3D Inversion of Time Domain Data with application to San Nicolas
Napier *, Oldenburg *, Haber †, and Shekhtman *

SUMMARY
We present a 3D time domain inversion algorithm and apply it to UTEM data at San Nicolas. Practical issues for inverting any time domain data set are discussed within the context of the San Nicolas deposit.

INTRODUCTION
The interpretation of surface time domain data has been hampered over the years because of the inability to invert data to recover a 3D conductivity model. Interpretations have often been limited to simplistic plate models, 2D inversions, or combining the results of 1D inversions.

For many geologic situations these approaches have proved to be insufficient. We show how time domain electromagnetic data collected in a field survey can be inverted to recover a 3D distribution of electrical conductivity. The earth is represented as a set of cuboidal cells of constant conductivity and the inversion methodology is similar to that used to invert gravity, magnetic, or DC/IP data, although the scale of computations is considerably larger. Receivers can be in the air, on the ground or in boreholes, and they can measure either electric or magnetic fields in the on-time or off-time of the transmitter. Because computation time increases with the number of transmitters, the algorithm is most effectively used in ground-based geophysics where there are only a few source locations.

To illustrate the effectiveness of the 3D inversion we focus upon the UTEM survey at the San Nicolas deposit in Zacatecas Mexico. This is an excellent study area since the geology is complex but well understood, and numerous other geophysical surveys have been performed and interpreted at the site. In this talk we present a brief description of our forward modeling and inversion algorithms, provide a basic workflow for inverting 3D time domain data, and provide the inversion results for San Nicolas. We discuss some of the practical challenges involved, and provide our views regarding data quality and background information that must be provided in order to make 3D inversions routinely applicable.

Figure 1: East-West cross section of the San Nicolas deposit from drilling (Phillips, 2001).

3D EM FORWARD MODELLING AND INVERSION
Forward Modeling
The details of our forward modeling techniques are outlined in Haber et al. (2004) and the references therein. Briefly, beginning with Maxwell’s equations, the initial conditions, boundary conditions and the inverse problem is defined as a minimization of the functional $\Phi$. The minimization is achieved by the Quasi-Newton approach and the conjugate gradient algorithm.

For the spatial domain, the earth is discretized into cuboidal cells and a finite volume technique is used to calculate the electromagnetic fields on a staggered grid where $E$ is defined on cell faces and $H$ is defined on cell edges. The electric field is decomposed into electromagnetic potentials $A$ and $\phi$ as shown in equation 2 using the Coulomb gauge condition, $\nabla \cdot A = 0$.

$$E = A + \nabla \phi \tag{2}$$

$A$ is defined on the faces of cells while $\phi$ is defined at each cell center. This allows the reformulation of the problem in terms of $A$ and $\phi$ as follows

$$\begin{pmatrix} \nabla^2 - \alpha \mu S(\sigma) \\ \nabla \cdot S(\sigma) \end{pmatrix} = \begin{pmatrix} \mu s \\ \nabla \cdot s \end{pmatrix} \tag{3}$$

where $S(\sigma) = \sigma + \alpha e$ and $s$ is a term containing sources and boundary conditions. The system of linear equations in 3 is solved with a conjugate gradient algorithm.

Inversion
The system of equations shown in 3 can be abbreviated to $A(m)u = q$ where $m$ is the conductivity model of the earth, $A(m)$ is the operation of Maxwell’s equations on that model, $u$ contains the fields and $q$ contains sources and boundary conditions. The predicted data are sampled from the forward modeled fields in $u$ by a transformation $Q$.

$$d^{pred} = Qu \tag{4}$$

The inverse problem is defined as a minimization of the functional $\Phi$. The minimization is achieved by the Quasi-Newton approach and the determination of the unknown regularization parameter $\beta$ is handled.
through a iterative cooling process. The algorithm is described in submitted, but as yet, unpublished work by Haber, Oldenburg and Shekhtman (2006).

The inversion algorithm has been shown to work well on synthetically generated data but there are numerous practical aspects that need to be dealt with when working with field data. In the following we outline some of these workflow elements as we apply the inversion to the San Nicolas data set.

BACKGROUND ON THE SAN NICHOLAS DEPOSIT

San Nicolas is a Cu-Zn massive sulphide deposit located in central Mexico in the state of Zacatecas. The deposit is a continuous but geometrically complex body of sulphides which is covered by 175-250 m of variable composition overburden. The local geology is somewhat complex and contains numerous sedimentary and volcanic units.

The San Nicolas deposit has a long history of being a test case for the application of geophysics to the exploration of massive sulphides. The deposit has been investigated in the past using a number of geophysical surveys including gravity, magnetics, DC resistivity, induced polarization, airborne EM, and ground-based CSEM/CSAMT surveys. The deposit has also been thoroughly drilled and, as a result, the geology and physical properties of the deposit are reasonably well understood.

The sulphide deposit presents a conductivity contrast with most of the geologic units in the area, however some of the overburden, the tertiary volcanic breccia, has a conductivity in the range of that found in the sulphide. Figure 1 shows an east-west cross section through the deposit. Figure 2 contains a summary of typical values of the various physical properties of the deposit and its hosts. These values are tabulated as a result of borehole surveys and core sample testing.

A UTEM survey was conducted over the San Nicolas deposit in December of 1998. The survey detected the deposit, however a more complete interpretation of the data was hampered, at least partly, due to the complexity of the local geology. In the following we apply our 3D forward modeling and inversion to these data. We detail some of the challenges encountered and attempt to develop a systematic method for inverting field data which can be applied to time domain surveys in general.

### INVERSION OF UTEM DATA

#### Discretization in Time

There are some key issues one must deal with in order to invert a field data set such as the San Nicolas UTEM survey. The first objective is to discretize the transmitter waveform in time. With an on-time system such as UTEM we must be especially careful about the time discretization as multiple cycles of the waveform may be required in the waveform for the electromagnetic fields to reach an equilibrium within the earth. To test this, a synthetic modeling exercise with multiple waveforms is carried out. We find, for the error level ascribed to the data, that data after $\frac{3}{4}$ of a UTEM cycle are sufficient to achieve equilibrium.

The inversion used a 38 time step waveform over $\frac{1}{4}$ UTEM cycles. The waveform is densely sampled in log-time on the final ramp and sampling on that ramp begins at least 1.5 decades before the first datum. We find that 4 samples per decade produces sufficiently accurate results and, at the same time, limits computational effort.

#### Discretization in Space

Discretization in space necessitates two considerations: first, mesh boundaries must be sufficiently distant from the survey area so that our boundary conditions are satisfied; and second, the cells in the center of the mesh should be fine enough to describe the early time data adequately. Ideally we let the mesh boundary extend away from the survey location to several diffusion distances, based on the latest time recorded. The size of the small cells in the mesh are specified to be a fraction of the diffusion distance of the earliest time. We define the diffusion distance as per Ward and Hohmann (1987) as being the quantity $L$ in equation 6.

$$L = \sqrt[4]{\frac{2r}{\mu \sigma}}$$

For the San Nicolas data set, the central volume of the mesh was discretized into 50 m by 50 m cells with a variable thickness, from 25 m
Understanding the Data

Time domain EM data are often normalized for the purposes of interpretation. This is particularly prevalent for UTEM data where the presented data are often reduced by subtracting the latest time channel and then normalizing by the primary field. Often there is other scaling that is incorporated and, in the worst cases, additional filtering is applied so that plotted anomalies or profiles can be more readily interpreted by eye. When inverting data however, it is essential that our mathematical representation of the data conform with the observations provided. Discrepancies between these two forms of “data” can spell disaster for the inversion. Considerable effort is often required to completely understand the data including carefully removing prior normalizations, confirming that units are known, that orientation conventions are clear and locations are in the same coordinate systems. These issues may seem trivial at first glance, however they are the cause of most errors in the inversion process. In fact, this is often the most important step in the inversion process and its importance to the results can not be underestimated. For the San Nicolas case considerable time was spent converting the data into a usable format for inversion.

Assigning Errors to the Data

The misfit functional used in the inversion requires that each datum be normalized by an estimated error. This is important to ensure that no datum contributes unduly to the solution, and this is another key stage in the inversion process. Unrealistic error estimates can degrade the solution or possibly cause the inversion program to stall long before it reaches a geologically useful result. Factors contributing to the error are instrumental errors, mislocations or mis-orientations of receivers, additive noise, and numerical discretization. These aspects arise in all inverse problems but are particularly difficult for time domain EM problems where we have many orders of magnitude change in the data as a function of distance from the source or with different time channels. For San Nicolas, data error assignment was complicated by zero crossings in the data, in both time and space. Data errors were assigned by separating time channels, and letting the data in each time channel have an error based on a percentage of the datum and a constant value. In consideration of the somewhat large cell size in the inversion mesh the percentage error was larger for early time channels and smaller for late time channels. The constant was based on a fraction of the median of the absolute value of the data in each time channel. This procedure results in a fairly effective and robust method of assigning errors to field data. An example of the observed and predicted data for UTEM channel 5 is presented in Figure 4. An observed and predicted decay curve for a selected receiver is shown, along with assigned errors, in Figure 5.

Providing a-priori information

Because of costs, most practical field surveys are deficient in the number of sources and the spatial distribution of receivers and the types of fields they can measure. Consequently non-uniqueness is a reality, and the quality of result that can be obtained from the inversion is greatly increased by additional information about the conductivity. Background conductivities, which can be used as reference models in the inversion, are particularly important. For San Nicolas we used drill core conductivity information as well as previous EM surveys to help define background conductivity. We also experimented with including an overburden layer in the background conductivity model. We found that the depth of overburden and its conductivities appear to be extremely variable and drilling does not define it adequately. In this case the overburden is best defined by inversion of the UTEM data itself. In the inversion result shown here, in Figure 6, we used a background reference model with a resistivity of 200 Ωm.

Dealing with computing limitations

A critical difference to other 3D inversion codes such as 3D DC resistivity inversion is the much larger scale of the computations required to invert a comparable survey. The 3D time domain inversion is a computational challenge that tests the limits of present day personal computers. Consider a typical inversion from the San Nicolas data set: a single transmitter with 249 recording stations each with 9 usable time channels. The inversion was performed on a spatial domain discretized into 90,000 cells and there were 38 time steps in the transmitter waveform. The inversion took roughly 5 days to complete on a personal computer. Multiple transmitter inversions can take much longer. While these computational times are not prohibitive they do require careful use of the inversion code as mistakes, or inadequacies
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in the operation of the inversion code, may not become apparent for several days into the process.

RESULTS

Figure 6: a) UTEM inversion results as cross section compared to a conductivity profile produced from an interpolation of drill core conductivity testing.

San Nicolas data from the individual loops were inverted but the best inversion result was achieved by simultaneously inverting data from multiple transmitters. That provided better definition of the shape of the deposit and attenuated some artifacts that the reference models tended to produce. In particular, the inversion of the east loop in conjunction with the large loop does a very good job of defining the deposit. The result is shown in Figure 6. Note that while the deposit is well located in horizontal directions, the depth is underestimated and there is no manifestation of a conductive overburden. The problem may be related to a non-uniqueness in San Nicolas UTEM data. We are investigating the situation to determine how the depth issue can be resolved.

CONCLUSIONS

The field data inversion of UTEM data appears to have been successful. Figure 6 clearly shows a conductive body in the location of the San Nicolas deposit with similar conductivities to those from from the drill core tests. The depth to the body is underestimated and we are carrying out further investigations to understand more precisely what has occurred. Nevertheless, whereas previous electromagnetic surveys have been unable to clearly define any of the deposit boundaries, the 3D inversion of the UTEM data has located it laterally with a high degree of accuracy.

To conclude, we find that 3D time domain inversion of field data sets is challenging, but with high quality data, well planned workflows, and careful attention paid to detail, very useful inversion results can be obtained. With continued development of these aspects cost-effective inversion will soon become routine.

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REFERENCES

EDITED REFERENCES
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REFERENCES