Applying UBC-GIF potential field inversions in greenfields or brownfields exploration (MAG3D and GRAV3D)

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Geologically realistic inversion of gravity and magnetic data
Melbourne – July 1, 2006
We wish we could…
But inverting the data alone gives…
Adding geological constraints gives...
What’s the difference?

- Infinite number of solutions
- Geophysical data and mathematics are the only constraints
- Model does not have to be geological
- Could be a decent first guess if nothing else is known
- If you don’t agree with the model you MUST have some other information

- Infinite number of solutions
- Geophysical data, mathematics, and expected geology are constraints
- May start to be predictive of geology, because model is based on some geological expectations
- If you don’t agree with the model you MUST have some other information
Types of geological constraints

- Greenfields → brownfields
- Greenfields → brownfields
- Greenfields → brownfields
- Brownfields
- Brownfields
- Greenfields → brownfields

Framework
- Geophysical data
- Surface information
- Drill holes
- Lithology volumes
- Cross sections
- Zones

Physical properties
- Orientations
- Positions & shapes

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Inversion

- A geophysical survey, measures:

  \[ d^{\text{obs}} = \varepsilon + d^{\text{true}} \]

  \[ d^{\text{true}} = Gm \]

- Inversion seeks to find \( m \) given \( d^{\text{obs}} \)
Inversion problems

• Several problems arise in obtaining a solution to the inverse problem:
  ► **Underdetermined**: More unknowns \((m)\) than data \((d^{\text{obs}})\)
  ► **Non-existence**: Noise may mean that there is NO model that can fit the data
  ► **Instability**: Small changes in the data, *especially noise*, can cause large changes in the recovered model
  ► **Non-uniqueness**:
    - Even a finite number of noise-free data can be reproduced in an infinite number of ways
    - Potential field data has no inherent depth information – single layer at surface could fit the data as well as a detailed 3D model

• Calculate the inverse model solution using optimisation
  ► Optimal model and optimal data misfit
Model objective function: Smallness

\[
\phi_m(m) = \alpha_s \int w_s \left( w_r(z) (m - m_{ref}) \right)^2 dV + ...
\]

- Attempts to match the recovered model to a reference model (small differences)
  - Specific cells can be matched more closely than others

- \( m \): Recovered model – our result

- \( m_{ref} \): Reference model – expected physical properties

- \( w_r(z) \): Depth weighting function
  - Balances decay of potential field response with increasing depth in the model

- \( w_s \): Cell smallness – confidence in reference model for each cell

- \( \alpha_s \): Model smallness – confidence in whole reference model
Property-based constraints

- Smallness weights ($w_s$)
  - Default is 1: “low confidence”
  - Weights are relative
  - $w_s = 10$ is $10 \times$ more confident than $w_s = 1$
  - Higher smallness weights $\rightarrow$ closer match
  - May compensate elsewhere in the model
Model objective function: Smoothness

\[ \phi_m(m) = \alpha_s \int_V w_s \left[ w_r(z)(m - m_{\text{ref}}) \right]^2 dV + ... \]

\[ \alpha_x \int_V w_x \left[ \frac{\partial}{\partial x} w_r(z)(m - m_{\text{ref}}) \right]^2 dV + ... \]

- Spreads differences between the recovered model and reference model over several cells in the \( x \)-direction
  - Allows specific pairs of cells to vary more or less smoothly
- \( w_x \): Smoothness across each cell boundary in the \( x \)-direction
- \( \alpha_x \): Model smoothness in the \( x \)-direction
**Position-based constraints**

- Default reference model is zero everywhere

- Smoothness weights \((w_x, w_y, w_z)\)
  - Default is 1: “moderately smooth”
  - Weights are relative
    - \(w_x = 10\) means smoothness is 10 \(\times\) more important than \(w_x = 1\)
  - Values < 1 indicate promote roughness
Model objective function: Smoothness

\[ \phi_m(m) = \alpha_s \int_V w_s \left[ w_r(z)(m - m_{ref}) \right]^2 dV + \ldots \]

\[ \alpha_x \int_V w_x \left[ \frac{\partial}{\partial x} w_r(z)(m - m_{ref}) \right]^2 dV + \ldots \]

\[ \alpha_y \int_V w_y \left[ \frac{\partial}{\partial y} w_r(z)(m - m_{ref}) \right]^2 dV + \ldots \]

\[ \alpha_z \int_V w_z \left[ \frac{\partial}{\partial z} w_r(z)(m - m_{ref}) \right]^2 dV \]

- Smoothes model differences in the y- and z-directions
- \( \alpha_y, \alpha_z, w_y, w_z \)
Orientation-based constraints

Smoothness \((w_x)\)

Recovered model \((m)\)

Reference model \((m_{\text{ref}})\)

Smoothness \((w_z)\)

Ref. model \((m_{\text{ref}})\)

Recovered model \((m)\)

Cross-section

Greenschist A

Greenschist B

Fault

Granite

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Balancing model smoothness and smallness

• Balance is defined for the whole model by 4 user-defined $\alpha$ parameters
  ► Fundamentally controlled by cell size-squared:

\[ \alpha_x = c_x \cdot \alpha_s (\Delta x)^2 \]
\[ \alpha_y = c_y \cdot \alpha_s (\Delta y)^2 \]
\[ \alpha_z = c_z \cdot \alpha_s (\Delta z)^2 \]

• $\Delta x$, $\Delta y$, and $\Delta z$: cell dimensions in each direction

• $\alpha_s$: Proportion of smallness: default = 0.0001

• $c_x$, $c_y$, and $c_z$: Model smoothness parameters in each direction
  ► Values of 1 should equally balance smallness versus smoothness
Model smoothness parameters: $c_x, c_y, c_z$

- Typically use values between 4 and 25
  - Equivalent to old length scales: $L_x = \Delta x \cdot \sqrt{c_x} = 2\Delta x$

- Geological strike or continuity can be reproduced with high $c$ values
  - Empirical rule of thumb: $c_x \geq 2 \cdot (n_x - 7)$ for $n_x \geq 9$

![Diagram showing the effect of different $c$ values on cell property](image-url)
Data misfit

\[ \phi_d = \left| W_d (Gm - d^{obs}) \right|^2 \]

\[ W_d = \text{diag} \left( \frac{1}{\sigma_i} \right) \]

- Ensures the response of the recovered model matches the observed data, to within uncertainty

- \( W_d \): Weights based on expected standard deviations (\( \sigma_i \)) for each data point
  - Large uncertainty \( \rightarrow \) fit that data point less closely

- \( Gm \): Predicted response of the recovered model

- \( d^{obs} \): Observed data
Obtaining a solution: Optimisation

\[
\begin{align*}
\text{minimize } & \phi = \phi_d + \beta \phi_m \\
\text{such that } & \phi_d = \phi_d^*
\end{align*}
\]

- \(\phi\): Total objective
- \(\phi_m\): Model objective function
- \(\phi_d\): Data misfit
- \(\phi_d^*\): Target data misfit
  - Usually equals the number of data points
- \(\beta\): Trade-off parameter
  - Balances data misfit and model-objective function
Trade-off parameter

High $\beta$: Poor data fit; feature-less model

Low $\beta$: Close data fit; noisy model

$\phi_d = N$

Just right...

$\phi_d$ (data misfit)

$\phi_m$ (model objective)
Additional non-linear constraints

- **Physical property bounds (lower bound and upper bound)**
  - Assigned for individual cells
  - Can be used with or without a reference model and smallness weights
  - Recovered model **MUST** lie between the lower and upper bounds
    - Extremely powerful for restricting possible model space
    - Can lead to convergence problems if bounds are too narrow over too much of the model

- **Positivity (only for magnetics)**
  - Enforces a lower bound of 0 SI
    - Only positive magnetic susceptibilities are recovered
Property-based constraints: another way

- Use wider bounds if less confident
- Use narrower bounds if more confident
Parameter summary

• Must set:
  ► Cell sizes ($\Delta x$, $\Delta y$, $\Delta z$)
    ■ Default model smallness and smoothness ($\alpha_s$, $\alpha_x$, $\alpha_y$, $\alpha_z$)
  ► Standard deviations of the data ($\sigma_i$)

• If geological information is available, set:
  ► Model smoothness parameters ($c_x$, $c_y$, $c_z$) $\rightarrow$ ($\alpha_x$, $\alpha_y$, $\alpha_z$)
  ► Cell reference model ($m_{ref}$) & smallness weights ($w_s$)
  ► Cell property bounds (lower, upper)
  ► Cell boundary smoothness weights ($w_x$, $w_y$, $w_z$)

• No need to change (but is possible):
  ► Depth weighting
  ► Trade-off parameter ($\beta$)
UBC-GIF inversion summary

• Minimum input required is geophysical data and a 3D mesh

• Resulting default recovered model will reproduce the data to within uncertainty, with a model that appears “small” and “smooth”

• 3 ways to include detailed geological information
  ► Property-based:
    ■ Reference model with smallness weights
    ■ Bounds
  ► Position-based:
    ■ Smoothness weights
  ► Orientation-based:
    ■ Smoothness weights

• Use which ever methods are most appropriate given the available geological information
Comparison of property-based constraints

<table>
<thead>
<tr>
<th>STYLE:</th>
<th>Default (Smooth model)</th>
<th>Geologically-constrained (Smooth model-difference)</th>
<th>Geologically-constrained (Smooth model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTRAINTS:</td>
<td>Default zero reference model</td>
<td>Non-zero reference model</td>
<td>Default zero reference model</td>
</tr>
<tr>
<td></td>
<td>Default smallness weights</td>
<td>Variable smallness weights</td>
<td>Default smallness weights</td>
</tr>
<tr>
<td></td>
<td>Default (wide) bounds</td>
<td>Any bounds</td>
<td>Narrow bounds</td>
</tr>
</tbody>
</table>

RESULT:
- Model is small and smooth
- Model has breaks where reference model changes
- Model has breaks where bounds change sharply

WHEN TO USE:
- No geological constraints are available
- Expected properties and full extent of bodies is known (full 3D model)
- Range of properties is known at scattered locations (surface, drill holes, cross-sections)
Position- and orientation-based constraints

Granite
Default smoothness:
- $w_x = 1$
- $w_y = 1$
- $w_z = 1$

Greenschist A
North-south strike, shallow dip:
- $w_x = 4$
- $w_y = 10$
- $w_z = 1$

Greenschist B
East-west strike, steep dip:
- $w_x = 10$
- $w_y = 1$
- $w_z = 4$

Vertical fault
No smoothness:
- $w_x = 0$
- $w_y = 4$
- $w_z = 4$
Options for geological constraints

Property information: $m_{\text{ref}}$ & $w_s$, and/or bounds

Position/orientation information: $w_x$
Brief inversion preparation checklist

1. Define problem to be addressed
2. Define volume of interest (depth, width and length of desired mesh)
3. Define data area
4. Define cell sizes
5. Pad the mesh to prevent boundary effects
6. Upward continue the potential field data to the width of the cells
   • Removes high frequencies that can only be attributed to smaller cells
7. Calculate and remove any regional data trend (Li and Oldenburg, 1998)
A synthetic geological example

• Simple geological model
  ► North-south-striking granite-greenstone belt
  ► Some outcropping ultramafic units and sulphides, but more expected under extensive cover
## Densities and density contrasts

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Actual (g/cm³)</th>
<th>Basic sampling Minimum (g/cm³)</th>
<th>Basic sampling Maximum (g/cm³)</th>
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<tr>
<td>Regolith and sediments</td>
<td>2.0</td>
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<td>2.4</td>
<td>1.85</td>
<td>2.15</td>
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<tr>
<td>Sulfides</td>
<td>3.8</td>
<td>3.4</td>
<td>4.2</td>
<td>3.65</td>
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<tr>
<td>Ultramafics</td>
<td>3.1</td>
<td>2.75</td>
<td>3.2</td>
<td>2.95</td>
<td>3.25</td>
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<td>2.6</td>
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<td>2.95</td>
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- Converting between density and density contrast → approximate
  - Estimate average density expected within inversion volume
    - Perhaps the average density of most abundant rock (→ median)
  - Subtract average density to get density contrast:
    \[ \Delta \rho = \rho - \bar{\rho}, \quad \bar{\rho} \approx 2.8 \text{ g/cm}^3 \]

- No conversion required for magnetic susceptibility
### Densities and density contrasts

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<th>−0.8</th>
<th>−1.1</th>
<th>−0.4</th>
<th>−0.95</th>
<th>−0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>−0.1</td>
<td>−0.15</td>
<td>+0.05</td>
<td>−0.25</td>
<td>+0.05</td>
</tr>
<tr>
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<td>+1.0</td>
<td>+0.6</td>
<td>+1.4</td>
<td>+0.85</td>
<td>+1.15</td>
</tr>
<tr>
<td>Ultramafics</td>
<td>+0.3</td>
<td>−0.05</td>
<td>+0.4</td>
<td>+0.15</td>
<td>+0.45</td>
</tr>
<tr>
<td>Metamorphics</td>
<td>0.0</td>
<td>−0.2</td>
<td>+0.2</td>
<td>−0.15</td>
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A synthetic geological example

- Synthetic gravity data, with noise added, upward continued to width of the cells

- Series of 3D gravity inversions incrementally adding geological constraints as they become available during exploration and development
Inversion setup

• Mesh (volume of interest): 42,000 cells (+ extra padding)
  ► East-west: 70 × 100 m cells (7 km)
  ► North-south: 30 × 100 m cells (3 km)
  ► Vertical: 20 × 50 m cells (1 km)

• Assigning $\alpha$'s
  ► Use default model smoothness parameters: $c_x = c_z = 4$
  ► Know north-south strike:
    ■ $c_y \geq 2 \cdot (n_y - 7)$: $n_y = 20$ cells (2 km) $\rightarrow c_y = 26$
    ■ $\alpha_s = 0.0001$, $\alpha_x = 4$, $\alpha_y = 26$, $\alpha_z = 1$
Default inversion

- No geological constraints
- Zero reference model
- Default bounds
Default inversion

- No geological constraints
- Zero reference model
- Default bounds
Add outcrop (smooth $m - m_{ref}$)

- Surface mapping
- Some surface density measurements
- Estimate background density = 2.8 g/cm$^3$

- Reference model for surface cells based on mapping and mean densities
- Default bounds
Add outcrop (smooth $m$)

- Surface mapping
- Some surface density measurements
- Zero reference model
- Narrow bounds for surface cells based on measured densities

Greenfields → brownfields
Add geological zones (smooth $m$)

- General geological concepts based on expected variability
- Narrow bounds for surface cells based on measured densities
- Broad bounds based on concepts

**Zones (not coloured by density)**
- Regolith or basement: -1.1 to +1.4 g/cm$^3$
- Basement rocks: -0.2 to +1.4 g/cm$^3$

**Recovered densities**

**True densities**

Greenfields → brownfields
Add detailed surface sampling (smooth $m$)

- Detailed surface sampling over small area
- Tight ($\pm 0.15 \, \text{g/cm}^3$) bounds over $1 \, \text{km} \times 1 \, \text{km}$ area at surface

Zones (not coloured by density)

- Regolith or basement: $-1.1 \, \text{to} \, +1.4 \, \text{g/cm}^3$
- Detailed density sampling
- Basement rocks: $-0.2 \, \text{to} \, +1.4 \, \text{g/cm}^3$
- Basic density sampling

Recovered densities

True densities

Greenfields → brownfields
Add partial 3D model (smooth \( m \))

- 3D model based on shallow drilling, sampling and structural interpretation
- Tight (± 0.15 g/cm\(^3\)) bounds over 3D model that is 1 km \( \times \) 1 km to a depth of 350 m

---

**Zones (not coloured by density)**

- Regolith or basement: -1.1 to +1.4 g/cm\(^3\)
- Detailed density sampling and structural interp.
- Basic density sampling
- Basement rocks: -0.2 to +1.4 g/cm\(^3\)

**Recovered densities**

**True densities**

Greenfields → brownfields
Add 1 deep drill hole (smooth $m$)...

- Deep drilling with sampling
- 1 drill hole, 2 km long

Greenfields $\rightarrow$ brownfields
...Or many drill holes (smooth $m$)...

- Drilling: 7 holes in section, 14 out of section (up to 1.7 km away)
- Tight ($\pm 0.15 \text{ g/cm}^3$) bounds along drill holes
- No detailed surface sampling or 3D model

**Zones (not coloured by density)**

- Regolith or basement: -1.1 to +1.4 g/cm³
- Detailed density sampling along drill holes
- Basic density sampling
- Basement rocks: -0.2 to +1.4 g/cm³

**Recovered densities**

**True densities**

**Density**

<table>
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<th>Density</th>
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<th>2.6</th>
<th>2.8</th>
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<td>Density Contrast</td>
<td>-0.4</td>
<td>-0.2</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
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**Greenfields → brownfields**
...and structural trends (smooth $m$)

- Drilling: 7 holes in section, 14 out of section (up to 1.7 km away)
- Tight ($\pm 0.15$ g/cm$^3$) bounds along drill holes
- Smoothness weights based on structural trends and contacts

![Diagram showing density zones and recovered densities with greenfields to brownfields transition]
Inversion example summary

- **Only** difference is that geological information *routinely* collected in exploration and development was included as inversion constraints as *it became available*
Inversion example summary

Default inversion densities

Final geologically-constrained inversion densities

True densities

<table>
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<tr>
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AESC 2006
Both fit the data equally well
Inversion example summary

- Constraints used (in approximate order of usefulness):
  - Partial 3D model
    - Only if trustworthy and good property measurements are available
  - Many drill holes
    - Drill holes remain only way to get actual geological information from depth
  - Surface maps with basic density estimates
    - Identifying zones of weathered material is especially important
  - Geological concepts
    - Wider bounds where more geological variety is expected
    - Narrower bounds where geology is expected to be more homogeneous
  - Smoothness weights based on observed structural trends and contact positions
  - Detailed surface property measurements
Geological constraints summary

• A basic model can be obtained using no geological information
  ► Identify lateral positions of anomalies
  ► Identify relative magnitudes and sizes of anomalies
  ► Basic depth estimates
  ► Low confidence in targets

• A large range of geological information can be readily included to improve recovered models
  ► Mapping, density measurements, drilling, 3D models
  ► No “special” data requirements
  ► Include new geological data as it becomes available to refine model
    ■ Don’t need to create a full 3D model!

• Where constraints are sparse, use bounds instead of a reference model
  ► Extrapolate the constraints out into the model
Some continuing research at UBC-GIF

• Other types of geological constraints
  ▶ How might structural style be imparted on a model?
  ▶ Supplying smoothness weights in non-orthogonal directions

• Other forms of regularisation
  ▶ “Blocky” models, sharp contacts

• How to ease the inclusion of constraint information?

• Volume-scaling of physical property measurements

• Interpreting and classifying physical property models in terms of geology
  ▶ How can we get geology instead of physical properties?
  ▶ Mapping mineralogy in 3D
Acknowledgments

Ph.D. project sponsors:
Geoscience Australia, *pm$d*CRC, UBC-MDRU, UBC-GIF, BHP Billiton

Ph.D. supervisors:
Prof. Dick Tosdal (UBC-Mineral Deposit Research Unit) &
Prof. Doug Oldenburg (UBC-Geophysical Inversion Facility)

UBC-Geophysical Inversion Facility:
Nigel Phillips, Roman Shekhtman, and Peter Lelievre
Depth weighting

\[ w_r^2(z) = \frac{1}{(z + z_0)^eta} \]

- \( z \): Depth to cell’s centre

- \( z_0 \): Adjustable parameter to match potential field’s decay with depth
  - Approximately half the cell height

- \( \beta \): Adjustable parameter to match potential field’s decay with depth
  - Magnetic data: Response of a spherical source decays with distance cubed → \( \beta = 3 \)
  - Gravity data: Response of a spherical source decays with distance squared → \( \beta = 2 \)

- Gives all cells, regardless of depth, equal likelihood of containing features