}\right].
\]  

(12)

Here \(u_b\) is the basal ice velocity associated with either decoupled sliding over the bed or subglacial sediment deformation. Vertically averaged velocities can be calculated from a vertical integration of (12),

\[
u = u_b - \frac{2B}{n+2} \rho g \alpha_s^{n-1} \left(\frac{\partial h_s}{\partial x}\right) H^{n+1},
\]

\[
u = v_b - \frac{2B}{n+2} \rho g \alpha_s^{n-1} \left(\frac{\partial h_s}{\partial y}\right) H^{n+1}.
\]  

(13)

[13] Basal flow is important at Vatnajökull, but is difficult to parameterize as the physical controls of both decoupled sliding and subglacial sediment deformation are poorly known. We use a simple relationship in this study, with basal velocity linearly proportional to vertical shear stress (equation (9)) at the bed, which we denote \(v_{b}\),

\[
u_b = -C_0 \gamma \sigma'_{zz}(h_b) = -C_0 \gamma \tau_{b,h},
\]

\[
u_b = -C_0 \gamma \sigma'_{xz}(h_b) = -C_0 \gamma \tau_{b,x}.
\]  

(14)

The parameter \(C_0\) is a constant, while \(\gamma(x, y, t)\) is a hydrological parameter that represents basal decoupling due to elevated subglacial water pressures. We apply a simple relationship to this study, with \(\gamma\) representing the flotation fraction: \(\gamma = p_w / p_t\), for subglacial water pressure \(p_w\) and glaciostatic ice pressure \(p_t = \rho g H\). Water pressure is internally calculated via a coupled hydrological model [Flowers et al., 2005]. This parameterization means that basal velocity in (14) is linearly proportional to the flotation fraction, with no basal flow when subglacial water pressure \(p_w = 0\).

[14] Vertically averaged ice velocities in (13) can be rewritten with (14) and in terms of the basal shear stress, \(\tau_{b,y}\),

\[
u = -C_0 \gamma \tau_{b,x} + \frac{2B}{n+2} \rho g \alpha_s^{n-1} H^n \tau_{b,x},
\]

\[
u = -C_0 \gamma \tau_{b,y} + \frac{2B}{n+2} \rho g \alpha_s^{n-1} H^n \tau_{b,y}.
\]  

(15)

Under the shallow ice approximation, basal shear stresses are taken from (9) and are equivalent to the gravitational driving stress at the base of the ice cap,

\[
\tau_{b,y} = \tau_{b,x} = -\rho g H \frac{\partial h_s}{\partial y}.
\]  

(16)

In this formulation, local gravitational driving stress and ice bed coupling (\(\gamma\)) are the only determinants of ice flux. Strain rates in the ice are therefore governed by local ice thickness, surface topography, and subglacial hydrological conditions, with no (instantaneous) influence from other sectors of the ice cap.

2.3. Ice Fluxes: Longitudinal Stress Coupling

[15] Exceptional progress has recently been made in the development of higher-order models of ice dynamics, which include the effects of longitudinal stress coupling and horizontal shear stresses in an ice mass [Blatter, 1995; Pattyn, 2002, 2003]. In contrast to a full high-order model of ice dynamics for Vatnajökull, we introduce longitudinal stress coupling as a perturbation to the shallow ice flow, essentially a two-dimensional analogue of the longitudinal stress theory developed by Kamb and Echelmeyer [1986].

[16] When longitudinal deviatoric stress gradients are retained in the momentum balance, the vertical shear stresses in (9) are adjusted to include a nonlocal stress coupling term [Budd, 1970; Kamb and Echelmeyer, 1986; Paterson, 1994, p. 263],

\[
\sigma_{zz}(z) = -\rho g (h_s - z) \frac{\partial h_s}{\partial x} + 2 \frac{\partial}{\partial x} \left(H \sigma'_{zz}\right),
\]

\[
\sigma_{xz}(z) = -\rho g (h_s - z) \frac{\partial h_s}{\partial y} + 2 \frac{\partial}{\partial y} \left(H \sigma'_{xz}\right).
\]  

(17)
Longitudinal deviatoric stress terms are vertically integrated in this expression,

\[
\sigma'_{xx} = \frac{1}{H} \int_z \sigma'_{xx} \, dz; \quad \sigma'_{yy} = \frac{1}{H} \int_z \sigma'_{yy} \, dz,
\]  

(18)

so the effect of these terms in (17) is to push or pull a local “column” of ice uniformly, as a result of upstream and downstream compressive or extensive flow.

Practical incorporation of (17) requires manipulation of the constitutive relationship for ice rheology. Glen’s flow law, as summarized in equations (3)–(6), can be inverted where \(E \) is the effective viscosity in Glen’s inverted flow law,

\[
A(T) = B(T)^{-1/a} E_z^{(1-a)/2a},
\]  

(20)

\( E_z \) is the second invariant of the strain rate tensor, calculated from local velocity gradients (including horizontal and vertical shear strains).

Using (19) and (20), the vertically averaged longitudinal deviatoric stresses can be calculated from

\[
\sigma'_{xx} = \frac{1}{H} \int_z A(T) \frac{\partial u}{\partial x} \, dz, \quad \sigma'_{yy} = \frac{1}{H} \int_z A(T) \frac{\partial v}{\partial y} \, dz.
\]  

(21)

In three-dimensional ice sheet modeling, \( E_z \) and \( B(T) \) are known at each vertical level and \((\sigma'_{xx}, \sigma'_{yy})\) can be estimated from (20) and (21) by numerical integration. For Vatnajökull, \( B(T) \) in (20) is a constant and it can be removed from the integral.

Kamb and Echelmeyer [1986] introduced longitudinal stress coupling effects through a splendid perturbation analysis where \((\sigma'_{xx}, \sigma'_{yy})\) terms in (17) are imposed on a background flow dominated by “standard” vertical shear deformation or basal sliding, governed by the gravitational driving stress. In this analysis, \( \Sigma'_2 \) in (7) can again be approximated from the dominant vertical shear stresses,

\[
\Sigma'_2 \approx |[\rho g (h_z - z) \alpha_z]|^2,
\]  

(22)

and expressions for horizontal shear velocity can be written

\[
\frac{\partial u}{\partial z} \approx 2B |\rho g (h_z - z) \alpha_z|^{n-1} \left[ -[\rho g (h_z - z) \frac{\partial h_z}{\partial x} + 2 \frac{\partial}{\partial x} (H \sigma'_{xx})] \right],
\]

\[
\frac{\partial v}{\partial z} \approx 2B |\rho g (h_z - z) \alpha_z|^{n-1} \left[ -[\rho g (h_z - z) \frac{\partial h_z}{\partial y} + 2 \frac{\partial}{\partial y} (H \sigma'_{yy})] \right].
\]  

(23)

Analogous with (12), integration of (23) yields expressions for the vertically averaged ice velocity associated with isothermal vertical shear deformation and parameterized basal flow, only with the addition of longitudinal stress coupling. This takes the identical form to (15) in our study,

\[
\tau_x = \left[ -C_0 \gamma + \frac{2B}{n+2} \rho g \alpha_z \gamma^{n-1} H^n \sigma'_{xx} \right],
\]

\[
\tau_y = \left[ -C_0 \gamma + \frac{2B}{n+2} \rho g \alpha_z \gamma^{n-1} H^n \sigma'_{yy} \right].
\]  

(24)

where the basal shear stresses \((\tau_x, \tau_y)\) are the vertical shear stresses in (17) evaluated at the ice sheet base,

\[
\tau_x = -\rho g H \frac{\partial h_z}{\partial x} + 2 \frac{\partial}{\partial x} (H \sigma'_{xx}),
\]

\[
\tau_y = -\rho g H \frac{\partial h_z}{\partial y} + 2 \frac{\partial}{\partial y} (H \sigma'_{yy}).
\]  

(25)

Our incorporation of longitudinal stress effects in the ice flux calculation reduces to a modification of basal shear stress in the spirit of Kamb and Echelmeyer, with longitudinal stress coupling introduced as a perturbation to the gravitational driving stress. The vertical integration in (20) represents the three-dimensional implementation of this strategy, and is generalizable to polythermal ice masses. Modification of basal velocity from sliding laws that incorporate basal shear stress proceeds in similar fashion, with nonlocal longitudinal stress coupling introduced as a linear perturbation to the gravitational driving stress.

Physically, our parameterization of longitudinal stress coupling modifies the local gravitational driving stress at a point to reflect the influence of extensive or compressive flow upstream or downstream of the point. Longitudinal coupling is proportional to the gradient of ice thickness and longitudinal deviatoric stress (hence longitudinal strain rates), and these nonlocal influences on glacier flow diminish with distance from the point. Kamb and Echelmeyer [1986] demonstrate that longitudinal stress coupling is dominated by stress gradients within four ice thicknesses in valley glaciers, and introduced an exponential weighting filter to describe the reduced influence of regional ice thickness, slope, and velocity gradients with distance. The zone of influence in ice sheets is believed to extend up to 20 ice thicknesses [Budd, 1970; Paterson, 1994, p. 264], or ~8 km for Vatnajökull, which has an average ice thickness of close to 400 m.

For Vatnajökull modeling, we incorporate far-field coupling effects from the deviatoric stress gradients \( \partial_i (H \sigma'_{xx}) \) and \( \partial_j (H \sigma'_{yy}) \) at all cells within a five-grid cell radius of the local point (~9 km). In practice, this involves superposition of the Cartesian \((x, y)\) projections of velocity gradients in several directions.

The two-dimensional analogue of Kamb-Echelmeyer can be interpreted as longitudinal coupling influences on local flow from all directions where extension or compression act. In effect, regional ice dynamics within a radius of influence, of order 10–20 ice thicknesses, exert pushes and pulls on a point, with the basal shear stress at the point modified from the local gravitational driving stress based on a weighted superposition of these coupling stresses. In the regular Cartesian grid, “off-axis” pushes and pulls can be...
expressed as projections onto a radial axis from each point, calculated as if in a cylindrical coordinate system (i.e., for an axisymmetric ice cap), \(2\partial_t (H \sigma_r)\). The horizontal stress coupling from each point can then be projected back onto the Cartesian grid for the solution of (24).

2.4. Isostatic Model

[24] The bed isostatic response to loading from ice and from subglacial and proglacial lakes is simulated through a local viscous response,

\[
\frac{\partial h_b}{\partial t} = \left( \frac{h_b - h_{b0}}{\tau} + \rho H + \rho_w \frac{\partial H_w}{\rho_b \tau} \right),
\]  

(26)

where \(h_{b0}\) is the equilibrium (unloaded) bed topography, \(\rho\), \(\rho_b\), and \(\rho_w\) are the densities of ice, bedrock, and water, and \(\partial H_w\) is the change in water layer or lake thickness associated with ponding in bedrock depressions. This becomes important during the retreat of Vatnajökull’s southern outlets, which exposes bedrock overdeepenings that fill in with water, forming proglacial lakes that delay the isostatic recovery. The exponential response time \(\tau\) in (26) is taken to be 210 years [Sigmundsson, 1991; Sigmundsson and Einarsdóttir, 1992], a rapid isostatic response associated with the low-viscosity mantle plume beneath Vatnajökull.

2.5. Model Numerics

[25] We simulate Vatnajökull ice dynamics on a spherical (latitude-longitude) finite difference grid, spanning the latitude range \(y \in 63.67°-65.33°\)N and the longitude range \(x \in 15.0°-18.33°\)W. Horizontal model resolution is \(\Delta y = 1\) arcmin (1/60°) and \(\Delta x = 2\) arcmin (1/30°), equivalent to 1.85 km by 1.55–1.64 km. The ice dynamics, subglacial hydrology, and isostatic rebound models and the climate input fields all share this grid. Vatnajökull never reaches the model boundary conditions in the simulations presented here (steady state and future climate change scenarios).

[26] Subglacial bed topography is calculated for the model grid from 200-m ice radar measurements of ice thickness and ice surface altitude over the entire ice cap [Björnsson, 1982, 1986; Björnsson and Einarsdóttir, 1993; Science Institute, University of Iceland, unpublished data, 2002]. Available measurements are averaged in each model grid cell.

[27] Ice dynamics are solved at time steps of 0.02 years, with isostatic solutions and climate field updates at 1-year time steps. The subglacial hydrology system is asynchronously coupled with the ice dynamics, with updates at 5-year time steps. The resulting flotation fraction patterns are assumed to hold for the next five years in the ice dynamics solution. Because Vatnajökull is isothermal, a vertically integrated, two-dimensional solution is used to accelerate the numerical solution. Since temperature is uniform, the temperature dependence of the ice effective viscosity is eliminated and the relationships derived above (e.g., equations (19)–(21)) are simplified. A three-dimensional discretization of ice cap velocity fields is used for the mass balance solution, however, to calculate internal melt (see equation (31)). We use a vertically stretched grid for the vertical discretization, with 20 vertical layers under an exponential grid transformation [Marshall et al., 2000].

3. Climatic Boundary Conditions

3.1. Ice Cap Mass Balance

[28] Given the equations governing ice cap velocities and fluxes, surface evolution can be simulated through the governing equation for conservation of mass,

\[
\frac{\partial H}{\partial t} = - \frac{\partial}{\partial x} (\bar{u}H) - \frac{\partial}{\partial y} (\bar{v}H) + b,
\]  

(27)

where \(\bar{b}(x, y, t)\) is the annual mass balance, calculated from the net annual accumulation (/\(\bar{a}\)) minus ablation (/\(\bar{m}\)) of ice over the ice column at point \((x, y)\),

\[
\bar{b} = \bar{a}_s - \bar{m}_s + \bar{a}_b - \bar{m}_b - \bar{m}_d.
\]  

(28)

Subscripts \(s\), \(b\), and \(d\) refer to surface, basal, and internal (deformational) terms in the net column mass balance. All mass balance terms have units of m yr\(^{-1}\) ice equivalent in this paper. At the surface of the ice sheet, ice is added to the system via snow accumulation (/\(\bar{a}_s\)), while mass is removed through surface melt, sublimation, the sensible heat flux associated with liquid precipitation, and calving on marine or proglacial lake margins. These combined mechanisms of mass loss are lumped into \(\bar{m}_s\). We prescribe calving fluxes to be proportional to water depth and ice thickness, following Marshall et al. [2000], while surface melt due to liquid precipitation is calculated from any rain that falls on the ice cap with a temperature greater than 0°C,

\[
m_p = \frac{c_w}{L} \left( P - \frac{\rho}{\rho_w} a_s \right) T_C,
\]  

(29)

where \(c_w\), \(L\), and \(\rho_w\) are the heat capacity, density, and latent heat of fusion of water, and \(T_C\) is the temperature in °C during a particular rain event. The amount of liquid precipitation in the event is calculated from the total precipitation, \(P\), minus the solid precipitation, \(a_s\), adjusted for water equivalence.

[28] At the ice sheet base, ice can be added or removed via basal refreezing (/\(\bar{a}_b\)) or melting (/\(\bar{m}_b\)). Basal melting is due to geothermal heat flux into the ice, \(Q_G\), and frictional heat generated by sliding over the bed,

\[
\bar{m}_b = \frac{1}{\rho_L} \left( Q_G + v_b\tau_{bg} \right).
\]  

(30)

Internal melting in temperate ice caps occurs due to deformational (strain) heating over a volume, and is concentrated near the base of the glacier where the stresses and strain rates are highest. For a unit area of the ice cap, the volume of a finite difference element at height \(z\) is characterized by finite difference cell height \(\Delta z\). This gives the strain heating per unit area \(\sigma_{ij}(z)\varepsilon_{ij}(z)\Delta z\), with the associated internal melt rates

\[
\bar{m}_d(z) = \frac{\Delta z}{\rho_L} \left[ \sigma_{ij}(z)\varepsilon_{ij}(z) \right].
\]  

(31)
We calculate total column internal melt from an integration of (31) over the ice column, and we assume that this meltwater is instantaneously transmitted to the basal water system.

[30] Geothermal heat flux to the base of the ice cap is prescribed based on available measurements, including active geothermal areas in several regions of western Vatnajökull, where \( Q_G \) reaches up to 50 W m\(^{-2}\) [Björnsson, 1988]. We carried out simulations with and without localized geothermal hot spots (\( Q_G = 50 \) W m\(^{-2}\) in the Grímsvötn and Skáftaf geothermal areas (Figure 1)). Following Flowers et al. [2003, 2005], a background geothermal heat flow value of 0.18 W m\(^{-2}\) was assigned for eastern Vatnajökull. Subsurface hydrothermal circulation and groundwater drainage beneath western Vatnajökull is believed to pump all of the geothermal heat away from the base of the ice cap (O. G. Flóvenz, personal communication, 2001), so we assume that the geothermal heat flux to the western part of the ice cap is confined to the geothermal hot spots.

### 3.2. Climate Coupling

[32] Degree day methodology is used to estimate the fraction of precipitation to fall as snow, \( a_s \), and the mean annual surface melt, \( m_s \) [Reeh, 1991; Jóhannesson et al., 1995]. We adopt degree day parameters for the melt modeling from Jóhannesson et al. [1995], which are in good agreement with parameters derived more recently for Vatnajökull [Gudmundsson et al., 2003]. This calculation requires the spatial distribution of mean annual air temperature, July air temperature, and total annual precipitation over the ice cap. Mass balance terms in (28) are estimated at 1-year time steps in the model, giving \( b(x, y) \) and allowing forward integration of the conservation of mass equation, (27), to predict ice cap evolution in response to changing climatic boundary conditions.

[33] Climate input fields in the modeling are described in detail by Flowers et al. [2005]. Modern-day, “reference” temperature fields over Vatnajökull are based on Gylfadóttir [2003] and Björnsson [2003], as constructed from available Icelandic Meteorological Office (IMO) station data for the period 1961–1990. These fields have been detrended for elevation, latitude, longitude and distance from coast and then spatially interpolated to give a representation of mean annual and mean July sea level temperatures for Iceland, \( T_{\text{msv}}(x, y, z = 0) \) and \( T_{\text{bif}}(x, y, z = 0) \). We bilinearly interpolated these sea level temperature fields onto the glaciological model grid. Screen level temperatures over the ice cap can then be estimated at each point from

\[
T_0(x, y, z = h_s) = T_0(x, y, z = 0) + \beta h_s(x, y),
\]

where \( \beta = -0.0053^\circ C \) m\(^{-1}\) after Jóhannesson et al. [1995]. We apply the same altitude correction in (32) to both mean annual and July temperatures, with \( h_s(t) \) varying in time in our experiments. Figure 2a plots mean annual air temperature over Vatnajökull for our 1961–1990 reference climatology.

[34] In addition to evolution of ice surface altitude, temperature fields evolve in time in the simulations through sea level climate change scenarios, as described below. Precipitation fields can also be expected to evolve as a function of both ice surface morphology and climate change, but we do not dynamically simulate precipitation in these simulations. For simplicity, most model experiments presented here assume a temporally fixed spatial precipitation pattern (Figure 2b), after Eythorsson and Sigtryggsson [1971]. Vatnajökull experiences high rates of precipitation, up to 4 m yr\(^{-1}\), on its southeastern flanks, with a notable precipitation shadow on its northern margins (\( P < 1 \) m yr\(^{-1}\)). We maintain this spatial pattern, but annual precipitation rates do vary in time in our simulations, as a function of temperature change relative to the reference 1961–1990 climatology. The precipitation sensitivity, \( dP/dT \), is taken as a free parameter in our study.

[35] This map of Eythorsson and Sigtryggsson [1971] is dated, but it is the most recent precipitation data available to us at the time of writing. A new precipitation map for Vatnajökull is in preparation (G. Adalgeirsdóttir, personal communication, 2005), and our studies would benefit from this updated information. However, our general approach to mass balance modeling based on fixed present-day precipitation patterns has severe limitations irrespective of the input precipitation map. As Vatnajökull grows or shrinks in response to different climate scenarios, the orographic forcing of precipitation over the ice cap would be heavily modified. These processes cannot be captured by a static or statistically based precipitation map, and physically based precipitation modeling [e.g., Hulton and Purves, 2000; Roe, 2002; Roe et al., 2003] is needed for improved mass balance estimates that reflect changing Vatnajökull surface geometry.

[36] We do not introduce a dynamical precipitation model here, but in an attempt to capture the potential feedbacks of

Figure 1. Map of the Vatnajökull ice cap in southeastern Iceland. The locations of Höfn, Akureyri, and Stykkishólmur are indicated on the inset map; we discuss long-term meteorological data from these three communities in sections 2 and 3. The locations of the Grímsvötn and Skáftaf geothermal hot spots are also indicated.
changing ice topography we experimented with multivariate statistical models of precipitation patterns for Vatnajökull. With precipitation as the dependent variable, we investigated statistical models for the entire region of southeast Iceland that falls within our model grid ($N = 10,706$) and for a reduced data set, restricted to just the present-day ice cap ($N = 2682$).

[36] Using stepwise multivariate regression, we explored a number of terrain variables expected to have an influence on precipitation, including distance from the nearest coastal point, altitude, latitude, surface slope, aspect, and a proxy for orographic uplift of air parcels under different prevailing air mass trajectories (e.g., increase in altitude for southeastern air masses, etc.). Independent variables with strong correlations ($r > 0.6$) were eliminated from the analysis, resulting in the emergence of five independent variables: altitude, $h$, surface slope, $s$, distance from coast, $d$, and slope components, $\partial h$s and $\partial_s h$. 

**Figure 2.** Steady state meteorological and mass balance fields over Vatnajökull for our reference climatology. (a) Air temperature, contour interval = 0.8°C. (b) Precipitation rates, contour interval = 0.3 m yr$^{-1}$. (c) Annual surface melt rates, contour interval = 0.7 m yr$^{-1}$. (d) Annual snow accumulation, contour interval = 0.26 m yr$^{-1}$. (e) Basal and internal melt rates, contour interval = 0.013 m yr$^{-1}$. (f) Annual mass balance, contour interval = 1 m yr$^{-1}$. Note that mass balance fields in Figures 2c–2f are represented as m yr$^{-1}$ ice equivalent.
Distance from coast, elevation, slope, and north-south slope gradients all emerge as potentially useful predictors of precipitation, with distance from coast explaining 64% of the variance in the precipitation on the ice cap. Improvements in this are achieved through stepwise multivariate regression, with distance from coast, elevation, and slope all significant at 99.9% in both the southeast Iceland and Vatnajökull-only regressions. $R^2_{adj} = 0.74$ and 0.82 in the two cases, respectively. This indicates that 74–82% of the variance in present-day precipitation rates in the region can be explained with these three variables.

A subset of experiments presented here includes spatially evolving precipitation patterns as the ice cap evolves, based on these regressions. Since we are interested in the effects of dramatic changes in surface geometry (e.g., precipitation fields in the absence of the ice cap), we choose the southeast Iceland regression as our favored model to capture these effects,

$$P(x, y, t) = [1.468 - 0.0192d(x, y) + 0.00175h_s(x, y, t) + 0.00435\alpha_s(x, y, t)] \text{m yr}^{-1}. \quad (33)$$

In this regression relationship, $d$ is measured in km and $h_s$ in m. Note that distance from coast is treated as temporally invariant here, as sea level change and coastal erosion/sedimentation changes are assumed to be negligible in the time window of interest in this study, at the ~1.5 km model resolution. In contrast, both elevation and slope evolve in time, with $dP/dh_s = 0.00175 \text{ yr}^{-1}$ and $dP/d\alpha_s = 0.00435 \text{ m yr}^{-1}$. Both of these relationships indicate increases in precipitation with slope and altitude, reflecting the influence of orographic forcing on air mass saturation.

### 4. Steady State Simulations

We first present Vatnajökull ice cap simulations derived from fixed (perpetual) climatic boundary conditions, allowing the ice cap to come to steady state for a given set of ice dynamics and mass balance parameters. Sea level climate is prescribed to reflect contemporary (1961–1990) air temperatures and precipitation rates over Vatnajökull, as described by Flowers et al. [2005]. Climatic fields (air temperature and precipitation) and mass balance fields at equilibrium for Vatnajökull are plotted in Figure 2, with values along a north-south transect at 17.2°W (through Skeidararjökull) shown in Figure 3.

The mass balance field over the ice cap is estimated from the sea level fields and the time-evolving surface topography, meaning that the ice cap mass balance itself is not prescribed or fixed. Rather, the ice cap and its surface climate/mass balance mutually evolve in a given steady state simulation. This approach precludes the guaranteed existence of an equilibrium solution, as positive mass balance feedbacks can lead to boundless growth or retreat of the ice cap. For instance, higher surface elevations lead to cooler temperatures, hence less surface melt, resulting in ice growth and further cooling. This simple feedback also operates in reverse when the ice cap thins. We witnessed this behavior during initial attempts to simulate a steady state ice configuration for Vatnajökull with freely evolving mass balance fields. It is physically reasonable and there is no a priori requirement that Vatnajökull should have an equilibrium state; climatic variability in Nature will preclude an equilibrium ice cap. However, further analysis revealed that our initial input climatology had significant departures from the actual temperature and precipitation conditions in the region. With improved reference climatology, equilibrium Vatnajökull configurations were found for all glaciological parameters that we explored.

#### 4.1. Illustration of Steady State Simulations

Figure 4 plots ice cap volume and area evolution over sample 4-kyr simulations, illustrating the approach to steady state in ~2 kyr. The solid and dashed lines depict ice volume and ice area time series, respectively. Thick lines correspond to the reference sea level climatology for Vatnajökull, as described by Flowers et al. [2005]. Starting from initial conditions close to the present-day ice cap configuration, the simulated ice cap evolves to an equilibrium area of 7766 km$^2$ and a volume of 3392 km$^3$, compared with present-day estimates of 8025 km$^2$ and 3149 km$^3$. Our present-day estimates are derived from fixed (perpetual) climatic boundary conditions in the region. With improved reference climatology, equilibrium Vatnajökull configurations were found for all glaciological parameters that we explored.
Figure 4. Time series of ice cap volume evolution (solid lines, left axis) and area (dashed lines, right axis) in sample 4000-year steady state simulations, showing the approach to steady state in 1500 years. The thick solid and dashed lines indicate the reference climatology [Flowers et al., 2005] and the thin solid and dashed lines are for the raw temperature fields of Björnsson [2003].

2-kyr values and unchanged from the 4-kyr values (to the first decimal) for the reference model experiments.

This case corresponds to ice rheological parameters in accord with the suggested values of Adalgeirsdóttir [2003] (see Table 1). For simplicity, our reference model adopts a spatially uniform basal flow parameterization, with \( C_0 = 0.0006 \text{ m yr}^{-1} \text{ Pa}^{-1} \) and a fixed basal flotation fraction, \( \gamma = 0.6 \). Relative to the actual present-day ice cap, this set of parameters gives an equilibrium area and volume that are 3.2% too low and 7.7% too high, respectively. As discussed in the next section, one significant result from the steady state simulations is that combinations of parameters that give a good fit to the present-day ice area invariably give an equilibrium ice cap that is too thick. That is, no steady state simulation that we explored could simultaneously fit the observed ice cap area and volume.

The thin lines in Figure 4 depict ice cap area and volume evolution for a fixed sea level climate corresponding to the raw 1961–1990 temperature reconstruction of Björnsson [2003] for southeast Iceland, as interpolated to our model grid. The ice cap is almost completely vanquished by these boundary conditions, with an equilibrium area and volume of 814 km² and 92 km³. The raw temperature reconstructions are believed to be too warm over the ice cap, as they are derived from long-term nonglacierized stations in Iceland and do not reflect regional cooling effects of the ice caps (H. Björnsson, personal communication, 2003). We derived our reference sea level temperatures by applying a mean annual and summer cooling offset of \(-1.12^\circ\text{C}\) and \(-2.24^\circ\text{C}\) to the raw Björnsson [2003] temperatures, as detailed by Flowers et al. [2005].

Experiments with different initial conditions indicate that, for modest differences (e.g., 10–20%, based on using the results of previous model simulations as initial conditions), the steady state ice solution is independent of initial ice cap configuration. Starting with no initial ice, the reference climatology still leads to the development of a full Vatnajökull ice cap that resembles our equilibrium reference model (area of 7618 km² and volume of 3348 km³, within 2% of the steady state values in Figure 4).

Figure 5 plots steady state ice dynamics fields for the reference model. The actual present-day ice margin is superimposed on the equilibrium ice surface topography as a solid white line in Figure 5a, illustrating the different degree of misfit to ice cap extent in different sectors of the ice cap. The largest discrepancy is for Síðujökull on the southwestern margin, which is retracted by \(\sim 10\) km in the equilibrium reconstruction. There are also significant departures from the present-day ice extent on the northern margin, where the northwestern flanks of Dyngjújökull and Brúarjökull are underpredicted and overpredicted, respectively, by \(\sim 5\) km.

Ice thickness differences relative to the present-day ice cap are plotted in Figure 5b, illustrating a systematic overprediction of ice thickness in some sectors of the ice cap and an underprediction in other sectors. The zero-difference contour is drawn in white on this plot. Brúarjökull, the major outlet lobe on the northern margin, is up to 300 m too thick in the equilibrium reconstruction, as are Skeidararjökull and Breidamerkurjökull on the southern margin. In contrast, the western margin outlets Kóludukvisljarjökull and Tungnárjökull are 100–200 m too thin, while the retracted outlet tongues of Síðujökull and Dyngjújökull are depleted by 300–400 m in the equilibrium simulation. Differences from the modern observed ice distribution are more variable in the complex topography of eastern Vatnajökull and on Öraefajökull in the south, reflecting our poor resolution of the narrow outlet valleys and topographic features (e.g., nunataks) in these regions. Figure 6 plots observed and modeled surface topography on a north-south transect through the ice cap, to better illustrate the discrepancies in the equilibrium simulation.

In the ice cap interior, there is an interesting trend toward an equilibrium ice distribution that is \(\sim 100\) m too thick on a northeast-southwest axis from Brúarjökull to Skeidararjökull, and \(\sim 100\) m too thin in western Vatnajökull. This pattern in the ice cap interior falls along drainage basin divides, and is probably driven by systematic differences in outlet glacier dynamics in the equilibrium simulation versus the actual ice cap. In addition, this control run does not include a parameterization of the subglacial

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Table 1. Ice Dynamics Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice dynamics time step (\Delta t)</td>
<td>0.02 years</td>
</tr>
<tr>
<td>Isostatic model time step (\Delta t_{is})</td>
<td>1 year</td>
</tr>
<tr>
<td>Hydrological coupling interval</td>
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</tr>
<tr>
<td>East-west grid dimension (\Delta x)</td>
<td>2 arcmin (1.55–1.64 km)</td>
</tr>
<tr>
<td>North-south grid dimension (\Delta y)</td>
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</tr>
<tr>
<td>Latitude range</td>
<td>63.67º–65.33ºN</td>
</tr>
<tr>
<td>Temperature lapse rate (\beta)</td>
<td>(-0.0053^\circ\text{C m}^{-1})</td>
</tr>
<tr>
<td>Degree-day parameter, snow (d_s)</td>
<td>0.003 m water eq (^\circ\text{C d}^{-1})</td>
</tr>
<tr>
<td>Degree-day parameter, ice (d_i)</td>
<td>0.008 m water eq (^\circ\text{C d}^{-1})</td>
</tr>
<tr>
<td>Glen law parameter (B_0)</td>
<td>(3.0 \times 10^{-3}) Pa(^{-1}) yr(^{-1})</td>
</tr>
<tr>
<td>Creep activation energy (Q)</td>
<td>60,900 J mol(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>Ideal gas law constant (R)</td>
<td>8.314 J K(^{-1}) mol(^{-1})</td>
</tr>
<tr>
<td>Glen law exponent (n)</td>
<td>3</td>
</tr>
<tr>
<td>Sliding law parameter (C_0)</td>
<td>0.0006 m yr(^{-1}) Pa(^{-1})</td>
</tr>
</tbody>
</table>
geothermal hot spots, explaining the locus of overthickened ice in the Grímsvötn region. Equilibrium basal velocity and surface velocity fields are plotted in Figures 5c and 5d, on a square-root scale to improve visualization. Velocities are difficult to compare with field observations, because flow rates on the ice cap experience significant seasonal and interannual variability.

The nonrandom nature of biases in the model is similar to that of the present-day simulations presented by Flowers et al. [2005]. No homogeneous change in model parameters (e.g., ice effective viscosity, basal sliding formulation) can simultaneously improve the fit to actual ice sheet geometry in all regions. Some of the discrepancy can be attributed to uncertainties in the climate fields and the simple mass balance model in these simulations. We believe that the main differences in the fit to the actual ice thicknesses is due to the lack of surge physics in our simulations. The ice cap reconstruction of Figure 5 is in equilibrium, with steady state basal flow (Figure 5c) but no surge cycles spontaneously simulated in the model. The hydrological regulation of surging behavior involves physical switches that are not fully understood and that we do not capture, even in the hydrologically coupled simulations. However, almost all of Iceland’s outlet glaciers are subject to surge cycles [Björnsson, 1998; Björnsson et al., 2003]. Surging acts to thin the ice cap relative to equilibrium ice thickness profiles, as surges advect ice from the interior of the ice cap to lower altitudes, where surface melt rates increase several-fold (see Figure 2c). The equilibrium reconstruction of Figures 5 and 6 underpredicts ice area but overpredicts ice volume, a result that is qualitatively consistent with what is expected due to a lack of surging behavior in our simulations.

4.2. Equilibrium Model Sensitivity Tests

Figure 7 plots illustrative 2000-year time series of Vatnajökull ice volume and area evolution for a suite of sensitivity experiments with different Glen flow law parameters, $B_0 \in [1, 6] \times 10^{-5}$ Pa$^{-3}$ yr$^{-1}$. Our reference value, $B_0 = 3.0 \times 10^{-5}$ Pa$^{-3}$ yr$^{-1}$, is shown with the thick line and is based on the simulations and field observations of Adalgeirsdóttir [2003] and Adalgeirsdóttir et al. [2000]. The basal sliding parameterization is fixed for these tests, with $C_0 = 0.0006$ m yr$^{-1}$Pa$^{-1}$ and a spatially uniform flotation fraction, $\gamma = 0.6$. Not all results have converged to a final equilibrium after the 2000-yr integration; in particular, cases with slow adjustment timescales (i.e. low values of $B_0$) are still adjusting after 2000 yr. However,
values are asymptotically converging and are within 2% of final equilibrium values, based on experiments carried out to 10 kyr. The relative differences between model experiments, and therefore all conclusions, are unchanged by carrying the integrations beyond the 2000-year snapshots.

The dashed lines in the two plots correspond to the estimated present-day ice volume and ice area of Vatnajökull, illustrating the tendency for equilibrium ice caps to be too thick, as noted above. With a good fit to the present-day ice cap area ($B_0 = 1.2 \times 10^{-5} \text{ Pa}^{-3} \text{ yr}^{-1}$), ice cap volume is 17.5% too high. With more deformable ice (higher $B_0$), ice velocities increase and a thinner ice sheet is predicted. The value $B_0 = 3.0 \times 10^{-5} \text{ Pa}^{-3} \text{ yr}^{-1}$ gives a good fit to present-day ice volume. However, the thinner, more mobile ice sheet ultimately leads to reduced ice fluxes in the major outlets and the modeled ice cap area in this case is 7615 km$^2$, about 5% less than the actual present-day ice cap.

Figures 8a and 8b plot steady state ice volume and area in a series of sensitivity tests with different Glen flow law parameters, for two different basal sliding coefficients, $C_0 = 0.0006$ and $0.0004 \text{ m yr}^{-1} \text{ Pa}^{-1}$ and with $\gamma = 0.6$. Consistent with the results above, better fits to the present-day ice volume can be achieved through increased sliding (the diamonds in Figure 8a), but ice cap area is underpredicted for all cases with this increased sliding. For the simple parameterizations of ice dynamics in this study, no combination of effective viscosity and basal flow tuning gives a good equilibrium fit to the modern geometry of Vatnajökull.

Figures 8c and 8d present analogous steady state experiments for different flotation fractions, with $C_0$ fixed at

![Figure 6. Steady state profiles along the N-S transect at 16.33°W. (a) Observed surface topography (thick solid line) and bed topography (thick shaded line), m. Simulated steady state surface and bed topography are plotted with the dotted lines. (b) Modeled surface (solid line) and basal (shaded line) N-S ice velocity, m yr$^{-1}$. Negative numbers indicate southward flow. (c) Difference between the simulated and observed ice thickness along this profile, m.](image)

![Figure 7. Time series of steady state Vatnajökull ice cap evolution for different flow law parameters, $B_0 \in [1, 6] \times 10^{-5} \text{ Pa}^{-3} \text{ yr}^{-1}$. (top) Ice cap volume, km$^3$. (bottom) Ice cap area, km$^2$. The present-day volume and area of Vatnajökull, as discretized on the model grid, are indicated with dashed lines. The thick solid lines are for the reference parameter value, $B_0 = 3.0 \times 10^{-5} \text{ Pa}^{-3} \text{ yr}^{-1}$, and the values of $B_0$ for the other simulations are indicated on the plot ($\times 10^{-5} \text{ Pa}^{-3} \text{ yr}^{-1}$).](image)
The first 11 cases in Figures 8c and 8d correspond to spatially uniform flotation fractions, \( g_\gamma \in [0, 1] \), representing the end member cases of no free water \( (p_w = 0) \) and full flotation \( (p_w = p_I) \) at the bed. The latter case gives enhanced basal flow, as we prescribe \( v_{bj} \) in equation (14), but there is no dynamical runaway because we have adopted a linear relationship in (14). As expected, equilibrium ice cap volume and area both decrease as subglacial water pressure rises and basal flow increases. Increases in ice deformation and basal flow, have similar effects on the ice sheet reconstruction, with increased ice velocity drawing down the surface but simultaneously decreasing the equilibrium areal extent.

The two points labeled “Var” in Figures 8c and 8d correspond to two different simulations with coupled ice dynamics and subglacial hydrology, with internally modeled, spatially variable \( g_\gamma \) values. The former case, plotted in Figure 9, gives an equilibrium ice volume of 3062 km\(^3\), 3% less than the modern ice cap. This case can be compared directly with that of Figure 5, which has identical model parameters but adopted a fixed spatial distribution of water pressure, \( g_\gamma = 0.6 \). The case with the coupled hydrology has an ice volume 11% less than that shown in Figure 5. The difference is a result of increased basal lubrication in the coupled simulation, leading to greater basal flow, a slightly thinner ice cap, and diminished ice cap area (7562 km\(^2\) versus 7784 km\(^2\) in Figure 5). The equilibrium spatial pattern of the flotation fraction, \( g_\gamma \), is plotted in Figure 9a, with a mean value of \( \bar{\gamma} = 0.79 \). Note in Figure 9a that superflotation \( (p_w > p_I) \) is predicted in a few

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**Figure 8.** The 2000-year (left) ice cap volume and (right) ice cap area for different sensitivity experiments. (a, b) Variable effective viscosity in Glen’s flow law, with \( B_0 \) varying from 1.5 to 6.0 Pa\(^{-1}\) yr\(^{-1}\). The crosses and diamonds correspond to fixed sliding law coefficients of 0.0004 and 0.0006 m yr\(^{-1}\) Pa\(^{-1}\), respectively. (c, d) Variable flotation coefficients, \( g_\gamma \in [0, 1] \). Here \( \gamma = 0 \) prohibits basal flow, while \( \gamma = 1 \) represents full flotation \( (p_w = p_I) \) over the entire bed. The final two points, labeled “Var,” correspond to two coupled ice dynamics and subglacial hydrology simulations, with variable (internally modeled) \( g_\gamma \) values. (e, f) Cases with different fixed sea level temperature offsets, with \( \Delta T = 0^\circ C \) for the raw Björnsson [2003] temperature fields. Our reference climatology is discussed in the text and corresponds to \( \Delta T = -1.1^\circ C \) on the plot. The final case, labeled \(-1.1^*\), applies a temperature depression of \(-1.1^\circ C\) year-round, whereas the summer temperature depression is twice that of the annual cooling in all other model experiments (i.e., \( \Delta T_{Jul} = 2\Delta T_{Ann} \)). The observed present-day volume and area of Vatnajökull are shown with the dotted line in each plot.
regions, in particular at the head of Skeidararjökull. In this situation the ice cap is locally decoupled from the bed, but adjacent pinning points prevent runaway basal flow.

Basal velocity, surface velocity, and the ratio $v_b/v_s$ are plotted in Figures 9b–9d, for comparison with the velocity fields with a fixed flotation fraction in Figure 5. Mean basal velocity is 26.0 m yr$^{-1}$ in the coupled-hydrology run, compared with 23.8 m yr$^{-1}$ in Figure 5c. There are significant regional differences, however, with peak basal velocities of 167 m yr$^{-1}$ in Figure 9b near the head of Skeidararjökull, compared with 74 m yr$^{-1}$ in Figure 9c. To facilitate comparison, Figure 9e plots the difference between the two cases, $v_b^{\text{hyd}} - v_b^{\text{ref}}$, with the zero-difference contour plotted in white. Some regions of Vat-
najökull experience little change in the two models, but Búiajökull, Skeidarárjökull, and eastern Bredamerkurjökull see enhanced basal flow as a consequence of elevated subglacial water pressures. This is consistent with the findings of Flowers et al. [2003], who report significant meltwater ponding in these regions as a result of bedrock overdeepenings.

[55] Surface velocities are similar in the two cases, with a mean ice cap value of \( v_s = 38.6 \text{ m yr}^{-1} \) in Figure 9d, with internally regulated subglacial hydrology, versus \( v_s = 39.8 \text{ m yr}^{-1} \) in the reference model, Figure 5d. Surface velocity includes both basal flow and internal deformation. The slightly reduced flow rates in the hydrologically coupled model are a result of thinner ice, lower slopes, and reduced internal deformation. The ratio \( v_b/v_s \) in Figure 9d provides an indication of the relative importance of basal flow versus internal deformation in the ice cap. The mean value in this plot is 0.77, indicating that 77% of the surface motion, on average, can be attributed to basal flow. This compares with 71% for the case in Figure 5, without the hydrological controls on basal flow.

[56] Figure 9f plots the difference in equilibrium ice thickness between the two cases, \( H_{\text{hydro}} - H_{\text{ref}} \), with the white line indicating zero difference. Averaged over the ice cap, the mean ice thickness in the hydrologically coupled simulation is 31 m less than that of Figure 5, 405 m versus 436 m without the hydrological coupling. The average ice thickness in the actual present-day ice cap is estimated to be 392 m, suggesting that the more realistic spatial pattern and magnitude of basal flow in the hydrologically coupled case gives a better representation of the ice cap. Skeidarárjökull provides an excellent illustration of the impacts of hydrological coupling, with increased ice thickness at the margin and thinner ice upstream of the terminus. The pattern reflects an increased transfer of mass from the accumulation area, concomitant with a reduction in surface slope and a generally greater role of basal flow versus internal deformation in ice fluxes. We return to the effects of hydrological coupling in Section 5.

4.3. Climate Sensitivity

[57] The sensitivity to glaciological parameters in Figures 8a–8d is of order 10% for equilibrium ice cap area and 20% for ice cap volume, for the range of ice rheological parameters that are consistent with observed deformation velocities [Adalgeirsdóttir et al., 2000; Adalgeirsdóttir, 2003; Flowers et al., 2003] and for an order-of-magnitude variation in the strength of basal sliding in our simulations. This stands in contrast to the large sensitivity of equilibrium ice cap reconstructions to changes in climatic boundary conditions. Figures 8e and 8f plot the steady state ice volume and area for a series of experiments with different temperature offsets, relative to the raw Björnsson [2003] sea level temperature fields. \( \Delta T = 0^\circ \text{C} \) refers to the raw Björnsson [2003] temperature field, as interpolated onto our model grid.

[58] Our reference climatology corresponds to \( \Delta T = -1.12^\circ \text{C} \) on the plot. Under this reference climatology, the mean annual temperatures of Björnsson [2003] have been cooled by 1.12\(^{\circ}\)C and July temperatures of Björnsson [2003] have been cooled by 2.24\(^{\circ}\)C. This adjustment is based on best fit simulations to the modern-day ice cap under a historical climate forcing scenario [Flowers et al., 2005]. The annual cooling and the greater degree of summer cooling represent the influence of the ice cap in cooling its surroundings, particularly during the summer season when the melting glacier surface cannot rise above 0\(^{\circ}\)C, while adjacent ice-free areas warm to July temperatures of \( \sim 10^\circ \text{C} \) [Björnsson, 2003]. This enhanced regional cooling in summer months is integral to the preferred reference climatology, as summer temperatures are the dominant control on Vatnajökull’s mean annual mass balance. The final case in Figures 8e and 8f, labeled \( -1.1^* \), applies a temperature depression of \( -1.12^\circ \text{C} \) year-round, in contrast with all other cases in these plots, where \( \Delta T_J = 2 \Delta T_u \). The impact on mass balance is dramatic; with only a 1.12\(^{\circ}\)C summer temperature depression, equilibrium ice volume and area are reduced by 53% and 65% relative to the reference model.

[59] The range of mean annual temperatures that gives good Vatnajökull reconstructions is extremely narrow, for both equilibrium reconstructions and for present-day reconstructions with a historical climate spin-up [Flowers et al., 2005]. Figure 10 plots the reconstructions from two cases that closely bracket our reference climatology: \( \Delta T_u = -1.5^\circ \text{C} \) and \( \Delta T_u = -0.5^\circ \text{C} \). The equilibrium ice volumes

\[ \Delta T = -1.5^\circ \text{C} \]

\[ \Delta T = -0.5^\circ \text{C} \]

Figure 10. Steady state ice surface topography for two different temperature offsets relative to the raw Björnsson [2003] temperature fields: (a) \( \Delta T = -1.5^\circ \text{C} \) and (b) \( \Delta T = -0.5^\circ \text{C} \). The reference model climatology is \( \Delta T = -1.11^\circ \text{C} \).
in the two cases are 4467 km$^3$ and 1675 km$^3$, close to 50% larger and smaller than the present-day ice cap, respectively. This sensitivity to mean annual and summer temperature differences of order 0.1°C must be kept in mind in interpreting all other sensitivity tests presented here and by Flowers et al. [2005]. Given the uncertainties in our climatic boundary conditions, it is not possible to provide rigorous constraints on glaciological and hydrological parameterizations for Vatnajökull.

4.4. Geothermal Cauldrons and Longitudinal Stress Coupling

Further sensitivity tests of interest explore the effects of longitudinal stress coupling and the inclusion of subglacial geothermal hot spots on the equilibrium ice geometry. Figure 11 plots time series of ice cap evolution with longitudinal stress coupling included in the equilibrium simulation (thin solid lines). This can be contrasted directly with our standard model solution with the shallow ice approximation (thick lines). We have also included a 2000-year simulation with geothermal hot spots parameterized in the ice cap, at the Grímsvötn and Skaftá cauldron regions (dashed lines). All cases have identical climatic boundary conditions, fixed subglacial hydrological and basal sliding parameterizations ($\gamma = 0.6$, $C_0 = 0.0006$ m yr$^{-1}$Pa$^{-1}$), and our reference model rheology ($B_0 = 3.0 \times 10^{-5}$ Pa$^{-3}$ yr$^{-1}$).

These model settings are in accord with the recommended parameter settings of Flowers et al. [2005], which are based on fits to present-day Vatnajökull with a realistic spin-up climatology. For all three cases in Figure 11 there is increased basal flow relative to the reference model of Figure 5, giving an ice cap volume close to present-day Vatnajökull but an ice cap area 5–10% less than the current ice cover. The ice cap has not reached an equilibrium in the simulation with the geothermal cauldrons, but we present the 2000-year snapshot for consistency with all other cases in this paper. The tremendous geothermal heat flux that is specified in the two interior regions leads to basal melt rates of up to 5 m yr$^{-1}$, drawing down the ice cap in the two regions and reducing overall ice volume to 3047 km$^3$ at 2000 years, relative to 3173 km$^3$ in the case without geothermal cauldrons. The cauldrons represent an area of less than 100 km$^2$, so their impact is significant relative to their extent. The drawdown in the ice cap interior reduces both ice thickness and surface slopes, creating a diminished ice flux to the margins. This results in a 2.6% reduction in ice cap area, from 7615 km$^2$ in the reference case to 7428 km$^2$ in the 2000-year snapshots.

Similar to the effects of softer ice and invigorated sliding, the geothermal hot spots therefore help to create a thinner equilibrium ice cap, but with a concomitant reduction in ice cap area. While parameters can be tuned to give good fits to the present-day volume of Vatnajökull, they simultaneously result in a poorer fit with the modern ice cap area. The case with the geothermal heat cauldrons provides a more realistic basal boundary condition and interior ice cap geometry than our reference model, but we do not explore it further in the sensitivity tests presented here. The subglacial meltwater generated by these geothermal hot spots is known to pond in subglacial lakes that drain episodically by catastrophic outburst floods [Björnsson, 1992, 2002]. The process physics underlying these episodic
meltwater releases are not included in these simulations; our subglacial hydrology model simulates sheet flow and exchanges with the groundwater system, but does not contain channelized drainage in the simulations presented here.

With longitudinal stress coupling, the modeled ice cap has a volume of 3078 km$^3$ and an area of 7543 km$^2$ at 2000 yr, again compared with the values of 3173 km$^3$ and 7615 km$^2$ for the equilibrium solution with identical dynamical parameters but without longitudinal stress coupling. The slight reduction in equilibrium ice cap extent is consistent with the expected effects of longitudinal stress coupling, with extensional flow in the major outlets exerting a “pull” that increases the transport of ice from the interior. The average ice thickness is 2.2% less as a result of the longitudinal coupling effects, 408 m versus 417 m in the reference case.

The noisy evolution of ice cap area under longitudinal coupling is noteworthy in Figure 11b. This can be contrasted with the ice cap volume time series, which approaches a true equilibrium. The area fluctuations are due to a small number of ice marginal cells that cannot achieve an actual equilibrium on the fixed geographical grid, once higher-order stress effects are included in the solution. The ice cap is seeking to establish ice margin positions between two grid cells, with oscillations between these two locations. Longitudinal stress coupling communicates the changes in marginal ice thickness and velocity upstream, effecting the flux of ice to the margins and establishing these minor oscillations for the duration of the model experiment. This is a consistent result in all higher-order ice dynamics simulations we have conducted; fixed boundary conditions do not necessarily lead to an equilibrium ice cap configuration. The area fluctuations are relatively minor in this case, with grid cells of ~3 km$^2$, but this result suggests the need for either as fine a resolution as possible or a subgrid scheme for ice margin location [e.g., Waddington, 1981, pp. 247–253] in glaciological simulations with higher-order stresses.

Figure 12 plots representative longitudinal stress fields from the 2000-year (near equilibrium) snapshot of the ice cap. The ice effective viscosity, $A_{ef}$ (equation (20)), is plotted in Figure 12a. The ice cap is isothermal, with a constant and homogeneous ice stiffness parameter $B$, so $A$ is governed by strain rates $E$. Even under isothermal conditions, effective viscosity varies by a factor of ~40 over the ice cap, with the softest ice in the major outlets on the southern margin. The most stiff ice is in the ice cap interior, where strain rates are low. Because longitudinal deviatoric stresses are proportional to effective viscosity (equations...
Table 2. Climate Change Scenario for Vatnajökull, NCAR CCSM Version 2.0

<table>
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<td>P, mm</td>
<td>T, °C</td>
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</table>

(19) and (21)), this opposes the effect of high horizontal strain rates in the outlet glaciers and leads to a complex distribution of longitudinal deviatoric stresses (Figure 12b) in the ice cap. \( \sigma_h \) in this plot denotes the vector magnitude of the horizontal longitudinal deviatoric stress,

\[
\sigma_h = \left( \sigma_{x}^2 + \sigma_{y}^2 \right)^{1/2}. 
\]

[66] In general, the highest values of longitudinal deviatoric stresses are in the ice cap interior, along the dynamical divides, a direct result of the high effective viscosity in these regions. Typical ice divide values are \( \sigma_h = 40–50 \) kPa, while \( \sigma_h = 10–20 \) kPa is more characteristic of longitudinal deviatoric stresses in the outlet glaciers. The high degree of variability is a result of Vatnajökull’s complex topography, which gives rise to large ice thickness gradients. The relative importance of longitudinal deviatoric stresses in the ice divide region is best illustrated in Figure 12d, which plots the ratio \( \sigma_h/\tau_d \), where \( \tau_d \) is the vector magnitude of the gravitational driving stress in (16).

[67] This result does not simply translate to longitudinal stress effects being important in the ice cap interior. Because their influence on ice dynamics comes through the gradient \( \partial(H\sigma_{ij}/\partial) \) (equation (17)), the actual longitudinal coupling term is small relative to the deviatoric stresses and the gravitational driving stresses at Vatnajökull. Figure 12c plots the vector magnitude of the horizontal longitudinal stress coupling,

\[
G = \left[ \frac{2}{dx} \left( H\sigma_{xx} \right) \right]^2 + \left[ \frac{2}{dy} \left( H\sigma_{yy} \right) \right]^2 \right]^{1/2}, 
\]

with a peak value of 23 kPa and a mean ice cap value of just 2.3 kPa. In contrast, the gravitational driving stress, the first term in equation (17), has a maximum value of 247 kPa and a mean value of 85.3 kPa. On average over the ice cap, \( G \) is 4.4% of \( \tau_d \). Highest values of \( G \) occur in the upper eastern catchment of Skeidararsjökull, an area with significant bedrock overdeepening (hence thick ice and high rates of flow).

5. Present Day and Future Reconstructions

5.1. Climate Spin-up, 1600–1990

[68] Simulations of present-day Vatnajökull and future climate change scenarios require a realistic historical climate spin-up; equilibrium solutions such as those presented above are not appropriate, as Vatnajökull is not in a state of equilibrium. Flowers et al. [2005] describe the development of a historical temperature forcing for the period 1600–1990, adopted as a perturbation to the mean 1961–1990 temperature fields over Vatnajökull [Gyldådottir, 2003; Björnsson, 2003], with a uniform cooling offset to parameterize the regional cooling effects of the ice cap, as described above. The historical record from 1823–1990 is derived from mean annual and July temperatures recorded at Stykkishólmur, southwest Iceland. Stykkishólmur has the longest available observational record in Iceland. For the period 1600–1823 we adopt the temperature reconstruction of Bergh thorsson [1969], which is based on sea ice distribution and other subjective quantities. Further details of this 1600–1990 spin-up forcing are provided by Flowers et al. [2005].

[69] We prescribe a fixed spatial pattern of precipitation rates for the climate spin-up, based on the map of Eythorsson and Sigtrygsson [1971], \( P_0(x, y) \), as described in Section 3.2. Local precipitation rates vary with temperature in a simple parameterization of the increase in saturation vapor pressure with temperature,

\[
P(x, y, t) = P_0(x, y) + \frac{\Delta P}{\Delta T} \Delta T(x, y, t),
\]

where \( \Delta T \) is the local temperature perturbation from the mean 1961–1990 reconstruction and \( dP/dT \) is a free parameter, which Jóhannesson et al. [1995] estimate to be 5% °C⁻¹.

5.2. Future Climate Scenarios

[70] Flowers et al. [2005] describe simulations of Vatnajökull ice cap retreat under a range of prescribed warming scenarios, from 1°C to 4°C per century. These scenarios have been guided by recent coupled ocean-atmosphere simulations from the NCAR-CCSM model (CCSM v2.0) [Kiehl and Gent, 2004; B. Otto-Bliessner, personal communication, 2003]. NCAR-CCSM future climate change simulations with a 1% per year CO2 increase for the period 1990–2100 produce a 21st century warming of ~2°C in Iceland, despite a slight weakening in meridional overturning circulation in the North Atlantic [Gent and Danabasoglu, 2004].

[71] We implement the NCAR-CCSM future warming scenarios as a perturbation on the observationally based 1961–1990 sea level temperature reconstructions for the Vatnajökull region, \( P_0(x, y) \). NCAR-CCSM temperature and precipitation forecasts for Iceland were calculated from the annual average temperature and precipitation in four NCAR-CCSM grid cells in the Iceland region, spanning the longitudes 13.25–20.25°W and the latitudes 59.4–66.8°N. Screen temperature values were adopted for this study, with NCAR-CCSM screen temperatures lapsed to sea level using the NCAR-CCSM orography and a constant altitudinal lapse rate as in equation (32). Table 2 presents seasonal and mean annual temperature and precipitation values in Iceland from a 150-year control run with the NCAR-CCSMv2.0 model, for conditions representative of the baseline period 1961–1990. Values in Table 2 are averages of the last 50 years of the control run simulation.

[72] Annual time series of sea level climate perturbations, \( \Delta T(t) \) and \( \Delta P(t) \), were calculated for the period 1990–2140.
based on differences from the control run. Future temperatures over the ice cap are then calculated from

$$T(x, y, z, t) = T_0(x, y) + \beta h_s(x, y, z, t) + \Delta T(t). \quad (37)$$

Mean annual temperature perturbations, $\Delta T_a(t)$, and July temperature perturbations, $\Delta T_J(t)$, were both derived for each year in the future climate change scenario. Table 2 presents the average climatology for the period 2086–2115, representative of the year 2100 in the simulation, along with the difference from present-day (1961–1990) conditions. The model forecasts a warming of 2.05 °C by 2100 relative to the 1961–1990 average, with a near-identical amount of summer warming. A 3.3 decrease in precipitation accompanies this warming, with the largest decrease in the summer and autumn, although the forecasted change in precipitation is not statistically significant annually or in any individual season, relative to the interannual variability.

To gain more insight on the temperature-precipitation relationships in Iceland, we examined the historical relationship between total annual precipitation and mean annual temperature from a number of Icelandic Meteorological Office stations with long-term records. Available data from Höfn, southeastern Iceland (1951–2001) and Akureyri, north central Iceland (1928–2001) offer examples of precipitation variability on the southeastern and northern coasts. There is a weak positive correlation between mean annual temperature and precipitation in Höfn, $r = 0.39$. A regression of $P$ versus $T_a$ is significant at 99%, with $dP/dT = 157$ mm °C$^{-1}$. The mean annual precipitation recorded at Höfn from 1951–2001 was 1401 mm, giving $dP/dT = 11$% °C$^{-1}$. The situation is opposite at Akureyri in north-central Iceland, with a weak negative correlation between mean annual temperature and precipitation, $r = -0.09$ and $dP/dT = 2.5$% °C$^{-1}$.

The relationships at several other sites that we examined were intermediate between these two cases. The results hint at regional patterns to the precipitation–temperature relationship in Iceland, presumably controlled by the different regional storm tracks, moisture sources, and precipitation mechanisms. Höfn on the southeast coast is nearest to Vatnajökull, and probably offers the best guidance on $dP/dT$ values appropriate to the ice cap.

5.3. Results of Climate Change Simulations

Figure 13 presents simulations that illustrate the historical and future climate forcing and explore the sensitivity of Vatnajökull ice cap evolution to several different precipitation scenarios. Figure 13a plots average ice cap air temperature, from equation (37). This is a combination of the sea level climate forcing scenario, $\Delta T(t)$, and the evolving ice cap surface height, $h_s(x, y, t)$. Several precipitation scenarios are plotted here, as discussed below, but the ice surface evolution does not differ enough to cause significant differences in surface temperature for the different cases. The change in pattern of temperature variability
after calendar year 1990 corresponds to the shift from the historical climate spin-up to the unsmoothed NCAR temperature forcing. We choose to use the raw annual NCAR fields, rather than a smoothed version, to provide a representation of interannual variability and its influence on the ice cap. The integration extends from 1600 to 2140.

[75] Mean ice field mass balance for the same cases is plotted in Figure 13b, with positive mass balance until ~1900, slightly negative mass balance through most of the 20th century, and a sharp decline in mass balance through the 21st century, to values exceeding 4 m yr$^{-1}$ of ice loss, averaged over the ice cap. Mass balance variability reflects that of the input temperature forcing, with large interannual fluctuations from 1990 onward. This includes several positive mass balance years in the period 1990–2030, despite a trend toward increasingly negative mass balance. Post-2030 the mass balance has dropped to low enough values that even the cool years in the NCAR simulation have a negative mass balance. At this point Vatnajökull has entered a state of monotonic and accelerating retreat, as illustrated in the ice volume and area times series of Figures 13c and 13d.

[77] The ice volume and area evolution reflect the mass balance time series of Figure 13b, and are temperature-driven to first order. However, the small but systematic differences in mass balance under the different precipitation models have cumulative impacts on ice volume and area, with a clear separation of the time series in Figures 13c and 13d. The dotted line corresponds to a case with $dP/dT = -5\% \ C^{-1}$, creating higher precipitation in the cool conditions of the Little Ice Age, pre-1990, and less precipitation under warmer conditions in the future. This is an unlikely scenario but it reflects the observations in Akureyri, north Iceland, and it offers a possible end-member of the $dP/dT$ sensitivity tests. The remaining cases correspond to $dP/dT$ values from 0 to 15\% C$^{-1}$, as well as a case with spatially variable precipitation response to the changing ice geometry, following equation (33) (case Pmod; shaded lines in Figure 13).

[78] As $dP/dT$ increases, more modest ice volume increases are predicted through the period of ice buildup from 1600–1900; cold conditions suppress moisture supply to the ice cap. For the setting $dP/dT = 15\% \ C^{-1}$ (dashed line), there is essentially no ice cap growth during this period. Decreased precipitation rates in this case balance the ice cap. For the setting $dP/dT = 0$ or 5\% C$^{-1}$, which permit significant ice cap growth from 1700–1900. For all cases with $dP/dT \in [0, 10] \% \ C^{-1}$, peak Vatnajökull area is predicted in 1888 and peak ice cap volume follows shortly thereafter, in the period 1892–1894.

[79] The variable precipitation model, which includes local slope and altitude effects on modeled precipitation rates, adopts in addition the relationship $dP/dT = 5\% \ C^{-1}$, and differences between these two cases are difficult to discern in these plots. In general, ice volume and area are less in the adaptive precipitation model, particularly in the 21st century; both slopes and altitudes on the ice cap decline in this period, reducing orographic forcing and thus reducing precipitation rates on the ice cap. Overall in Figures 13c and 13d, however, it is apparent that the predicted warming swamps the effects of different precipitation scenarios for future ice cap mass balance. Even a 15\% C$^{-1}$ increase in precipitation makes little difference to the overall negative mass balances of the 21st century, delaying but not staving off the ice cap collapse.

[80] The interval from circa 1860 to 1890 featured the highest mass balance in the historical period, as modeled here, with the ice cap mass balance and volume in a near-equilibrium state from circa 1890 to 1930. The period 1930–1960 featured persistently negative mass balance and ice cap decline, followed by cooler temperatures and stabilization through to circa 2000. Mass balance in the last decades of the 20th century is characterized by a mix of positive and negative years, qualitatively similar to available observations [Björnsson et al., 1998].

[81] In the future projection the cumulative mass balance is increasingly negative, although the annual mass balance time series is still punctuated by positive years in the early part of the 21st century. The positive balance years have discernible short-term impacts on the ice cap area (Figure 13d), as the ice cap area responds through margin advance and the temporary expansion of perennial ice fields. There is much less impact on ice volume (Figure 13c), which integrates the interannual mass balance variability and provides a better indication of cumulative mass balance. As the 21st century progresses, Vatnajökull retreat rates accelerate and the mass balance regime moves far below the oscillations around equilibrium that characterized the 20th century in our model.

[82] Our climate forcing pre-1990 is based on historical temperature data, and direct annual or decadal comparisons with Vatnajökull observations are possible. Post-1990, the model is forced by NCAR CCSM predictions, which should be taken “statistically” rather than literally. That is, years 1995 or 2004 in the simulation bear no true relationship with the weather patterns in Iceland in those calendar years. Rather, the temperature times series (hence mass balance) reflects that of the input temperature forcing, with large oscillations around equilibrium state from circa 1860 to 1890 featured the highest mass balance in the historical period, as modeled here, with the ice cap mass balance and volume in a near-equilibrium state from circa 1890 to 1930. The period 1930–1960 featured persistently negative mass balance and ice cap decline, followed by cooler temperatures and stabilization through to circa 2000. Mass balance in the last decades of the 20th century is characterized by a mix of positive and negative years, qualitatively similar to available observations [Björnsson et al., 1998].

[83] Sensitivity tests for the same time period, 1600–2140, are plotted in Figure 14 for several different ice dynamics parameterizations. Ice volume is plotted in Figures 14a and 14b, and ice area evolution is plotted in Figures 14c and 14d. All cases adopt $dP/dT = 5\% \ C^{-1}$ and uniform ice rheological and sliding parameterizations, $B_0 = 3.0 \times 10^{-5} \ Pa \ yr^{-1}$ and $C_0 = 0.0006 \ m \ yr^{-1} \ Pa^{-1}$. A uniform flotation fraction, $\gamma = 0.6$, is adopted for all experiments except the hydrologically coupled simulation (case Hydrol, thin solid line). These sensitivity tests therefore isolate the effects of (1) basal flow governed by freely evolving subglacial hydrology, (2) geothermal cauldrons, and (3) longitudinal stress coupling on the present-day and future ice cap reconstructions.

[84] Table 3 compiles the 1990 volume and area for the suite of tests in Figure 14, for comparison with the observed present-day values of 3149 km$^2$ and 8025 km$^3$. As seen in Figures 14b and 14d, the standard model (thick solid line) has the highest present-day ice cap volume of these cases,
3245 km$^3$ (calendar year 1990). The ice cap area for the standard model is 8017 km$^2$ at this time. This corresponds to an average ice thickness of 405 m, compared with an estimate of 392 m for the actual ice cap circa 1990 (as interpolated on our model grid). Average ice cap thickness is therefore 3.3% too high, with the differences exceeding this in some regions of the ice cap.

All of the model experiments with realistic climate spin-ups provide an improved fit to the present-day ice cap geometry relative to the equilibrium reconstructions. For identical settings to those in the standard model above, the equilibrium simulation predicted $H = 417$ m, 6.4% too thick relative to the estimated present-day value. The improvements in the fit to present-day Vatnajökull are actually greater than this comparison suggests, as the equilibrium ice cap for these settings has an area that is 5.1% too small (7615 km$^2$). This reduces the average ice thickness relative to an analogous 8025-km$^2$ ice mass, so $H > 417$ m could be expected for an ice cap area closer to that of present-day Vatnajökull.

The improvement in the reconstruction of present-day Vatnajökull under realistic historical climate forcing is consistent with glaciological expectations [e.g., Marshall et al., 2000], and is a result of the fact that actual ice masses are rarely, if ever, in a state of equilibrium. Ice masses are continually responding to past changes in mass balance as well as ice dynamical excursions (e.g, outlet glacier surges), and glaciers or ice caps are typically thinner than an equilibrium ice mass due to the slow response time of ice cap volume relative to area. The result also indicates that climatic disequilibrium plays a role that is comparable to surging behavior in dictating Vatnajökull’s present-day geometry.

The differences in historical and future simulations are relatively small between model experiments, relative to the overall climate-driven evolution during this period. However, there are interesting differences, on the order of

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<th>$A_{1990}$, km$^2$</th>
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Figure 14. Time series of Vatnajökull ice volume and area evolution under different ice dynamics parameterizations. (a) Ice volume, 1600–2140, km$^3$. (b) Ice volume, 1900–2050, km$^3$. (c) Ice area, 1600–2140, km$^2$. (d) Ice area, 1900–2050, km$^2$. The legend is provided in Figure 14a, and the model experiments are explained in the text.
a few % in ice volume and area, when hydrological coupling, a more realistic pattern of basal melt, and longitudinal stress coupling are added to the simulation. Modeled ice volume in different model experiments has a relatively uniform offset post-1900 (Figure 14b). Differences in ice cap area are more variable and occasional rapid excursions in Figure 14d reflect the short-term expansion of perennial snow fields during cold years in the NCAR-CCSM climate forcing, as noted above. These effects are transient, as the expanded snow fields quickly retreat during subsequent warm years.

The relationship between ice volume and area is not always predictable. The hydrologically coupled simulation exhibits the greatest variability, with an interesting transition circa 1950 to greater ice cap area than the standard model, despite a lower ice volume. We interpret this as a response to the high rates of ice cap retreat in the period 1930–1950, which thin the ice cap and increase meltwater delivery to the bed. Both effects lead to lower effective pressure (higher flotation fractions, γ), increasing basal flow in our simulations.

The standard model and the hydrologically coupled model have similar ice cap areas throughout the 21st century in the simulation, despite the 100–150 km² difference in ice volume. Because of the reduced average ice thickness in the experiment with hydrologically governed basal flow, this simulation provides the best overall fit to the present-day ice cap area and volume, within 0.5% for each. Average ice thickness in 1990 is 390 m, within 2 m of the actual ice cap. There are still systematic regional discrepancies: areas where the simulated ice cap is of order 100 m too thick or too thin (Figures 15a, 15d, and 15f). However, the hydrologically coupled simulation offers the best representation of the present-day ice cap that we have been able to simulate.

Similar to the equilibrium simulations, some sectors of the ice cap experience enhanced basal flow when governed by subglacial water pressures, notably the Skeiðarárjökull and Breidamerkurjökull outlets. Figure 15b plots the differences in ice thickness for calendar year 1990 relative to the standard model, showing the systematic thinning in areas where subglacial water accumulates. Figure 15c depicts the difference in modeled basal velocity relative to the standard model. The maximum basal velocity in this simulation is 146 m yr⁻¹, relative to 84 m yr⁻¹ in the standard model. However, there is little difference in the average and maximum surface velocities in the two model experiments. Figures 15d–15f plot these results in the N-S transect through Breidamerkurjökull. In this transect, both the standard and hydrologically coupled models predict too much ice on the southern and northern flanks for the ice cap, but the latter case is improved.

Future climate change scenarios were carried out for the full suite of model parameterizations that we have introduced. The climate scenario used in this simulation adopts the NCAR CCSM temperature predictions until 2140 and a 2°C per century warming scenario beyond this time, in close accord with the NCAR CCSM rate of warming in the period 1990–2140. Figure 16 plots time series of mean ice cap air temperature and ice cap volume for this scenario. Air temperature over the ice cap includes the effects of both the sea level climate forcing and the additional warming due to the lower elevations of the collapsing ice cap. The dashed line is for the spliced NCAR CCSM/2°C per century climate change scenario, as noted above, and the thick solid line plots the results for a direct sea level warming scenario of 2°C per century from 1990–2300.

The accelerated warming in the 23rd century is due to disappearance of the ice cap, as seen in Figure 17. Figure 17a plots modeled 1990 ice surface topography for this hydrologically coupled reconstruction, while Figures 17b–17d illustrate the collapse of the ice cap in future climate change scenarios that have been extended to year 2300. Ice cap declines by by 2100, relative to the 1990 value, but Vatnajökull is thinning throughout this period, creating a volume loss of 22% by 2100 (case Hydrol in Table 3). The demise of the ice cap accelerates through the 22nd century and the ice cap disappears entirely by 2300 in the simulation.

Change in ice volume and area from 1990–2200 are also shown in Table 3 for several ice dynamical parameterizations of interest. As noted above, the contrasts in ice cap evolution that result from different ice dynamics treatments are minor relative to the influence of air temperature on Vatnajökull. However, it is worth noting that each of the improvements to the model physics that we explored made Vatnajökull more sensitive to climate warming; the standard model experiences the least retreat of any of the simulations in Table 3. Volumetrically, the hydrologically coupled model is the most sensitive to climate warming, due to basal lubrication and ice thinning feedbacks. The interior of the ice cap has a lower average altitude in this case, as well as in the model experiments with longitudinal stress coupling and geothermal cauldrons. This increases the mass balance impact of climatic warming and accounts for much of the increased sensitivity in these experiments. In superposition, the effects of geothermal cauldrons, hydrological coupling, and longitudinal stresses appear to be additive, leading to progressively faster ice demise as each effect is introduced.

Experiment Pmod, which includes a parameterization of orographic precipitation feedbacks on annual precipitation rates, is also more sensitive to climate warming (Table 3). In particular, the areal retreat of the ice cap is accelerated when this feedback is included. This is an expected consequence of the reduced altitude and slope that accompany ice cap demise.

6. Conclusions

Given climatic boundary conditions close to modern conditions in southeast Iceland, our parameterizations of mass balance and glacier dynamics predict the existence of equilibrium solutions for Vatnajökull ice cap dynamics. With fixed climatic boundary conditions but freely evolving mass balance, including altitudinal feedbacks, near-equilibrium solutions are reached within 2000 yr of integration. Ongoing adjustments beyond this time are minor (less than 2%) and do not affect the conclusions of the sensitivity tests. Equilibrium solutions have systematic misfits with the observed present-day ice cap, particularly in the southwestern and eastern portions of Vatnajökull, but most major outlets of the ice cap are captured in the simulations.
We found that ice cap configurations resembling the modern ice cap exist for a very narrow window of climatic boundary conditions. Temperature differences of order \(0.1 ^\circ C\) have a large impact on equilibrium ice cap volume and area. Temperatures \(1 ^\circ C\) warmer or colder than our reference climatology lead to the complete demise and the unbounded growth of the ice cap, respectively, indicating a limited domain of stable equilibrium solutions.

For the entire range of ice rheological parameters and basal flow parameterizations that we explored, equilibrium reconstructions of Vatnajökull had an average ice thickness greater than that of the present-day ice cap. No combination of parameters provided a simultaneous fit to both the modern ice cap area and volume, with systematic over-predictions and underpredictions of ice cap thickness in different regions of the ice cap. We interpret this deficiency of the equilibrium solutions as an expected consequence of an artificial climatic equilibrium and the lack of surging behavior in our simulation. Most of Vatnajökull’s major outlets undergo frequent surge cycles, which serve to draw down the ice in the interior of the ice cap while maintaining or expanding Vatnajökull’s areal extent.

Transient model experiments with a realistic climate spin-up from 1600–1990 provide a better fit to the present-day ice cap than equilibrium solutions. This indicates that climatic disequilibrium also plays an important role in
dictating the aspect ratio of the ice cap; because of the slow response time of ice cap volume to climate change, relative to ice marginal (areal) response, disequilibrium ice caps are generally thinner.

Longitudinal stress coupling creates a slightly thinner ice cap, in better accord with the present-day configuration of Vatnajökull in both equilibrium and present-day (transient climate spin-up) simulations. The differences from the shallow ice approximation are on the order of a few %, with no significant difference in the ice cap margins. While longitudinal deviatoric stresses are significant in some regions on the ice cap, in particular the interior divide, deviatoric stress gradients are much less. On average over the ice cap, longitudinal deviatoric stress gradients ($G$) are

![Figure 16](image_url). Time series of (a) mean annual air temperature over the ice cap, °C, and (b) ice cap volume, km$^3$, from 1800 to 2300. The thick solid line indicates a sea level warming scenario of 2°C per century, and the dashed line is for the NCAR CCSM climate change scenario. The simulation including longitudinal stress coupling is shown with a thin solid line, indiscernibly different from the thick solid line for most of the simulation.

![Figure 17](image_url). Modeled ice cap surface elevation for the hydrologically coupled simulation for (a) 1990, (b) 2100, (c) 2200, and (d) 2250. The observed present-day ice distribution is indicated with the white outline. The ice cap disappears by 2300 under the 2°C per century warming scenario.
4% of the gravitational driving stress ($\tau_d$). This confirms that, for Vatnajökull modeling at this resolution, it is valid to introduce the effects of longitudinal stress coupling as a perturbation to the mean flow, which is driven by local gravitational driving stress and is captured by the shallow ice approximation. However, the improvements to model skill gained through longitudinal stress, at least as we implement it, are modest, and may not be warranted at our operational resolution of $\approx1.7\text{ km}$. This conclusion may not hold at finer resolutions, which would entail less smoothing of the topography.

[106] A more realistic representation of Vatnajökull was achieved in simulations with fully coupled ice dynamics and subglacial hydrological evolution. With basal flow proportional to the flotation fraction, $p_c/p_s$, increased basal flow is predicted in bedrock overdeepenings of some of the major Vatnajökull outlets, drawing down interior ice and creating a thinner ice cap that is in close accord with the actual present-day ice cap area and volume. Some regions of the ice cap are systematically too thick or too thin in all reconstructions, including that with hydrological coupling, but the systematic biases were reduced with the more realistic basal sliding controls offered by the hydrological evolution. The average ice thickness misfit in all model grid cells decreased from 98 m in the standard model to 77 m with hydrological coupling.

[101] We explored the impacts of a simple parameterization of the heat flux from the Grímsvötn and Skáftafell geothermal cauldrons. While fundamentally different in nature, these have a similar overall impact to the effects of longitudinal stress coupling. Ice volume and average ice thickness are reduced due to high rates of basal melt ($\sim6\text{ m yr}^{-1}$) and surface drawdown in the interior of the ice cap, for both equilibrium and present-day reconstructions. The effect is again on the order of a few %, and even a simple parameterization of Vatnajökull’s geothermal hot spots, as included here, is recommended to capture their influence on overall ice cap dynamics. We did not fully explore their role in Vatnajökull in this paper, as the basal meltwater produced in these geothermal hot spots typically drains by episodic outburst floods that we do not yet have the capacity to simulate. Additional process physics are needed to examine the potential role of the subglacial lakes and jökulhlaups on ice cap dynamics.

[102] Flowers et al. [2005] provide a detailed examination of future climate change scenarios for Vatnajökull, more comprehensive than what is explored here. Our simulations confirm that Vatnajökull is very sensitive to small, sustained temperature shifts. The modeled ice volume response time to climate perturbations is of order several decades to one temperature shifts. The modeled ice volume response time confirms that Vatnajökull is very sensitive to small, sustained climate perturbations, probably underestimating the vulnerability of Vatnajökull’s future warming.

[104] We are not able to simulate surge cycles in the present model, in part due to an incomplete understanding of the subglacial processes that trigger a surge. They do not arise spontaneously in the hydrologically coupled model, which points to missing process physics with respect to the drainage configurations (or transient failures in the drainage system) that give rise to surge events. This may be due to our oversimplified characterization of basal flow, which does not properly address the processes of decoupled sliding or sediment deformation. Our drainage system is also oversimplified, as we do not have channelized drainage in these simulations. Since 100% of basal water drainage is through the groundwater system or diffusive sheet flow, these drainage mechanisms may be tuned to be too efficient in our simulations. This would suppress the large-scale ponding that is necessary to induce surge events. We also restrict our numerical experiments to 5-year updates of the hydrological system, therefore neglecting seasonal cycles of subglacial drainage evolution and basal flow. Since the seasonal evolution of the hydrology system may be important to surge dynamics, this needs to be explored in future studies of Vatnajökull ice cap dynamics.

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