Electromagnetic coupling in frequency-domain induced polarization data: a method for removal

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SUMMARY
Electromagnetic (EM) coupling is generally considered to be noise in induced polarization (IP) data and interpretation is difficult when its contribution is large compared to the IP signal. The effect is exacerbated by conductive environments and large-array survey geometries designed to explore deeper targets. In this paper we present a methodology to remove EM coupling from frequency-domain IP data. We first investigate the effect of EM coupling on the IP data and derive the necessary equations to represent the IP effect for both amplitude and phase responses of the signal. The separation of the inductive response from the total response in the low-frequency regime is derived using the electric field due to a horizontal electric dipole and it is assumed that at low frequencies the interaction of EM effects and IP effects is negligible. The total electric field is then expressed as a product of a scalar function, which is due to IP effects, and an electric field, which depends on the EM coupling response. It is this representation that enables us to obtain the IP response from EM-coupling-contaminated data. To compute the EM coupling response we recognize that conductivity information is necessary. We illustrate this with a synthetic example. The removal method developed in this work for the phase and the per cent frequency effect (PFE) data are applicable to 1-D, 2-D and 3-D structures. The practical utility of the method is illustrated on a 2-D field example that is typical of mineral exploration problems.

Key words: electromagnetic coupling, induced polarization, inversion, mineral exploration, noise.

1 INTRODUCTION
Electromagnetic (EM) coupling can be defined as the inductive response of the Earth, which manifests itself as a response over the induced polarization (IP) signal. EM coupling is a major impediment to the interpretation of complex resistivity and IP data (Wait & Gruszka 1986; Wynn & Zonge 1977). The effect is increased when the survey is carried out over a conductive Earth or when the dipole length and dipole spacings are large. This has practical consequences because the depth of exploration depends upon the largest dipole length and dipole spacings. For many surveys it is necessary to remove EM coupling from IP data in order to retrieve meaningful information about the chargeable zones in the subsurface.

The removal of EM coupling has been a long-standing problem in IP data interpretation. As in most geophysical exploration surveys, the initial step is to eliminate the noise at the data acquisition level. This is achieved by sampling the later time channels in time-domain surveys or by using only low frequencies in the frequency domain. However, the objective is not achieved even at low frequencies when the medium is conductive or when the dipole lengths are large. The contamination can be significant and can produce artefacts in the inversion results, and this has prompted us to try and develop a practical methodology to remove EM coupling so that the resultant data can be inverted with standard IP inversion algorithms.

Three decades of research on EM coupling removal has evolved principally in three directions. The first method is based on the assumption that the IP phase is constant (Hallof 1974; Coggon 1984) or a linear function of the logarithm of frequency (Song 1984), and that the phase due to EM coupling is represented by a power series in frequency. The observed phases at two or three frequencies were used to determine the coefficients of the linear or quadratic equation. Since the IP phase was assumed constant, the zeroth power coefficient was then the measure of the decoupled IP phase. This is sometimes referred to as the three-point decoupling scheme.
techniques assume a particular polynomial form of frequency to represent the EM coupling response, and treat the IP phase as constant or linear. However, previous investigations suggest that the IP phase is not always constant, nor linear, and can vary depending on various factors such as mineral texture, grain size and type of mineral (Pelton et al. 1978; Van Voorthuis et al. 1973; Seigel et al. 1997). Since these methods do not take into account all of the physics associated with the process, their application can be questionable. Their principal advantage, however, is that they are simple to use.

The second method is based on approximating the low-frequency IP data contaminated by the EM coupling response by a double Cole–Cole model (Pelton et al. 1978; Brown 1985). Investigations with this method have revealed that the time constant for the polarization effect ($\tau_{ip}$) is greater than the time constant obtained for the EM coupling response ($\tau_{EM}$), and the relaxation constant for IP ($\tau_{ip}$) is less than the relaxation constant for EM coupling ($\tau_{EM}$). Major & Silic (1981) have pointed out the limitations of using this model, which is based on an empirical approach to fitting the data, and have indicated that the frequency dependences of the IP and EM effects are, in general, more complicated than a multiplication of two Cole–Cole models.

The third approach is to assume that the Earth is one dimensional and compute the EM coupling response by evaluating the mutual impedance of the grounded circuits at the desired frequency (Millett 1967; Dey & Morrison 1973; Hohmann 1973; Wynn & Zonge 1975, 1977; Wait & Gruszka 1986). Wait & Gruszka (1986) computed the impedance due to EM coupling for a homogeneous half-space model and subtracted this from the observed impedance, which has both IP and EM coupling responses. They showed that this is valid at low frequencies where the interaction of EM and IP effects is negligible, or, equivalently, when the propagation constant of the diffusive EM waves is not greatly altered by the chargeability of the medium.

Our work follows from the third approach but the formulation and removal methodology are substantially different from theirs. We formulate the problem in terms of the electric field and show that the observed electric field of a half-space can be expressed approximately as a product of an IP response function and an electric field due to the EM coupling response. This approach is different from the subtraction of the EM coupling impedance from the observed impedance proposed by Wait & Gruszka (1986). Either the phase or the amplitude of the IP response function is an indicator of chargeability, and our representation of the EM fields enables us to remove EM coupling from the per cent frequency effect (PFE) and phase data. Our methodology is in principle applicable to 3-D earth models.

This paper is laid out as follows. We first present the effects of EM coupling on IP data and define the various electric fields for a homogeneous half-space model, and discuss the factors affecting the EM coupling responses. Using these electric fields we derive an IP response function that is valid when the propagation constant of the diffusive EM waves is not greatly altered by chargeability. The validity of this approximation is discussed for various combinations of the parameters. Next we apply the removal procedure to phase and PFE data in a 1-D synthetic example. The methodology of EM coupling removal in two dimensions is outlined and the procedure is applied to a field data set. The paper concludes with a discussion.

2 EFFECTS OF EM COUPLING ON IP DATA

The problem with EM coupling is that both IP and EM coupling effects occur simultaneously in a certain frequency bandwidth. To be able to distinguish the different responses we first define the various electric fields that are associated with these effects. We consider a half-space complex conductivity model of the Earth. The electric field in the x-direction due to an x-directed horizontal electric dipole (HED) at non-zero frequency is given by (Sunde 1968; Ward & Hohmann 1988, p. 233, eq. 4.153)

$$E_{(io)} = \frac{-ids}{2\pi r^3\sigma_{(io)}} \left[ 1 - (1 + i\omega) e^{-ikr} \right] + \frac{Ids}{2\pi \sigma_{(io)}} \left( \frac{2x^2 - y^2}{r^5} \right),$$

(1)

where $k^2 = i\omega \sigma_{(io)}$ is the propagation constant of the medium, $\omega$ is the angular frequency, $\mu$ is the magnetic permeability, $\sigma_{(io)}$ is the complex conductivity, $r = \sqrt{x^2 + y^2}$ is the distance from the dipole and $Ids$ is the current element. The complex conductivity $\sigma_{(io)}$, or its reciprocal complex resistivity $\rho_{(io)}$, can have many forms. A general representation is

$$\rho_{(io)} = \rho_0 \left[ 1 - m(\omega) \right],$$

(2)

where $\rho_0$ is the DC resistivity, or the resistivity at zero frequency. $m(\omega)$ is the complex chargeability, which depends on the intrinsic chargeability, frequency and additional parameters. For example, in the Cole–Cole model, $m(\omega)$ is given by

$$m(\omega) = \eta \left( \frac{1}{1 + \frac{\omega}{\omega_p}} \right),$$

(3)

where $\eta$ is the intrinsic chargeability ($0 \leq \eta \leq 1$), $\omega$ is the frequency, $\tau$ is the time constant and $\epsilon$ is the relaxation constant. We note that $\lim_{\omega \to 0} m(\omega) = 0$ and $\lim_{\omega \to \infty} m(\omega) = \eta$, which implies that the DC conductivity $\sigma_0 = \sigma_{\infty}(1 - \eta)$, where $\sigma_{\infty}$ is the conductivity in the absence of chargeability.

The electric field due to the source will vary with frequency and with intrinsic chargeability. It is convenient to introduce the four different electric fields explicitly.

(i) $E_{DC}$ is the electric field that exists when $\omega = 0$ but there are no effects of chargeability ($\eta = 0$). The resistivity of the medium is therefore equal to $\rho_{DC} = \frac{1}{\sigma}. Setting the propagation constant $k = 0$ in eq. (1) and replacing the conductivity by $\sigma_{\infty}$ yields

$$E_{DC} = \frac{Ids}{2\pi \sigma_{\infty}} \left( \frac{2x^2 - y^2}{r^5} \right),$$

(4)

(ii) $E_{DCIP}$ is the electric field that exists when $\omega = 0$ and the medium is chargeable ($\eta \neq 0$). The resistivity $\rho_{DCIP} = \frac{1}{\sigma_{\infty}}$. Setting the propagation constant $k = 0$ in eq. (1) and replacing $\sigma_{(io)}$ by $\sigma_0$ yields

$$E_{DCIP} = \frac{Ids}{2\pi \sigma_0} \left( \frac{2x^2 - y^2}{r^5} \right).$$

(5)

(iii) $E_{EM}$ is the electric field in eq. (1) when $\sigma_{(io)}$ is any complex conductivity function. It includes electromagnetic induction and chargeability effects at non-zero frequency. The propagation constant $k_{EM} = \sqrt{i\omega \sigma_{(io)}}$ depends on the conductivity and chargeability of the medium, and the electric field
is written as
\[ E^{\text{EMIP}}(io) = \frac{-Ids}{2\pi r^2\sigma_0}\left[1 - (1 + ik^{\text{EMIP}}) e^{-ik^{\text{EMIP}}}r\right] + \frac{Ids}{2\pi r}\frac{(2x^2 - y^2)}{r} \]  
(6)

We note that \( E^{\text{EMIP}} \rightarrow E^{\text{DCIP}} \) as the frequency approaches zero \((\omega \rightarrow 0)\).

(iv) \( E^\text{EM} \) is the electric field obtained at non-zero frequency \((\omega \neq 0)\) when the chargeable effects are not considered to alter the conductivity \((\eta = 0)\). As in the definition of \( E^{\text{DC}} \), we have \( \rho(\omega) = \rho_\infty = 1/\sigma_\infty \). However, since \( \omega \neq 0 \), the wavenumber \( k^\text{EM} = \sqrt{i\omega\sigma_\infty} \) and the electric field is given by
\[ E^\text{EM}(io) = \frac{-Ids}{2\pi r^2\sigma_\infty}\left[1 - (1 + ik^\text{EM}) e^{-ik^\text{EM}}r\right] + \frac{Ids}{2\pi r\sigma_\infty}\frac{(2x^2 - y^2)}{r^2} \]  
(7)

The expression for \( E^\text{EM} \) is similar to eq. (6) except that the complex conductivity \( \sigma(\omega) \) is replaced by \( \sigma_\infty \). We note that \( E^\text{EM} \rightarrow E^{\text{DC}} \) as \( \omega \rightarrow 0 \).

In an IP survey, finite dipole lengths are used and the measured quantity is impedance rather than electric field. Formulation in terms of impedance can be obtained by integrating the electric field along the transmitter and receiver dipole lengths and normalizing by the current. For example, the measured EMIP impedance can be written as
\[ Z^{\text{EMIP}}(io) = \frac{1}{I} \int_{TX} \int_{RX} E^{\text{EMIP}}(io) rdrds, \]  
(8)

where \( dr \) and \( ds \) are the elemental lengths of the receiver and transmitter respectively, and \( E^{\text{EMIP}}(io) \) is the electric field due to a horizontal electric dipole. Typically, the IP data are represented in terms of the PFE and/or phase responses. The PFE responses are generated from amplitude measurements at two widely separated frequencies \( \omega_1 \) and \( \omega_2 \). If \( \omega_2 > \omega_1 \), then the observed PFE response is given by
\[ PFE^{\text{OBS}} = 100 \left(1 - \frac{|Z^{\text{EMIP}}(io)|}{|Z^{\text{EMIP}}(io)|_1}\right). \]  
(9)

The phase response is defined as the phase of the measured impedance, \( Z^{\text{EMIP}}(io) \). This is a single frequency measurement and is given by
\[ \phi^{\text{OBS}}(\omega) = \tan^{-1}\left(\frac{\text{Im}[Z^{\text{EMIP}}(io)]}{\text{Re}[Z^{\text{EMIP}}(io)]}\right). \]  
(10)

In order to gain some insight into how the EM coupling responses affect the IP data we compute the PFE and phase responses for a homogeneous half-space model in the absence of chargeability but with EM coupling. The corresponding electric field is given by eq. (7), and the factors affecting the EM coupling responses are (i) dipole length, (ii) dipole spacing, (iii) conductivity of the medium and (iv) frequency of the transmitting signal. We consider a dipole–dipole survey over a half-space of conductivity \( 0.05 \) S m\(^{-1} \) (200 ohm m). The basic parameters are shown in Fig. 1(a). The test concentrates on the phase data, but similar conclusions hold for the PFE data. Fig. 1(b) shows that the phase response increases with dipole length and dipole spacing. For a grounded wire, the endpoints of the wire contribute to galvanic currents in the ground and the rest of the wire acts as an inductive source. Thus, with larger dipole lengths, the magnitude of the inductive source increases and this results in the increase of the EM coupling response. This implies that for pole–dipole and pole–pole survey geometries, the EM coupling contribution will be large compared to dipole–dipole surveys since the lengths of the grounded wire are much larger. Figs 1(c) and (d) show that EM coupling increases with increasing conductivity and frequency. Because the propagation constant depends upon the product of frequency and conductivity, \( k = \sqrt{i\omega\sigma_\infty} \), the inductive effect increases when either conductivity or frequency increases. These conclusions are in agreement with comments made by Dey & Morrison (1973), Wynn & Zonge (1975) and Ward (1990). These high phase responses that arise as a result of EM coupling can be misinterpreted as IP responses and can mask the IP anomaly. In the next section we discuss the removal issue and derive an expression for decoupling the EM coupling responses from the data.

3 THE REMOVAL OF EM COUPLING

In this section we derive the EM coupling removal equations based on the electric fields defined in the previous section. The formulation can be extended to include the effects of finite length grounded wires by integrating the electric field along the transmitter and receiver dipole lengths. Substituting eq. (2) into eq. (6) and using the relation \( \sigma_0 = \sigma_\infty (1 - \eta) \), we obtain
\[ E^{\text{EMIP}}(io) = \frac{1 - m(io)}{(1 - \eta)} \left(\frac{-Ids}{2\pi r^2\sigma_\infty}\left[1 - (1 + ik^{\text{EMIP}}) e^{-ik^{\text{EMIP}}r}\right] + \frac{Ids}{2\pi r\sigma_\infty}\frac{(2x^2 - y^2)}{r^2}\right) \]  
(11)

If we assume that the propagation constant is not greatly altered by the chargeability of the medium then we can write
\[ k^\text{EM}(\sigma(io), \omega) \approx k^\text{EM}(\sigma_\infty, \omega). \]  
(12)

A quantification of this approximation is presented in Appendix A. Substituting the approximation given in eq. (12) into eq. (11), we obtain a relation between the measured field \( E^{\text{EMIP}} \) and the computed field \( E^\text{EM} \). This is given by
\[ E^{\text{EMIP}}(io) \approx \frac{1 - m(io)}{(1 - \eta)} E^{\text{EM}}(io) = \psi(io) E^{\text{EM}}(io). \]  
(13)

Eq. (13) is the fundamental relationship between the fields with and without chargeability. It shows that the measured response \( E^{\text{EMIP}}(io) \), which is a combination of the IP effects and the EM induction effects, can be approximately expressed in a product form in which the quantity \( \psi(io) = (1 - m(io))/(1 - \eta) \) is the IP response, and \( E^{\text{EM}}(io) \) is the response due to induction and DC effects. So, with assumptions in eq. (12), it is possible to write the measured response \( E^{\text{EMIP}}(io) \) as a product of a complex scalar \( \psi(io) \) and the electric field that would exist in the absence of any polarization.

Fig. 2 shows the nature of the function \( \psi(io) \) for a Cole–Cole model with \( \eta = 0.2, \tau = 1.0 \) and \( \epsilon = 0.5 \). Fig. 2(a) indicates that the amplitude \( 1/|\psi(io)| \) monotonically increases from \((1 - \eta)\)–its low-frequency limit—to unity at high frequencies. The function \( \psi(io) \) is unity if there is no IP response, hence any deviation from unity is an indication of chargeable material.
It can also be unity if the frequency is sufficiently high, even if the medium is chargeable. This is expected, since at higher frequencies the charging process is interrupted so quickly that the chargeable material does not get sufficient time to build up the charges. This observation can also be explained using the electric field \( E_{\text{EMIP}} \) in eq. (6). If the medium is chargeable and if the frequency is high enough such that \( \sigma(\omega) \rightarrow \sigma_{\infty} \), then the electric field \( E_{\text{EMIP}} \rightarrow E_{\text{EM}} \). Thus, at high enough frequencies, the measured response does not have any IP effect.

If the real part, or the amplitude of \( y(\omega) \), is measured, then its greatest magnitude is at zero frequency. The phase of the function \( y(\omega) \) in Fig. 2(b) indicates that the IP phase is zero as frequency approaches zero or infinity and peaks near the frequency \( 1/\tau \). Thus, if a phase measurement is made to determine the maximum IP effect, the measurement is best made at an intermediate frequency near \( 1/\tau \).

Performing the integration on the fields in eq. (13), we obtain

\[
\psi(\omega) = \frac{1 - m(\omega)}{(1 - \eta)} \approx \frac{Z_{\text{EMIP}}(\omega)}{Z_{\text{EM}}(\omega)} \tag{14}
\]

In order to estimate the IP effect we can choose any measurement that quantifies how far \( \psi(\omega) \) deviates from unity. This can be represented in a number of ways but the two that are useful here are

\[
\text{IP amplitude} = 1 - \frac{1}{|\psi(\omega)|},
\]

\[
\text{IP phase} = \text{phase}(\psi(\omega)) \tag{15}
\]

We note that as \( \omega \rightarrow 0 \), the IP amplitude in eq. (15) \((1 - 1/|\psi(\omega)|) \rightarrow \eta\), which explains the choice of this particular form. In the next subsections we consider the amplitude and phase due to IP effects and discuss how they compare to the field measurements.
3.1 Amplitude response
The amplitude at any frequency $\omega$ can be obtained by substituting eq. (14) into eq. (15). This yields

$$\text{IP}_{\text{AMP}} = 100 \left( 1 - \left| \frac{Z_{\text{EM}}(i\omega)}{Z_{\text{IP}}(i\omega)} \right| \right), \quad (16)$$

Multiplication by 100 converts the response to a percentage. The IP amplitude response in eq. (16) is obtained at a single frequency. We note that the impedance $|Z_{\text{EM}}|$ in eq. (16) can be measured in the field at frequency $\omega$, whereas the impedance $|Z_{\text{EM}}|$ cannot be measured in the field. However, $|Z_{\text{EM}}|$ can be numerically computed at frequency $\omega$ if the conductivity structure $\sigma_s$ is available. The implication of this formulation is that if we collect amplitude data at a single frequency, and if we have knowledge about the conductivity of the ground, then eq. (16) can be used to obtain an estimate of the IP response of the chargeable material in the medium.

3.2 Phase response
In phase-domain IP surveys, the measured signal is the phase of the impedance at a particular frequency. The IP phase response according to eq. (15) is $\phi_{\text{IP}} = \text{Phase}(\psi(i\omega))$. Using eq. (14), this can be written as

$$\phi_{\text{IP}}(\omega) = \text{arg} \left( \frac{Z_{\text{EM}}(i\omega)}{Z_{\text{IP}}(i\omega)} \right) = \phi_{\text{EM}}(\omega) - \phi_{\text{EM}}(0). \quad (17)$$

Eq. (17) is used to correct for the EM coupling effects in the data. The phase due to EM coupling, $\phi_{\text{EM}}(\omega)$, is computed at the desired frequency $\omega$ using the conductivity structure $\sigma_s$.

3.3 The PFE response and its relation to $\text{IP}_{\text{AMP}}$
In this section we discuss the PFE response that is generated from amplitude measurements at two frequencies and outline its relationship to the IP amplitude response $\text{IP}_{\text{AMP}}$ defined in eq. (16). Traditional field practice is to acquire data at sufficiently low frequencies so that EM coupling is not important. This frequency range typically lies between 0.01 and 10 Hz and is sometimes referred to as the exploration frequency range. If EM coupling effects are negligible, then at low frequency, the amplitude of the impedance contains a greater IP signature than at higher frequency. Thus, at higher frequencies when the chargeable material is not sufficiently charged, the impedance response $Z_{\text{IP}}$ will be due to $\sigma_s$. At low frequencies the response will be due to $\sigma_0$, and the measured impedance $Z_{\text{DCIP}}$ has effects due to conductivity and chargeability. Ideally, field data are thus routinely collected at two different frequencies that are widely separated within the exploration frequency range (at least by one or two decades). Thus, if the ratio of the high-frequency amplitude to the low-frequency amplitude departs from unity, it indicates the presence of chargeable material in the ground. This is given by

$$\text{PFE}_{\text{IP}}(\omega_2 : \omega_1) = 100 \left( 1 - \frac{Z_{\text{DCIP}}(\omega)}{Z_{\text{DCIP}}(\omega_1)} \right), \quad (18)$$

Substituting the expression for $Z_{\text{DC}}$ and $Z_{\text{DCIP}}$ for a homogeneous half-space from eqs (4) and (5), we obtain $\text{PFE}_{\text{IP}} = 1000$. Next we compare the PFE in eq. (18) with the IP amplitude in eq. (16). We note that if the frequency $\omega \rightarrow 0$ then $Z_{\text{EM}} \rightarrow Z_{\text{DC}}$ and $Z_{\text{EMIP}} \rightarrow Z_{\text{DCIP}}$. This shows the relation between the two formulae. However, there is a fundamental difference between eqs (18) and (16). The equation for the IP amplitude in eq. (16) is obtained at a single frequency, whereas the PFE equation in (18) is obtained by taking measurements at two frequencies. It is because of this difference that we denote the response in eq. (18) as the IP amplitude instead of the PFE response.

The PFE response in eq. (18) is theoretically generated from amplitude measurements at zero and at ‘infinite’ frequency, where the chargeability effects are reduced to zero. However, in practice the amplitudes are measured at finite non-zero frequencies. The PFE response is then given by

$$\text{PFE}_{\text{IP}}(\omega_2 : \omega_1) = 100 \left( 1 - \frac{Z_{\text{DCIP}}(\omega_2)}{Z_{\text{DCIP}}(\omega_1)} \right), \quad (19)$$

where $\omega_2 > \omega_1$ and $Z_{\text{DCIP}}(\omega_2)$ is the impedance measured in the field when EM coupling effects are negligible. As $\omega_1 \rightarrow 0$, $\sigma(\omega_1) \rightarrow \sigma_0$ and $Z_{\text{DCIP}}(\omega_2) \rightarrow Z_{\text{DCIP}}$ given in eq. (5). For $\omega_2 \rightarrow \infty$, $\sigma(\omega_1) \rightarrow \sigma_s$ and $Z_{\text{DCIP}}(\omega_2) \rightarrow Z_{\text{DC}}$. This reproduces the PFE response in eq. (18). The impedance $Z_{\text{DCIP}}(\omega_2) = \psi(\omega_2) Z_{\text{DC}}$ can be obtained by solving the DC resistivity equation with complex conductivity $\psi(\omega_2) = \text{exp}[\text{arg}(\psi(i\omega)) i\omega]$. Integrating eq. (20) along the transmitter and receiver dipole lengths we obtain

$$Z_{\text{DCIP}}(\omega_2) = \psi(\omega_2) Z_{\text{DC}}. \quad (21)$$

3.4 Removal of EM coupling from the PFE responses
In frequency-domain IP, most of the work on EM coupling removal has been focused on phase data and there are not many studies that have addressed the removal problem for the PFE data (Wynn & Zonge 1977; Wang et al. 1985). Recognizing that the PFE data are generated from the amplitude measurements at two frequencies, we consider a practical scenario when EM coupling is present in the data. For data collected at two separate frequencies, $\omega_1$ and $\omega_2$, with $\omega_2 > \omega_1$, the PFE is given by

$$\text{PFE}^{\text{OBS}} = 100 \left( 1 - \frac{\text{IP}_{\text{AMP}}(\omega_2)}{\text{IP}_{\text{AMP}}(\omega_1)} \right), \quad (22)$$

The goal is to correct the data given in eq. (22) so that they are compatible with the PFE equation given in (19). We use eq. (21) and substitute the expression for $\psi(\omega_2)$ from eq. (14). This yields

$$Z_{\text{DCIP}}(\omega_2) = \psi(\omega_2) Z_{\text{EM}}(\omega_1) Z_{\text{DC}}. \quad (23)$$
Thus, to obtain the PFE response due to the IP effect, we substitute the expression for $Z_{DC\text{IP}}(i\omega)$ in eq. (23) into eq. (19) and denote the response as the corrected per cent frequency effect (PFE$^\text{CORR}$). This is given by

$$\text{PFE}^\text{CORR} = 100 \left(1 - \frac{|Z_{\text{EMIP}}(i\omega_2)||Z_{\text{EM}}(i\omega_1)|}{|Z_{\text{EMIP}}(i\omega_1)||Z_{\text{EM}}(i\omega_2)|}\right). \quad (24)$$

Eq. (24) is the fundamental equation for correcting the PFE responses. We note that if $\omega_1 \to 0$ then $Z_{\text{EMIP}}(i\omega_1) \to Z_{\text{DCIP}}$, $Z_{\text{EM}}(i\omega_1) \to Z_{\text{EM}}(i\omega_2)$, and if $\omega_2 \to \infty$ then $Z_{\text{EMIP}}(i\omega_2) \to Z_{\text{EM}}(i\omega_2)$ since there is no IP effect at very high frequencies. This gives eq. (18), as expected.

To compute the expression in eq. (24) we need to compute $|Z_{\text{EM}}(i\omega_1)|$ and $|Z_{\text{EM}}(i\omega_2)|$ using the conductivity structure $\sigma_s$; the responses $|Z_{\text{EMIP}}(i\omega_1)|$ and $|Z_{\text{EMIP}}(i\omega_2)|$ are available from the field measurements. An estimate of $\sigma_s$ can be obtained by inverting the amplitude data collected at the lowest frequency using a DC resistivity inversion algorithm. The conductivity

\begin{align*}
\sigma_s & = 0.125 \text{ S/m} \\
\sigma & = 0.01 \text{ S/m} \\
\sigma & = 0.02 \text{ S/m}
\end{align*}

Figure 3. (a) A synthetic three-layer model used for testing the EM coupling removal method for PFE and phase data. (b) The PFE data due to IP effects and EM coupling are indicated by the EMIP curve generated from amplitude data at 0.1 and 5 Hz. The PFE due to pure IP effects is indicated by the IP curve. The corrected PFE indicated as CORR shows a reasonable match with the true IP curve. (c) The phase of the corrected IP response is compared with the true IP response generated at 0.5 Hz. The EM coupling phase (denoted by EM) is subtracted from the observed phase (EMIP) to give the corrected IP phase. (d) The corrections obtained using different conductivity models. A homogeneous half-space of 0.01 S m$^{-1}$ (CORR$_{0.01}$) shows an undercorrection and a homogeneous model with 0.1 S m$^{-1}$ (CORR$_{0.1}$) shows an overcorrection of the PFE. Correction with a DC conductivity model obtained by inverting the amplitude data at 0.1 Hz (CORR$_{dc}$) indicates a close match with the IP curve. Similar conclusions hold for the phase data corrections in (e).
The EM coupling removal equation in (14) is derived using a homogeneous earth model. We expressed the IP response function \( \psi (io) \) as a ratio of the EMIP and EM impedances for a homogeneous earth model. For a general earth model we can extend the expression in eq. (14) in terms of apparent quantities, that is, the responses due to an equivalent homogeneous half-space. This is given by

\[
\psi_a (io) \approx \frac{Z_{\text{EMIP}}^a (io)}{Z_{\text{EM}}^a (io)} = \frac{\mathcal{F}_{\text{EM}} (\sigma (io); \omega)}{\mathcal{F}_{\text{EM}} (\sigma^e; \omega)},
\]

where \( \psi_a (io) \) is an apparent IP response function and \( Z_{\text{EMIP}}^a (io) \) and \( Z_{\text{EM}}^a (io) \) are the apparent impedances due to EMIP and EM effects respectively for an equivalent half-space. In a general mathematical framework we can write the apparent impedances \( Z_{\text{EMIP}}^a (io) \) and \( Z_{\text{EM}}^a (io) \) in terms of a non-linear operator \( \mathcal{F}_{\text{EM}} \) acting on the conductivities \( \sigma (io) \) and \( \sigma^e \) respectively. The conductivities \( \sigma (io) \) and \( \sigma^e \) can be 1-D, 2-D or 3-D conductivities with or without topography. Thus the removal equations for the PFE data in eq. (24) and phase data in eq. (17) can theoretically be obtained for an inhomogeneous earth. This is given by

\[
PFE_a^{\text{CORR}} = 100 \left( 1 - \frac{|Z_{\text{EMIP}}^a (io2)||Z_{\text{EM}}^a (io1)|}{|Z_{\text{EMIP}}^a (io1)||Z_{\text{EM}}^a (io2)|} \right),
\]

\[
\phi_a^{\text{IP}} (\omega) = \phi_a^{\text{EMIP}} (\omega) - \phi_a^{\text{EM}} (\omega),
\]

where \( PFE_a^{\text{CORR}} \) and \( \phi_a^{\text{IP}} \) are the corrected apparent PFE and phase data respectively. In the next section we investigate our removal methodology on 1-D synthetic examples.
5 REMOVAL IN ONE DIMENSION: SYNTHETIC EXAMPLES

In this section we consider a 1-D synthetic example to test our EM coupling removal methodology. A three-layer earth model, with the parameters for each layer, is shown in Fig. 3(a). The data are generated using a dipole–dipole array with a dipole length of 200 m for \( N = 1, \ldots, 10 \). The forward computation is carried out by computing the electric field due to a point horizontal electric dipole. The electric field in the \( x \)-direction for an \( x \)-directed HED over a layered earth is given by (Ward & Hohmann 1988)

\[
E_{x}^\text{EMIP} (i\omega) = \frac{I_0 s^2}{4\pi} \int_0^\infty \left( (1 - r_{TM}) \frac{u_0}{i\omega_0} - (1 + r_{TE}) \frac{i\omega u_0}{u_0} \right) \times J_0 (i\omega_0 r_0) \frac{\lambda}{u_0} (1 + r_{TE}) J_0 (i\omega_0 r_0) \, d\lambda,
\]

where \( r_{TE} \) and \( r_{TM} \) are the TE and TM reflection coefficients, \( u_0 = \sqrt{\lambda^2 - i\omega^2\mu_0\sigma_0} \) and \( \lambda \) is the integration variable. The reflection coefficients are functions of the complex conductivities of the layered earth model and are computed using a propagator matrix solution approach (a semi-analytic method). \( J_0 \) is the Bessel function of the zeroth kind. Eq. (27) is used to generate the EMIP electric field. The impedance is then computed by integrating the electric field along the transmitter and receiver dipole lengths given by eq. (8). Eq. (27) is an exact formulation for computing the electric field when both EM coupling and IP effects are present. We note that in the absence of EM coupling, the true IP responses are obtained by solving the DC resistivity equation with complex conductivity, so that the corrected PFE and phase data after the removal of EM coupling can be compared to the true PFE and phase data. The method for EM coupling removal developed in the previous section is applied to both PFE and phase data.

5.1 Synthetic test for PFE data

The model in Fig. 3(a) is used to generate the PFE data due to the IP effect using eq. (19) at \( \omega_1 = 0.1 \) Hz and \( \omega_2 = 5.0 \) Hz. This is denoted by IP in Fig. 3(b); it indicates a decrease in PFE with \( N \) spacing. This is expected because only the first layer is chargeable and the large dipole spacings are sensitive to material below this layer. Next we generate the PFE data using eq. (22) at two frequencies, 0.1 and 5 Hz. This is plotted as EMIP in Fig. 3(b). We note that the amplitude of the EMIP curve increases with dipole spacing; this is an indication that the responses are contaminated with EM coupling. The correction

![Figure 6](image-url) (a) The observed pseudo-section for the undecoupled phase for the dipole-dipole array at 0.125 Hz. (b) The recovered 2-D chargeability model after inversion.

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for the EM coupling contribution in the PFE data is carried out using eq. (24). The impedance due to EM coupling was computed using the true conductivity structure \( \sigma_{\infty} \). The corrected PFE data, denoted by CORR, are plotted along with the PFE data due to the IP effect in Fig. 3(b). The corrected amplitude is in good agreement with the true IP response and thus the EM coupling response has been removed from the observed responses.

5.2 Synthetic test for phase-domain data

The synthetic model in Fig. 3(a) is used to compute the phase response at 0.5 Hz. The EMIP phase data were generated from the impedance using eq. (27). This is indicated by EMIP in Fig. 3(c), which contains both the IP response and the EM coupling effect. The true IP phase, denoted by IP in Fig. 3(c), is computed by solving the DC resistivity equation with complex conductivity at 0.5 Hz and taking the argument of the impedance function \( Z_{\text{DCIP}}(i\omega) \). To perform the correction, the phase due to EM coupling (denoted by EM) is computed at 0.5 Hz by using the true conductivity model \( \sigma_{\infty} \). The corrected phase response, \( \phi^{\text{CORR}} \), is obtained by subtracting the EM coupling phase from the EMIP phase following eq. (17). The corrected phase response, \( \phi^{\text{CORR}} \), is a close match with the IP phase, \( \phi^{\text{IP}} \), which indicates that the EM coupling response has been effectively removed.

5.3 Effect of conductivity on EM coupling removal

The knowledge of conductivity structure is important in computing the EM coupling response. We consider two test cases for this layered earth example to emphasize the necessity of conductivity information. In the first case the conductivity model used to compute \( Z^{\text{EM}} \) is a homogeneous half-space with a conductivity of 0.01 S m\(^{-1}\). With this model both the PFE data and the phases are undercorrected, as shown in Figs 3(d) and (e) respectively. In the second example we increase the half-space conductivity to 0.1 S m\(^{-1}\). With this conductivity, the PFE data and the phases are overcorrected, as shown in Figs 3(d) and (e) respectively. Figs 3(d) and (e) indicate that some negative PFE and phase values are obtained due to inappropriate correction. This indicates that adequate conductivity information is necessary to compute the response due to EM coupling.

In practice, the true conductivity \( \sigma_{\infty} \) is not known \textit{a priori}. However, an estimate of the conductivity can be obtained.
by inverting the amplitude data at the lowest frequency. To investigate this, we next compute the EM coupling response using a DC conductivity model obtained by inverting the amplitude data at 0.1 Hz using a 1-D DC resistivity inversion algorithm. The amplitude data shown in Fig. 4(a) are inverted to obtain a 1-D conductivity model in Fig. 4(b). The corrected PFE responses denoted by CORR_{dc} in Fig. 3(d) show a good match with the PFE data due to IP effects and show significant improvement compared to the corrections using the half-space conductivities. The same DC conductivity model is used to compute the EM coupling phases at 0.5 Hz. The corrected phases show a close match with the IP phases shown in Fig. 3(e).

6 REMOVAL PROCEDURE IN TWO DIMENSIONS: AN APPROXIMATE METHOD

In a 2-D IP or complex resistivity survey, both the amplitude and the phase of the voltage are measured at each location over a frequency range using a pole–dipole or dipole–dipole array geometry. The amplitude at the lowest frequency is converted to apparent resistivity data. For interpretation purposes, the apparent resistivity data at the lowest frequency are inverted to recover a 2-D conductivity structure \( \sigma(x, z) \) and that conductivity is then used to generate the sensitivity for the IP inversion problem (Oldenburg & Li 1994). The IP data are then inverted to recover chargeability.

Our goal in this paper is to remove the effects of EM coupling and obtain a response that can be inverted for a chargeability structure of the subsurface. This requires the computation of the EM coupling responses \( Z^{EM} \) with a known conductivity structure \( \sigma \) at the desired frequency \( \omega \). From the previous section it is evident that the conductivity information is vital to the computation of the coupling response, so using a simple homogeneous earth model or a single layered earth model might lead to an inappropriate correction of the coupling-contaminated data. Therefore, we use an approximate technique that utilizes the 1-D assumption but also takes into account the lateral variation in conductivity. The 2-D conductivity structure obtained from the inversion is averaged over a region that spans a particular transmitting dipole and the

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Figure 8. (a) The corrected pseudo-section obtained by subtracting the EM coupling phase (Fig. 7b) from the undecoupled phase (Fig. 7a). (b) The recovered chargeability model obtained after inverting the corrected data.

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associated receiving dipoles. The averaging is carried out in the horizontal direction such that for each transmitter position \( T_j \) we obtain an averaged 1-D conductivity model \( \sigma_j(z) \). The thicknesses of the layers are equal to the vertical dimensions of the rectangular cells used in the 2-D inversion. Since the conductivity can vary over large orders of magnitude, the logarithm of conductivity is averaged in each layer. We define

\[
\log(\sigma_{\text{avg}}^j) = \frac{1}{N_k} \sum_{k=k_1}^{k=k_2} \log(\sigma_k^j),
\]

(28)

where \( \sigma_{\text{avg}}^j \) is the averaged conductivity of the \( j \)th layer in the 1-D conductivity model for the \( j \)th transmitter position. \( k \) is the horizontal cell number of the 2-D conductivity model. The cells included in the average extend from the transmitting dipole \( (k_1) \) to the furthest receiver dipole \( (k_2) \), and \( N_k \) is the number of cells between them. The averaging process is then rolled along as the transmitter moves to the next position. A 1-D forward modelling is carried out to compute the EM coupling response. First the electric field is computed using a point dipole on a layered earth at the desired frequency using eq. (27) assuming the medium is not chargeable. The impedance \( Z(i\omega) \) is then computed using eq. (8). For PFE data the amplitude \( |Z_{\text{EM}}(i\omega)| \) is used to correct the observed responses given in eq. (24) and the phases are corrected using eq. (17). In the next step the corrected phases are inverted using a 2-D IP inversion algorithm (Oldenburg & Li 1994).

7 Field Data Inversion

Our method for EM coupling removal is applied to a field data set obtained from a mineral exploration survey in Australia. The data were acquired using a dipole–dipole array with a dipole length of 600 m. The length of the survey line was 18 km and 26 current electrode locations were used. There are three drill-hole logs along this line. The data were collected at 11 frequencies between 0.125 and 110 Hz.

The apparent conductivity pseudo-section at 0.125 Hz (Fig. 5a) is inverted using a 2-D DC resistivity inversion algorithm. The 2-D recovered conductivity structure, shown in Fig. 5(b), indicates a thin resistive cover overlying a background conductivity of 20 mS m\(^{-1}\) (or 50 ohm m). Sensitivities for the IP inversion are generated using the 2-D conductivity

![3 point Decoupled Pseudosection](image)

Figure 9. (a) The corrected phase pseudo-section obtained by a three-point decoupling method. (b) The recovered chargeability model obtained after inverting the three-point decoupled data.

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structure in Fig. 5(b). The pseudo-section of the observed phase at 0.125 Hz is shown in Fig. 6(a) and phases up to 125 mrad are observed in the data. These data were input into the IP inversion algorithm and the recovered 2-D chargeability structure $g(x, z)$ is shown in Fig. 6(b). The model indicates high chargeable zones near $x = 4000$ and $x = 12000$ m in a depth range of 1.5–2.0 km. The high recovered chargeability, of the order of 200 mrad, indicates that there is a major contribution due to EM coupling. This is well exhibited by the linearly increasing trend of the observed phases with $N$ spacing in the undecoupled pseudo-section shown in Fig. 6(a).

The conductivity structure obtained in Fig. 5(b) is used to generate the 1-D averaged conductivity models. 26 1-D conductivity models are generated for each of the 26 transmitter locations using the multilayered averaging method described in the previous section. The phases due to EM coupling, shown in Fig. 7(b), are computed at 0.125 Hz using the 1-D averaged models. They indicate that EM coupling is a major contribution to the observed phases. Comparing the observed phases in Fig. 7(a) with the EM coupling phases in Fig. 7(b), it is seen that both exhibit a linearly increasing trend with $N$ spacing.

The amplitude of the phases suggests that the IP phase is dominated by the EM coupling contribution. The observed phases in Fig. 7(a) are corrected by subtracting the phases due to EM coupling in Fig. 7(b). This removes the linear trend and reduces the data to about one-third of their original magnitude. The corrected phases in Fig. 8(a) are inverted using the 2-D IP inversion algorithm, and the recovered chargeability is shown in Fig. 8(b). Significant differences are observed between the models in Figs 5(b) and 8(b). Overall, the high chargeability regions for the decoupled data have a lower amplitude and are closer to the surface than the high chargeability regions generated by the undecoupled data.

The borehole locations are indicated on the recovered section in Fig. 8(b). Unfortunately, no geophysical logging is available, so the information is only from geological logging. There is, however, a qualitative agreement between the logs and the inversion results. Of the three borehole logs, BH6 has the largest indicators of chargeable materials and it coincides with the region of highest chargeability predicted from the inversion. The BH6 log indicates altered and veined granites that can be polarizable. It also indicates chalcopyrite and pyrite veins with

![Manual Decoupled Pseudosection](image1)

![Inverted Model](image2)

Figure 10. (a) The corrected phase pseudo-section obtained from a manual decoupling method. (b) The recovered chargeability model obtained after inverting the manual decoupled data.

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some dissemination extending from 600–720 m. BH1 coincides with a flank of a chargeable high seen in the inversion. The BH1 log indicates haematite–magnetite breccias (massive iron oxides) that are generally polarizable. Generally, haematite–magnetite breccias are polarizable depending on the degree of alteration. The log also indicates the presence of chalcopyrite in vugs, along with haematite–magnetite breccias, at depths of 720–950 m. The BH9 log coincides with a region of no chargeability in Fig. 8(b). This seems to be consistent with the log information, which indicates felsic volcanics with sparse mineralization. Felsic volcanics in this region tend to have low polarizability compared to haematite–magnetite breccias. In summary, all three logs indicate minor dissemination but none of them contains significant sulphides. It is likely that the polarizability may be caused by alteration. The alteration products seen in the geological logs indicate that BH6, BH1 and BH9 might be associated with progressively lower amounts of chargeable material. This conjecture is in accordance with the inversion result, but beyond this rather weak qualitative agreement, there is little that can be said.

We next compare our results with the standard three-point decoupling and manual decoupling methods obtained from a contractor. The decoupled data using the three-point method are shown in Fig. 9(a). The linear trend is strongly evident. The recovered model, shown in Fig. 9(b), exhibits high chargeability below a depth of 1.2 km and at the surface. The corrected phases using the manual decoupling technique (proprietary of contractor) are shown in Fig. 10(a). The recovered model in Fig. 10(b) suggests that boreholes BH1, BH9 and BH6 are associated with progressively smaller amounts of chargeable materials. This is contrary to our results discussed above.

We have shown, in an earlier section of this paper, that having a good estimate of conductivity is crucial to the success of EM coupling removal. In order to examine the importance of conductivity information for the field example, we have considered two different conductivity models. The first model is a homogeneous half-space with a conductivity of 20 mS m\(^{-1}\). The average value of the conductivity in Fig. 5(a) is about 20 mS m\(^{-1}\). The phase due to EM coupling computed at 0.125 Hz is subtracted from the measured response to obtain

![Inversion with \(\sigma=0.02\) S/m](image)

![Inversion with \(\sigma=0.01\) S/m](image)

Figure 11. The EM coupling phase is computed using a homogeneous half-space model and the corrected data are inverted using the 2-D IP inversion code. (a) The recovered model after the correction is carried out with a conductivity of 0.02 S m\(^{-1}\). (b) The recovered chargeability model using a conductivity of 0.01 S m\(^{-1}\).

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the corrected phases. The recovered model is shown in Fig. 11(a). The amplitude of the recovered chargeability is 60 mrad and the depth range of the anomalous chargeable zone is less than 1.5 km. In the second model, we decrease the conductivity to 10 mS m\(^{-1}\). The corrected phases are then inverted and the recovered chargeability model is shown in Fig. 11(b). The recovered model indicates the presence of a high chargeability zone at a depth of 1.5 km, and the structure of the chargeability zone is similar to the model obtained for the undecoupled inversion in Fig. 6(b). We note that decreasing the conductivity by a factor of two results in a factor of three increase in the recovered phases. This shows that accurate conductivity information is necessary to compute the response due to EM coupling at a desired frequency, and reinforces the conclusion drawn in Section 5.3.

In the previous example for \(f=0.125\) Hz, the EM coupling phase was greater than the IP phase and yet the removal technique still produced a reasonable result. EM coupling effects continue to increase with increasing frequency but the removal methodology can still be applied. To show this we tried a more extreme example with \(f=1.0\) Hz. The phase data collected at 1.0 Hz along this line are shown in Fig. 12(a). They show a linear trend and achieve values of 700 mrad for \(N=6\). The inversion in Fig. 12(b) of undecoupled data shows a very high chargeability of 850–950 mrad in the depth range 1.5–1.7 km. The observed data are corrected by computing the EM coupling phase at 1.0 Hz with the conductivity obtained from the 2-D DC inversion. The corrected data are shown in Fig. 13(a) and the recovered model from the inversion is presented in Fig. 13(b). The maximum amplitude of recovered chargeability is 70–80 mrad and the structural location of the anomaly is more or less in agreement with the drill-logs and the model obtained in Fig. 8(b). This example illustrates that the decoupling method presented here is reasonably robust even when EM coupling is severe.

8 DISCUSSION AND CONCLUSIONS

EM coupling has been a long-standing problem in IP data interpretation because the conventional forward modelling to generate IP responses does not take into account the inductive effects of the ground. There are two approaches to solving the

![Undecoupled Pseudosection at 1Hz](image1)

![Inverted Model at 1Hz](image2)

Figure 12. (a) The pseudo-section of the undecoupled phases at 1 Hz. (b) The recovered model after inverting the undecoupled data at 1 Hz.
problem: (i) using a forward modelling solution that models both IP and the EM coupling effects, and (ii) removing EM coupling responses from the data and using the traditional DC resistivity formulation to model the IP effects. The first approach requires the solution of Maxwell’s equations for electric fields due to finite-length grounded wires; this is computationally intensive to solve, especially in two and three dimensions. The second approach, using conventional DC forward modelling, requires that EM coupling be removed from the IP data. In this paper we have developed a practical algorithm for removing EM coupling responses from IP data in the low-frequency regime. The methodology is general and can be applied to 1-D, 2-D and 3-D conductivity models. We also note that this method does not assume any particular form of complex conductivity model.

The foundation of the removal method is based upon theoretical derivations with the electric field generated from a horizontal electric dipole in a complex conductivity medium. We show that the observed electric field can be expressed as a product of two frequency-dependent responses. The first is an IP response function that depends on the chargeability of the ground. The second is an electric field due to inductive and DC resistivity effects that depends on the conductivity of the ground. Recognizing that both the amplitude and the phase of the IP response function are indicators of chargeability allowed us to develop removal methodologies for the PFE and phase data. To correct the PFE responses, we normalized the measured amplitudes with the expected EM responses computed at the desired frequency. For the correction of phase data, we computed the phase due to EM coupling and subtracted it from the measured responses. We presented 1-D synthetic examples to illustrate EM coupling removal. Synthetic examples show that it is important to know the conductivity structure of the ground for computing the EM coupling responses. In practice, the true conductivity of the ground is not known a priori. Our best option is to invert the amplitude data at the lowest available frequency using a DC inversion algorithm. Our hope is that this model is sufficiently close to the true conductivity that the procedure outlined here will work to a good approximation. The examples presented make us optimistic that this is valid.

To apply our methodology to 2-D problems we ideally need a 2-D EM forward modelling. This was not available to us so
we have designed an approximate method that uses a localized averaged 1-D conductivity model that has been constructed from the 2-D conductivity obtained from the DC resistivity inversion. Each transmitter position is associated with its own 1-D conductivity. The advantage of using a local averaged conductivity model, compared to a simple half-space or a single layered model, is that the lateral conductivity variation along the survey line is approximately accounted for. However, a full 2-D EM forward modelling would be ideal for computing the phases due to EM coupling, especially when there are topography and significant lateral conductivity variations. The application of our method to a field example has shown significant improvement in the recovered model compared to the existing methods such as three-point decoupling and manual decoupling techniques. Overall, the improvement in the signal-to-noise ratio in all of the examples presented in this paper is substantial, and this will lead to much greater interpretability of the IP data.

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REFERENCES


APPENDIX A: VALIDITY OF THE APPROXIMATION $K_{EMIP} \approx K_{EM}$

Being able to write the electric field $E_{EMIP}$ as a product of an electric field $E_{EM}$, which is not affected by chargeability, and $\psi(i\omega)$, which depends only on chargeability, is crucial to our work. The derivation of eq. (13) assumes $K_{EMIP} \approx K_{EM}$. In this Appendix we investigate the validity of this approximation. In order to quantify this we consider a homogeneous half-space model for a finite length dipole source and receiver and plot the amplitude and phase of $\zeta(i\omega)$, which is given by

$$\zeta(i\omega) = \frac{Z_{EMIP}(i\omega)}{\psi(i\omega)Z_{EM}(i\omega)}, \quad (A1)$$

where $\zeta(i\omega)$ depends on $\sigma_x$, $\eta$, $\tau$, $c$, $\omega$, the dipole length and the dipole spacings. If the approximation holds well, then the amplitude of $\zeta(i\omega)$ should equal unity and the phase will be zero. Figs A1a and (b) show that the discrepancy increases with increasing value of the conductivity $\sigma_x$ for a dipole–dipole survey with a dipole length of 300 m. For a conductivity of 1 S m$^{-1}$ the error in the amplitude is less than 12 per cent (Fig. A1a) and for phase it is less than 15 mrad (Fig. A1b) in the frequency range 0.01–1000 Hz. In most IP surveys, the frequencies used rarely exceed 100 Hz, so we are mostly interested in the low-frequency regime. The effect of intrinsic chargeability $\eta$ is shown in Figs A1c and (d). We note that for the same frequency range (0.01–1000 Hz) and for intrinsic chargeability $\eta \leq 0.3$, the error in amplitude is less than 2 per cent (Fig. A1c) and the error in phase is less than 15 mrad (Fig. A1d). Figs A2a and (b) shows the validity of the approximation as a function of time-constant $\tau$. As $\tau$ decreases the discrepancy increases in both amplitude and phase. For $\tau \geq 0.1$ s, the error in amplitude is less than 3 per cent and that
Figure A1. The validity of the approximation $k_{EMIP} \approx k_{EM}$ is tested by plotting the amplitude and phase of the quantity $\tilde{\zeta}(\omega) = f(\sigma, \eta, \tau, \epsilon, \omega)$ for a finite dipole source and receiver. (a) Amplitude and (b) phase of $\tilde{\zeta}(\omega)$ for varying conductivity $\sigma$ with fixed $\eta = 0.2$, $\tau = 10$, $\epsilon = 0.3$, $N = 1$, $a = 300$ m. (c) Amplitude and (d) phase of $\tilde{\zeta}(\omega)$ for various intrinsic chargeability $\eta$ with fixed $\sigma = 0.1$ S m$^{-1}$, $\tau = 10$, $\epsilon = 0.3$, $N = 1$, $a = 300$ m.

Figure A2. (a) Amplitude and (b) phase of $\tilde{\zeta}(\omega)$ for varying time-constant $\tau$ with fixed $\sigma = 0.1$ S m$^{-1}$, $\eta = 0.2$, $\epsilon = 0.3$, $N = 1$, $a = 300$ m. (c) Amplitude and (d) phase of $\tilde{\zeta}(\omega)$ for various relaxation constant $\epsilon$ with fixed $\sigma = 0.1$ S m$^{-1}$, $\eta = 0.2$, $\tau = 10$, $N = 1$, $a = 300$ m.
in phase is less than 25 mrad for the same frequency range. Figs A2(c) and (d) indicate that the discrepancy increases with decreasing \(c\). For \(c \leq 0.2\), the error in amplitude is less than 2 per cent (Fig. A2c) and that in phase is less than 20 mrad (Fig. A2d) in the same frequency range. Figs A3(a) and (b) show that the discrepancy increases with dipole length. For dipole lengths of less than 300 m, the error in amplitude is less than 1 per cent and the error in phase is less than 10 mrad. Figs A3(c) and (d) show that the discrepancy increases with dipole spacing. For a dipole length of 300 m, if \(N \leq 6\) then the error in amplitude is less than 2 per cent and the error in phase is less than 20 mrad.

Figure A3. (a) Amplitude and (b) phase of \(j(\omega)\) for varying dipole length \(a\) with fixed \(\sigma_a = 0.1 \text{ S m}^{-1}\), \(\eta = 0.2\), \(\tau = 10\), \(c = 0.3\), \(N = 1\). (c) Amplitude and (d) phase of \(j(\omega)\) for varying dipole spacings or \(N\)-spacings with fixed \(\sigma_a = 0.1 \text{ S m}^{-1}\), \(\eta = 0.2\), \(\tau = 10\), \(c = 0.3\), \(a = 300\) m.