3-D inversion of magnetic induced polarization data

Jiuping Chen  
UBC-Geophysical Inversion Facility  
Department of Earth & Ocean Sciences  
University of British Columbia  
Vancouver, B.C., V6T 1Z4, Canada.  
chen@geop.ubc.ca

Douglas W. Oldenburg  
UBC-Geophysical Inversion Facility  
Department of Earth & Ocean Sciences  
University of British Columbia  
Vancouver, B.C., V6T 1Z4, Canada  
doug@geop.ubc.ca

SUMMARY

The magnetic induced polarization (MIP) method is an exploration technique used to obtain information relating to the induced polarization characteristics of the subsurface through measurements of the primary magnetic field associated with steady-state current flow in the earth. According to Seigel, the secondary magnetic field due to polarization current can be expressed as a sum of the products of chargeability and the derivative of primary magnetic field, due to ohmic current, with respect to the logarithmic conductivity (or sensitivity). The magnetic field and the sensitivity matrix can be computed by subsequent solving Poisson's equation and a magnetostatic problem in terms of potentials using a finite-volume algorithm. The MIP response is a function of chargeability difference (η-η0) and relative conductivity (σ/σ0), where η0 and σ0 are constants.

When solving the inverse problem we need to impose positivity of the solution but the fact that MIP responses depend only upon the difference in chargeability means we have options regarding how we set up the inversion. We can: (1) invert for η without constraints and add a constant to the final result, (2) invert for η while imposing positivity, or (3) work with ln η. We compare all three methods here. Our inversion problem is formulated as an optimization problem where the objective function of the model is minimized subject to the constraints that the model adequately reproduces the data. We use a Gauss-Newton method to obtain the model perturbation at each iteration. The system of equations is solved using a conjugate gradient least squares method. In order to make the inversion produce depth or distance information, a depth weighting or sensitivity-based weighting is required.

Through synthetic model studies, we have shown that the conductivity ratio between a target and its host has a large effect on the MIP response. Ratios greater than two orders of magnitude difference will eventually make the MIP response undetectable. However, if the ratio is in the range of 0.1 to 10, the effect on the recovered chargeability is limited. The inversion algorithm is demonstrated by inverting the data set from Binduli, Australia.

Key words: 3-D, Inversion, Magnetic induced polarization, Magnetometric resistivity, Mineral exploration.

INTRODUCTION

The magnetic induced polarization (MIP) is used as an alternative method to derive information relating to the induced polarization characteristics of the subsurface through measurements of the magnetic field, rather than measuring the electric field as in electric induced polarization (EIP) method (Seigel and Howland-Rose, 1990). Since both MIP and magnetometric resistivity (MMR) are associated with the magnetic fields due to current flow in the ground, there is an intimate relationship between these two methods that parallels the way that the EIP method is related to the DC method. In other words, the MIP method makes use of observations of the secondary magnetic field associated with the secondary (or polarization) current, whereas the MMR makes use of the primary magnetic field due to the primary (or ohmic) current. The theoretical foundation of MIP has been established on this relationship, and was first introduced by Seigel (1974; 1959), Howland-Rose et al. (1980a,b) presented some additional mathematical development over dikes, and an excellent summary was given by Seigel and Howland-Rose (1990), in which theoretical formulae, instrumentation, field procedures and case studies were detailed.

According to Seigel's formulation, the secondary magnetic field due to polarization current in the MIP survey can be calculated in three consecutive steps. The first step is to solve a corresponding steady-state Poisson's equation to determine the distribution of the primary current flow J through the earth; the second is to integrate \( J \times (r - r')/(r - r')^3 \) over the whole earth and then differentiate the integrals with respect to ln \( \sigma_i \), where \( \mathbf{r} \) and \( \mathbf{r}' \) are vectors pointing to an observation point and an integral source point, respectively; \( \sigma_i \) is the conductivity of \( \mathbf{r} \)th domain of the earth; and the third is to multiply each derivative by its corresponding IP quantity, and sum for all domains. This integral form for calculation of the MIP response is conceptually elegant and easy to understand. However, it is not generally efficient to implement numerically. Except for some simple geometries, such as vertical dikes and a buried spheroid (Seigel, 1974; Howland-Rose et al., 1980a), explicit formulae are difficult to derive. Even having used the finite-difference method to obtain the distribution of the current flow, Boggs (1999) still requires a number of MMR forward modelling runs to compute the MIP response. In his algorithm, the differentiation of the integral with respect to ln \( \sigma_i \) needs two forward models of slightly varying conductivities for each domain having non-zero chargeability, resulting in a heavy computational load even in the forward modelling problem.

The difficulty in calculating the MIP response naturally hinders development of a method for directly inverting MIP data. Type-curve matching and 'eyeball' interpretation are
often applied in practical use of field data (Howland-Rose et al., 1980b; Hishime et al., 1993). The type curves allow a rapid qualitative interpretation to be made in terms of simple targets but they are of little use in quantitative field situations. Trial and error forward modelling in 3-D has also been used, but it too suffers in practical implementation. This has motivated us to develop a 3-D inversion algorithm for MIP. As far as we know, no such algorithm has been published.

Chen et al. (2002) developed numerical techniques to model the MMR response of an arbitrary 3-D conductivity structure and to invert the MMR data. In the forward modelling, the magnetic field due to galvanic current flow in the ground is obtained by solving two related sub-problems; the first is to solve an electrostatic equation for the scalar potential of the electric field, and the second is to solve a magnetostatic equation for magnetic field. The scalar potential obtained from solving the first sub-problem serves as the source term (galvanic current) for the magnetic field in the second sub-problem. It is evident to see that this technique for computing magnetic field is based upon solving a differential equation and is different from the integral procedure in the Biot-Savart law as mentioned above. The differential form is appealing compared to the integral form because the differentiation of magnetic field with respect to the logarithmic conductivity (or sensitivity) has been explicitly formulated and can be easily manipulated. This means the calculation of MIP response can be performed without much extra effort once the MMR response has been obtained.

The goal of this paper is to develop a numerical technique to invert 3-D MIP data. It is based upon our previous research regarding the MMR method. We first outline the equations and finite-volume solution for calculating the MIP response over 3-D conductivity and IP structures. The computed MIP response is compared with published results over a vertical contact and a vertical dike model, and the relationship between the MIP and the IP quantity is briefly discussed. Next we address the inversion of MIP data, particularly focusing on choice of model parameter, weighting scheme, and conductivity ratio effect with synthetic example. We then apply the algorithm to the Binduli data set, and conclude the paper.

COMPUTATION OF 3-D MIP RESPONSE

Magnetic field by using finite-volume method

In the MIP survey, an external current is impressed into the ground through a pair of current electrodes C1 and C2, which is connected by a wire laid-out in a U-shape (Figure 1). The magnetic field associated with the current flow in both the wire and ground can be measured by a magnetometer located on the surface or in drill-holes. Since the frequency in the transmitter current waveform is low, usually less than 4 Hz, we neglect inductive effects. This means we only need to deal with a steady-state problem, in which the electric field \( E \) and magnetic field \( H \) satisfy

\[
\nabla \times E = 0 \tag{1a}
\]

\[
\nabla \times H - \sigma E = J^e \tag{1b}
\]

\[
\nabla \cdot (\mu H) = 0 \tag{1c}
\]

where \( J^e \) is the external electrical current density in \( \text{A/m}^2 \), and \( \sigma \) and \( \mu \) are the electric conductivity and magnetic permeability, respectively. From equation (1a), we can define a scalar potential \( \phi \), which can make \( E \) expressed as

\[
E = -\nabla \phi . \tag{2}
\]

From equation (1c), a magnetic vector potential \( A \) can be introduced such as

\[
\mu H = \nabla \times A \tag{3}
\]

along with a gauge condition for \( A \), such that

\[
\nabla \cdot A = 0 . \tag{4}
\]

By using this (\( A, \phi \)) formula (Haber, 2000; Chen et al., 2002), equation (1b) can be formulated into a well-known Poisson’s equation for potential \( \phi \)

\[
\nabla \cdot (\sigma \nabla \phi) = \nabla \cdot J^e , \tag{5}
\]

and a magnetostatic problem for potential \( A \)

\[
\nabla \times \mu^{-1} \nabla \times A = J^e - \sigma \nabla \phi . \tag{6}
\]

Note the scalar potential \( \phi \) on the right hand side of equation (5) serves as a source term (current flow in the ground) for solving for \( A \). Thus \( \phi \) must be known from solving equation (6) before dealing with the magnetostatic problem.

Numerical solutions of equation (6) require introduction of a stabilizing term \(-\nabla \mu^{-1} \nabla \cdot A\) (which is identically zero). Equation (6) is rewritten as

\[
\nabla \times \mu^{-1} \nabla \times A - \nabla \mu^{-1} \nabla \cdot A = J^e - \sigma \nabla \phi \tag{7}
\]

The two systems of equations (5) and (7) can be solved by using a finite-volume method on a staggered grid. The 3-D medium is discretized into a large number of rectangular cells whose conductivity and permeability values are constants at each cell. We define \( A \) and current density \( J \) at centers of cell faces, \( H \) at centers of cell edges, and \( \phi \) at cell centers. With discretization, equations (4) and (7) can be assembled as a matrix system for unknowns \( A \) and \( \phi \). The unified matrix system is

\[
\begin{pmatrix}
\nabla^e \times \cdot M_e \nabla^e \times - \nabla^h \cdot M_h \nabla^h \cdot S \nabla^h \cdot V^h \nabla^h \cdot S V^h \phi \\
0
\end{pmatrix} \begin{pmatrix} A \\ J^e \end{pmatrix} = \begin{pmatrix} \nabla^e \cdot J^e \\ \nabla^h \cdot J^e \end{pmatrix} \tag{8}
\]

where the matrices \( \nabla^e \times \) and \( \nabla^h \times \) are assembled from the discretization of the curl operator, projecting from cell edges to faces and from faces to edges, respectively; matrices \( \nabla^h \cdot \) and \( \nabla^h \cdot \) correspond to the discretization of the div and grad operators. When assembling the discrete first-order partial differential equations, we impose certain boundary conditions: the normal components of \( A \) and \( J \), and the tangential components of \( H \) on the outer boundaries are set to zero. The material matrix \( S \), arising from the discretization of
the conductivity $\sigma$, harmonically averages of values at cell faces. The matrices $\mathbf{M}_c$ and $\mathbf{M}_e$ result respectively from an arithmetic averaging of permeability $\mu$ at cell edges and the permeability itself at the cell centers. The superscript -$1$ represents the inverse of the matrix.

It is not necessary to solve equation (8) for $\mathbf{A}$ and $\phi$ from the above system simultaneously. Rather we first solve Poisson's equation for $\phi$, then substitute $\phi$ into the right hand side of subsystem for $\mathbf{A}$, and eventually solve for $\mathbf{A}$. Both subsystem equations are solved using the bi-conjugate gradient stabilizing (BiCGSTAB) algorithm, combined with a symmetric successive over relaxation (SSOR). Once $\mathbf{A}$ has been computed, the magnetic field $\mathbf{H}$ elsewhere can be obtained by the matrix operation

$$\mathbf{H} = \mathbf{M}_c^{-1} \nabla(f) \times \mathbf{A}. \quad (9)$$

Derivative of $\mathbf{H}$ with respect to conductivity

The derivative of magnetic field with respect to logarithmic conductivity (or sensitivity matrix), required to compute the MIP response in the next section, can be derived as follows. Assume the model vector $\mathbf{m}$ represents the logarithmic conductivity. For convenience, the forward modelling matrix systems given in equation (8) can be simplified as

$$\mathbf{A}(\mathbf{m}) \cdot \mathbf{u} = \mathbf{f} \quad (10)$$

where $\mathbf{A}(\mathbf{m})$ denotes the coefficient matrix, the vector $\mathbf{u} = \left( \begin{array}{c} \phi \\ J \end{array} \right)$, and the source vector $\mathbf{f} = \left( \begin{array}{c} J^T \\ \nabla_h \cdot J^T \end{array} \right)$. At the same time, the magnetic field can also be represented as

$$\mathbf{H} = \mathbf{Q} \mathbf{u}. \quad (11)$$

where $\mathbf{Q}$ is a matrix which projects the potentials to the observation sites. From equation (9), it is easy to see that $\mathbf{Q}$ can be obtained by multiplying a linear interpolation matrix with the matrix $\nabla_h / \times$. Therefore, $\mathbf{Q}$ is independent of the model $\mathbf{m}$.

As shown by Haber et al. (2000), the sensitivity matrix $\mathbf{S}$ can be symbolically written as

$$\mathbf{S}_m = \frac{\partial \mathbf{H}}{\partial \mathbf{m}} = -\mathbf{Q} \mathbf{A}^{-1}(\mathbf{m}) \mathbf{G}(\mathbf{m}, \mathbf{u}) \quad (12)$$

where $\mathbf{A}^{-1}(\mathbf{m})$ is the inverse matrix of $\mathbf{A}(\mathbf{m})$; $\mathbf{G}(\mathbf{m}, \mathbf{u})$ is an intermediate matrix, which can be obtained by

$$\mathbf{G}(\mathbf{m}, \mathbf{u}) = \frac{\partial [\mathbf{A}(\mathbf{m}) \mathbf{u}]}{\partial \mathbf{m}} = \begin{pmatrix} \frac{\partial (\mathbf{S} \mathbf{V} \phi)}{\partial \mathbf{m}} \\ \nabla_h \cdot \frac{\partial (\mathbf{S} \mathbf{V} \phi)}{\partial \mathbf{m}} \end{pmatrix}. \quad (13)$$

Since the conductivity matrix $\mathbf{S}$ is a diagonal matrix with elements that are the harmonic average of conductivities at adjacent two cells, elements in both $\mathbf{S}$ and $\mathbf{G}(\mathbf{m}, \mathbf{u})$ can be analytically derived.

Formulae for MIP response

The presence of induced polarization effects in the earth may change the current distribution and therefore the resultant magnetic fields can be measured in a MIP survey. Following the expression derived by Seigel (1974), the polarization magnetic field (or secondary field) in vector form can be regarded as a perturbation on the magnetic field, which obeys the Biot-Savart law, as follows

$$\mathbf{H}_s = \Delta \mathbf{H} = \frac{1}{4\pi} \int \int \int \Delta \mathbf{J}(\mathbf{r}) \times (\mathbf{r} - \mathbf{r}')/(r - r')^3 \, dv. \quad (14)$$

In equation (14), the perturbation current is approximated by

$$\Delta \mathbf{J}(\mathbf{r}) \equiv \sum_i \frac{\partial \mathbf{J}(\mathbf{r})}{\partial \ln \sigma_i} \Delta \sigma_i \quad (15)$$

where $\sigma_i$ is the conductivity of the $i$th domain (in this case the $i$th cell) in the earth, and

$$\Delta \sigma_i = -\eta_i \sigma_i \quad (16)$$

where $\eta_i$ is the chargeability of the $i$th cell (Seigel, 1959).

Thus when we work with individual components of the field, we find that

$$H_s = -\mathbf{3}_a \eta = \mathbf{Q} \mathbf{A}^{-1} \mathbf{G} \eta \quad (18)$$

where $\mathbf{Q}$ is a vector of chargeability. For MIP data it is customary to measure the ratio of polarization field to the corresponding steady-state field to obtain the apparent chargeability ($\eta_a = H_s / H_s, \ i = x, y, \ or \ z$ )

$$\eta_a = \mathbf{H}^{-1} \mathbf{Q} \mathbf{A}^{-1} \mathbf{G} \eta \quad (19)$$

where $\mathbf{H}^{-1}$ denotes a diagonal matrix with elements being the reciprocal of the magnetic fields.

In the frequency domain, the pertinent MIP quantities are often apparent percent frequency effect (PFE), which measures the change in amplitude between the fundamental and the third harmonic component, and apparent relative phase shift (RPS), which measures the relative phase shift between the fundamental and third harmonic component. Both quantities can be similarly written in a matrix form as

$$\text{PFE} = \mathbf{H}^{-1} \mathbf{Q} \mathbf{A}^{-1} \mathbf{G} \ pfe \quad (20)$$
where $pfe$ is a vector of the usual resistivity percent frequency effect for each cell, and

$$RPS = \frac{1}{2} \arctan(\mathbf{H}^T \mathbf{Q} \mathbf{A}^{-1} \mathbf{G} \tan \theta)$$

where $\theta$ is a vector of the phase angle of the conductivity (assumed complex due to induced polarization). To be consistent with equations (18) and (20), functions $\tan$ and arctan are directly operating on a vector, and the operations are also a vector. The factor $2$ is due to the definition

$$RPS = 3 \theta - \theta_0 = 2 \theta$$

where $\theta_0$ is the mean IP phase angle over the frequency range represented by the fundamental and third harmonic (Seigel and Howland-Rose, 1990).

Numerical check

We have checked our MIP forward modelling code with two simple models: a vertical contact and a vertical dike. Analytical solutions for these two models are presented in Howland-Rose et al. (1980a). As shown in Figure 2, the vertical contact represents two conducting quarter-spaces of conductivity $\sigma_1$ ($x<0, z>0$) and $\sigma_2$ ($x>0, z>0$), where $z$ is positive into the earth. The chargeabilities of region 1 and 2 are $\eta_1$ and $\eta_2$, respectively. The source and sink electrodes are located at (0,-L,0) and (0,L,0), and each carries a current of 1.0 Ampere. $L=600$ m was used in the numerical computation. The contact model, 4x4x4 km$^3$, including an air layer of 2 km, was unevenly discretized into 56x56x42 cells. A mesh size of 20 m was used to partition the center region of the model and a larger mesh size was used for the remainder. Calculation of the apparent chargeability (corresponding to the $x$-component of the polarization magnetic field) was performed along the $x$-axis. Two cases were considered: $\sigma_2 = 0.1 \sigma_1$ and $\sigma_2 = 10 \sigma_1$. In both cases, $\eta_2$ was assumed to be $10 \eta_1$. The normalized chargeability ($\eta_a = \eta_a/\eta_1$) vs normalized distance is plotted in Figure 2. These can be compared to the analytical results shown in Figure 3 in Howland-Rose et al. (1980a).

The same mesh and electrode locations used for the contact model were also applied to the vertical dike which is shown in Figure 3. The two regions having conductivities $\sigma_1$, $\sigma_2$ and percent frequency effects $pfe_1$, $pfe_2$, are shown. For a case that $\sigma_2 = 10 \sigma_1$ and $pfe_1=0\%$, $pfe_2=10\%$, the computed apparent PFE (again, from the $x$-component of $\mathbf{H_i}$) is displayed in Figure 3, and found to be in a good agreement to the analytical solutions.

Relationship between MIP response and IP quantity

Since we want to develop an inverse algorithm, it is necessary to have a good understanding of the relationship between the measured MIP response and the IP characteristic that is to be recovered. From equations (19), (20), or (21), it seems that this relationship is simple: linear (for $\eta_a$ and PFE) or quasi-linear (for $RPS$). Unfortunately there is a complication that impacts upon interpretation. There is no MIP response over a 1-D polarizable earth. This can be derived from the unique feature of MIP:

$$\sum \frac{\partial \log H_i}{\partial \log \sigma_i} = 0,$$

which holds irrespective of the conductivity structure (Howland-Rose et al., 1980a). In other words, only lateral variation of IP quantities can give rise to an MIP anomaly. In this regard, surface MIP is similar to surface MMR in which the response is insensitive to the layered conductivity. Therefore, it is not possible to obtain information about the IP quantities of a layered structure directly from surface MIP data.

We may obtain some insight about the relationship between MIP response and IP parameters from the analytic solution to a vertical contact model used in the previous section. According to Howland-Rose et al. (1980a), the polarization magnetic field is given by

$$H_i = \frac{200I}{L\pi x^2 + 1} \log \left( \frac{\sqrt{x^2 + 1} + 1}{\sqrt{x^2 + 1} - 1} \right) \frac{2 \sigma_1 \sigma_2}{(\sigma_1 + \sigma_2)} (\eta_1 - \eta_2)$$

where $x$ is the distance between an observation point to the origin. Besides the geometry terms, this expression clearly shows that $H_i$ is proportional to the difference between $\eta_1$ and $\eta_2$, and at the same time, is also related to the conductivity ratio (or contrast) $\sigma_1/\sigma_2$. This suggests that the MIP response can only reveal the difference between $\eta_1$ and $\eta_2$, not individual values. For a general 3-D earth made of different media, we assume there exists a reference medium which has a constant chargeability of $\eta_0$ and a constant conductivity of $\sigma_0$. By using equation (23) it is easy to show that the MIP response can be formulated as

$$\eta_a = -\sum \frac{\partial \log H_i}{\partial \log \sigma_i} (\eta_i - \eta_0) .$$

Recall the magnetic field is only sensitive to the relative conductivity. Therefore, we observe that the MIP response is a function of the chargeability difference $(\eta_i - \eta_0)$ and conductivity ratio $(\sigma_1/\sigma_0)$ with respect to a reference medium.

It should be noted that the MIP response does depend upon the layered IP characteristic structures when the measurement is made in boreholes. This is similar to downhole MMR in which the magnetic field is sensitive to the layered conductivity structures (Chen et al., 2003). Even so, the relationship between the MIP response and the IP properties shown above (Eq.25) holds in the downhole situation.

3-D INVERSION OF MIP DATA

Inversion methodology

The inversion methodology for MIP is essentially that used by Chen et al. (2002) for surface MMR data. The inverse
problem is formulated as an optimization problem, and we find a 3-D earth model that minimizes \( \Phi(m) = \phi_j(m) + \beta \Phi_m(m) \), where \( m \) is the model parameter vector, and we will discuss it in the following section. The first term \( \phi_j \) is the typical \( l_2 \)-norm measure of data misfit:

\[
\phi_j = \left\| W_j(d - d^{\text{obs}}) \right\|,
\]

where \( d^{\text{obs}} \) is the vector containing the \( N \) observations, and \( d \) is the vector of predicted data from the current 3-D model. The matrix \( W_j \) is a diagonal matrix whose elements are the reciprocals of the estimated standard deviations of the noise in the observations. The second term is the model objective function: \( \Phi_m = \left\| W_m(m - m_0) \right\| \), where \( W_m \) is the finite-difference discretization of a desired regularization functional. The choice of the reference model \( m_0 \) reflects our prior knowledge on the study area, and can be obtained from other geological and geophysical information. The trade-off parameter \( \beta \) is chosen such that the data are adequately fitted.

We construct an iterative procedure, and at each iteration we solve a linearized approximation of the true inverse problem. The model perturbation can be solved using a standard Gauss-Newton approach. The solution to this perturbation equation is also equivalent to its least-squares system, and we apply the conjugate gradient least-squares algorithm (CGLS) to solve this system. In the CGLS algorithm, we only require evaluation of the products of the sensitivity (or Jacobian) matrix and its transpose with a known vector. This has proved to be efficient.

Unified data and choice of the model parameter

As discussed in the forward modelling section, MIP data may have three forms: apparent chargeability (\( \eta \)), percent frequency effect (PFE), and relative phase shift (RPS). To simplify the inversion formulation, we make an approximation on RPS. Since \( \theta \) (in radians) is usually much smaller than 1, we have \( \tan \theta = \theta \). Thus RPS (also in radians) can be approximated as

\[
\text{RPS} \approx -2H^{-1}Q\Lambda^{-1}G\theta.
\]

By doing so, we have a unified and linearized data formulation

\[
d = \alpha \mathcal{Z}_y \eta
\]

where \( d \) is the data vector, the kernel (or sensitivity) matrix

\[
\mathcal{Z}_y = H^{-1}Q\Lambda^{-1}G
\]

and \( \alpha \) is a constant, depending upon the data type. For example, if the data are apparent chargeability, then \( \alpha = 1 \). For PFE data in percentage, \( \alpha = 100 \), while for RPS data in degrees, \( \alpha = \frac{360}{\pi} = -114.6 \). The IP quantity \( \eta \) can represent chargeability, pfe, (although not in percentage; a pfe = 1% must be represented as 0.01), or IP phase angle \( \theta \) (in radians). Below we focus on the chargeability only.

Within our inversion framework, we have yet to choose the form for model parameter \( m \). There are three options for imposing positivity. We can let (1) \( m = \eta \) and shift the final result, (2) \( m = \ln \eta \) and explicitly impose positivity, (3) \( m = \ln \eta \). The flexibility arises because MIP data depend upon the differences of chargeabilities (\( \eta \)) with respect to a reference medium (\( \eta_0 \)). In option 1, if we choose a reference model \( m_0 = \eta_0 = 0 \), then the objective function we want to minimize is

\[
\Phi_1 = \left\| W_d(\mathcal{Z}_y \eta - \eta_0) \right\| + \beta \left\| W_m(\eta - \eta_0) \right\| \quad \text{.} \tag{29}
\]

By using the iterative procedure mentioned above, the recovered chargeability may change around zero. This means, some are positive, some are negative. Once we have obtained all \( \eta \), the practical chargeability values can be obtained by adding the smallest constant that makes all of the chargeabilities positive.

In option 2, we can choose a reference model \( m_0 = \eta_0 = 0.001 \) and impose positivity by adding a logarithmic barrier term. The objective function to be minimized is

\[
\Phi_2 = \left\| W_d(\mathcal{Z}_y \eta - \eta_0) \right\| + \beta \left\| W_m(\eta - \eta_0) \right\| - 2\lambda \sum \ln \eta_i \quad \text{.} \tag{30}
\]

Determination of \( \lambda \), and more details about this algorithm, can be found in Li and Oldenburg (2003).

In option 3, \( m = \ln \eta \). The corresponding objective function is

\[
\Phi_3 = \left\| W_d(\mathcal{Z}_y e^m - \eta_0) \right\| + \beta \left\| W_m(m - m_0) \right\| \quad \text{.} \tag{31a}
\]

If we choose \( \eta_0 = 1 \), then the reference model \( m_0 = 0 \), and \( \eta \) will fluctuate around this reference value, i.e., \( \eta = 1 + \tilde{\eta} \), where \( \tilde{\eta} \) is a small value (possibly negative). Since the absolute value of \( \tilde{\eta} \) is much less than 1, \( m \) may be approximated by \( m = \ln \tilde{\eta} = \ln(1 + \tilde{\eta}) \equiv \tilde{\eta} \). This means, eventually, Eq. (31a) can be formulated as

\[
\Phi_3 = \left\| W_d(\mathcal{Z}_y \tilde{\eta} - \eta_0) \right\| + \beta \left\| W_m(\tilde{\eta}) \right\| \quad \text{.} \tag{31b}
\]

which is very similar to the objective function shown in Eq. (29) for option 1. By doing so, the merit is that we can work with \( \ln \tilde{\eta} \) but avoid the problem of smaller \( \eta \) having more weight as mentioned above. Again, once we have obtained all \( \eta \), the practical chargeability values can be made positive and in the range of 0 to 1 by adjusting by a constant.

We have implemented the three options in our inversion algorithm. Through synthetic model study, we found the three options work almost equally well. So far we do not have a preference regarding which model parameterization to use. In the following examples, we used option 3 to be consistent with our MMR inversion.

Weighting schemes

It is well known that potential field data, such as magnetic and gravity data, have no inherent depth resolution. This is also true for MMR and consequently for MIP data. The chargeability distribution tends to be concentrated near the
surface since the kernel function for the galvanic magnetic field in the Biot-Savart law has larger values there than at depth. To counteract the geometric decay of the kernels and distribute the chargeability with depth, we follow the practice of Li and Oldenburg (1996) and incorporate a depth weighting scheme into the model objective function. A feasible approach is to apply the roughness penalty to the weighted model. By doing so, the model objective function has to be altered to include the depth weighting function inside both the smallest and smoothest components. Accordingly, the model matrix $W_m$ is changed to reflect the incorporation of a depth weighting. For a surface MIP survey, the depth weighting may take the form of (Chen et al., 2002)

$$w(z) = \frac{2z_0}{(z + z_0)^{0.95}},$$

where $z_0$ is the half cell height of the cells just below the surface, and $z$ is the depth of the cell to be evaluated.

When MIP data are acquired in boreholes, the depth weighting is no longer suitable. Distance weighting removes artifacts at potential electrodes but still leaves artifacts associated with current source electrodes (Chen et al., 2003). A better solution is to employ a sensitivity-based weighting function, defined by Li and Oldenburg (2000a), and further refined as (Chen et al., 2003)

$$w_j = \left\{ \begin{array}{ll}
\frac{1}{M} \sum_{i=1}^{N} \gamma^{1/4} & \text{if } w_j \geq \gamma, \\
1.0 & \text{if } w_j < \gamma
\end{array} \right.  \quad (33)$$

where $\gamma$ is the threshold determined by the maximum and minimum of $\left( \frac{1}{M} \sum_{i=1}^{N} \gamma^{1/4} \right)$. Chen et al. (2003) have given a method to compute the approximate sensitivity $\gamma$ by using a current dipole formulation. This approximate $\gamma$ can be easily adapted into equation (33) by normalizing by the magnetic field at observation sites.

**Effect of conductivity ratio**

In the forward modelling section, we have seen that the MIP response is not only sensitive to chargeability difference, but also related to the conductivity ratio of the target with respect to a reference host. We take a simple 3-D cube buried in a uniform half-space as an example. As shown in Figure 4a, the cube is located right below the origin of the coordinates, with a top depth of 75 m and side length of 200 m. The chargeabilities of the cube and the host are set to be 0.15 and a top depth of 75 m and side length of 200 m. The conductivity of the host is 0.01 S/m and that of the cube is 0.1 S/m at an interval of one order of magnitude, corresponding to ratios of 100, 10, 1, 0.1 to 0.01. The source and sink electrodes are located at -600 m and 600 m along the x-axis. For comparison, the MIP responses computed are displayed in a profile running along the y-axis, on which the maximum MIP response is observed.

Figure 4b shows the apparent chargeabilities normalized by $\eta_i(0.01)$ for the 5 different conductivity ratios along the profile. The absolute MIP response reaches a maximum (0.4) when there is no conductivity contrast (ratio 1). If the cube is more conductive (ratio 10), the peak response is reduced by about 37%. If the cube is more resistive (ratio 0.1), the peak response is decreased by 75%. Two orders of conductivity ratios will result in the MIP response being almost undetectable. This observation clearly reflects that the MIP method is more suitable for problems associated with low conductivity contrasts.

In practice, the problem might be that the conductivity ratio is usually unknown before making an inversion of the MIP data. In the EIP inversion, Li and Oldenburg (2000b) investigated the effect of different approximations to the background conductivity using five different conductivity models, including a uniform half-space, conductivities recovered from one-pass 3-D inversion, composite 2-D inversions, limited approximate inverse mapping (AIM), and full 3-D nonlinear inversions of dc resistivity data. Their conclusion was that when the background is simple, reasonable IP results are obtained without using the best conductivity estimated derived from full 3-D inversion of the dc resistivity data. In general, this applies to MIP situation as well. In this study, we focus on the effect of the magnitude of the conductivity ratio on the MIP recovery.

We investigate this by experimenting with the cube model using true and approximate contrasts. To produce the synthetic data for inversion, the true contrast is assumed to be 10, the other parameters are the same as above (a central section is shown in Figure 5a). The synthetic data cover an area from -400 m to 400 m in both x and y directions, with 25 lines and 25 sites on each line, resulting in a total number of 625 data. Gaussian noise with a standard deviation which is equal to 1% of the absolute peak response (see Figure 5b), is added. We invert these noise-contaminated data to recover the chargeability of the cube model parameterized by 38x38x22 cells (the air layer is excluded). Option 2 for the model parameter m and the reference model $m_0 = 0.01$ was used. When the true conductivity contrast (10) is applied, the inversion took 11 iterations and the final data misfit was 576. The cross-section of the recovered model is shown in Figure 5c. When the conductivity contrast is assumed to be 1, a final misfit of 603 was reached after 9 iterations, and Figure 5d is the section view of the inverted model. In both cases, the synthetic data are fit almost equally well and the recovered chargeability (0.09) is much smaller than the true value (0.15). This undershooting, typical for MIP and EIP inversions, may be attributed to two factors. The first one is the existence of small chargeability near the surface, and right above the cube. Although the chargeability difference is only 0.01, the contribution to the MIP data can be significant because of the close proximity of the cell to the observation points. The second factor is that the recovered chargeable body has a larger volume than the true cube.

At the first glance, the recovered results seem to be not consistent with Figure 4b, from which a conductivity contrast of 1.0 should result in a significant lower chargeability, compared with a contrast of 10. Our explanation is that: although apparent chargeability response is a function of both chargeability difference and conductivity contrast, the effect from the chargeability difference seems to be much stronger than that from the conductivity contrast, provided that the contrast is, say, less than 10. In other words, a slight difference in chargeability will produce the same amount of contribution in the response as does a significant conductivity difference in conductivity.
contrast. This happens in Figure 5c and 5d. On a positive side, this may suggest that the effect of choosing a conductivity ratio in performing an inversion be not so critical for MIP response, if the ratio is less than 10.

FIELD DATA EXAMPLE

It is well known that one of the benefits from MIP measurement over EIP is that MIP works best in conductive areas where it is normally difficult to measure voltages with EIP. This scenario can be envisioned in the schematic shown in Fig. 6, which was studied with a scale model in Seigel and Howland-Rose (1990). Since the overburden is highly conductive, it is expected that the applied current on the surface can hardly penetrate this layer and energize the target below. Both MIP and EIP might have difficulties to obtain good responses. One remedy is to locate the source electrodes down below the overburden so that the galvanic current will be channelled directly into the target. Due to the masking effect of the overburden, EIP still has a problem to pick up the weak electrical signal. However, if we measure the magnetic signal instead, then it seems that there is no obstacle at all. This idea is substantiated in the next field example.

MIP data were collected at Binduli project, 12 km west of Kalgoorlie, Western Australia, by Placer Dome Asia Pacific and its joint venture partner Crousus Mining. The Binduli deposit is situated within a large mineralization system with the potential to host a large, medium to high grade gold deposit. Total gold production and resources at Binduli exceed 1 M oz. Most of the production to date has come from the Centurion-Ben Hur line. Crousus has exploited mainly low grade vein deposits within the porphyries to a depth of 50-70 m, however the most significant style was the high grade ECM (Eastern Contact Mineralization) within the 150 m deep Centurion pit. Since the areas are covered by transported materials, including salt lakes at the southern end of the tenements, which are quite conductive, EIP was not a preferred method in this case. The objective of this project was to use the MMR/MIP method to locate significantly larger zones of this style of mineralization, which is polarizable but not so conductive, according to previous work.

There were four dipoles for the source electrodes, and data were collected along 33 survey lines. We have worked with the data associated with the first Tx dipole. Fig 7 is the survey location map, and Fig 8a is the measured RPS data. The current electrodes were set on 8400E at 600N and 2200N, down to 90m below the surface, in order to maximize the current channeling. An approximate area of 800 m by 800 m was surveyed. Nine survey lines, perpendicular to the current electrodes, were surveyed with a line spacing 100 m. On each line, the station interval was 25 m. The total data number of points was 275. A major RPS anomaly, shown in red and yellow, with a maximum magnitude of about 2.6 degrees, is located at east side of profile 8600E, almost parallel to the north direction. Theoretically, this positive RPS anomaly suggests that there is material which is more polarizable than its host.

A 3-D model, 4x2x4 km³ (including the air), was designed and discretized horizontally at a non-uniform interval, from 50 m in the central 800 m by 800 m area, to 100 m outside. In the vertical direction, the first 300 m was divided in 25 m intervals so that the shallow structure could be adequately modeled. Below that cell size of 50m to 100 m was used.

This resulted in a mesh with 42x34x22 cells (excluding the air). The inverse problem was therefore formalized by inverting 275 data to recover the chargeabilities in these 31,416 cells. Since the MMR data were also available to us, the background conductivities were obtained by applying a full 3-D MMR inversion. In addition, we also tried a uniform half-space conductivity (0.002 S/m) as the background model. We found the recovered chargeability models were similar. We inverted by working with \( m = \ln \eta \), and then by \( m = \eta \) and imposing positivity using the logarithmic barrier. Both options gave similar results. The recovered chargeability model presented here is from the logarithmic barrier with the conductivities obtained from the MMR inversion. The noise in the data was assumed to be a constant (0.2 degrees). After 15 iterations, the final misfit was 1330. The predicted data are shown in Fig. 8b, and agree fairly well with the observations.

The recovered chargeability model is shown in Fig. 9 in three cross-sections. On each section, a chargeability high is observed, located at the east side, about 100 m deep. This chargeability high (about 0.3) possibly corresponds to either the sulphidic black shale or to ECM (20% sulphide) style ore. The areas of chargeability high are coincident with a conductivity low in this case. Therefore, from previous knowledge, this suggests that this anomaly is potentially an ECM style deposit. Further studies will help clarify this interpretation.

CONCLUSIONS

We have developed an algorithm to invert 3-D MIP data, that is an extension of previous work we have done to invert MMR data. The theoretical formulation follows Seigel's derivation, that is, the MIP response can be written as a linear combination of IP quantities, such as chargeability, pfe or phase shift angle \( \theta \), with the sensitivity (derivative of magnetic field with respect to conductivity) as its coefficients. The sensitivity can then be expressed as a multiplication of three matrices, which are known when solving for the potentials. We have tested this forward modelling procedure against analytical solutions for a vertical dike and a vertical contact model. We have further observed that the MIP response is sensitive not only to relative conductivity, but also to chargeability difference. This interesting relationship allows the flexibility to work with any of the three model parameters, i.e., \( m = \eta \), or \( m = \eta \), together with positivity constraint, or \( m = \ln \eta \) with a reference value of unity.

The inverse problem is solved by minimizing a global objective function composed of the model objective function and data misfit. The final chargeabilities are obtained by carrying out the Gauss-Newton method for model perturbation at each iteration. Depending upon the option of the model parameter, the final chargeability is obtained by shifting the inverted the chargeabilities to a level on which the minimum value of model is zero. We do not have a preference for the choice of model parameters. Three options work almost equally well. For pursuing a joint inversion of EIP and MIP data, the second option might be more consistent with the EIP inversion, where chargeability with positivity constraints has been adopted (Oldenburg and Li, 1994).

Two key components for a successful implementation of the inverse method have been investigated. In order to produce depth or distance information about the target, it is necessary to use a depth weighting or a sensitivity-based weighting.
depending upon the measurements are made on the surface or in boreholes. The second component is related to the effect of conductivity ratio. The conductivity ratio between a target with its host has a significant effect on the MIP response. According to Seigel's assumption, the ultimate effect of chargeability is to alter the effective conductivity when current is applied. If the conductivity ratio between the target and the host is more than two orders of magnitude, it is unlikely to produce polarization current and eventually there is no MIP response. However, if this ratio is in the range of 0.1 to 10, there is an MIP response, and moreover, the background conductivity required for inversion of MIP data might be not so critical.

The Binduli MIP project is a good example to show that we can benefit from applying of the MIP method in cases where the conventional EIP may fail. Another such case might be in an area where a lake or a river covers most of the zone of interest. We hope the 3-D inversion algorithm developed in this research will lead to a rejuvenation of the MIP method for mineral exploration.

ACKNOWLEDGMENTS

We would like to thank Placer Dome Asia Pacific and Croesus Mining for permission to use the Binduli data, which were made available to us by P. Kowalczuk and S. Massey for this research. The work presented here was funded by NSERC and the "IMAGE" Consortium, of which the following are members: AGIP, Anglo American, Billiton, Cominco, Falconbridge, INCO, MIM, Muskox Minerals, Newmont, Placer Dome, and Rio Tinto.

REFERENCES


Li, Y., and Oldenburg, D. W., 1996, 3-D inversion of magnetic data: Geophysics, 61, 394-408.

Li, Y., and Oldenburg, D. W., 2000a, Joint inversion of surface and three-component borehole magnetic data: Geophysics, 65, 540-552.


Figure 1. A schematic of the surface MIP survey. C1 and C2 are current electrodes.
Figure 2. (a) The relative apparent chargeability \( \frac{\eta_2}{\eta_1} \) of numerical and analytic solutions along the x-axis for a vertical contact shown in (b). Two cases were compared: \( \sigma_2 = 0.1\sigma_1 \) and \( \sigma_2 = 10\sigma_1 \).

Figure 3. (a) The apparent PFE of numerical and analytic solutions along the x-axis for a vertical dike shown in (b).

Figure 4. Effect of conductivity ratio \( \frac{\sigma_2}{\sigma_1} \) on the MIP response. (a) The relative apparent chargeability along the y-axis, and the conductivity ratio is labelled on the respective profile. (b) The 3-D cubic model used.
Figure 5. Effect of conductivity ratio \( \sigma_z / \sigma_0 \) on recovery of chargeable target. (a) Cross-section view of the true cube; (b) The synthetic relative apparent chargeability produced by the cube model. Uncorrelated Gaussian noise, with a standard deviation of 3% of the datum magnitude plus 0.01, is added to form the data; (c) Cross-section of the recovered chargeability with \( \sigma_z / \sigma_0 = 10 \); (d) Cross-section of the recovered chargeability with \( \sigma_z / \sigma_0 = 1 \).

Figure 6. A schematic diagram showing the difference between MIP and EIP in a scenario where there is a conductive overburden over the target of interest. MIP current electrodes are placed below the overburden in boreholes.
Figure 7. Binduli MIP survey map.

Figure 8. Comparison of the observed RPS data (a) and the predicted data (b) for the first Tx dipole in the Binduli MIP survey.
Figure 9. Three cross-sections of the recovered chargeability 3-D model.