Summary

We develop algorithms to forward model and invert magnetometric resistivity (MMR) responses over an arbitrary 3-D conductivity structure. The observed data can be at the surface or in the borehole. In the forward modelling algorithm, the second-order partial differential equations for the scalar and vector potentials are discretized on a staggered-grid using the finite-volume technique. In the inversion method, we discretize the 3-D model into a large number of rectangular cells of constant conductivity, and the final solution is obtained by minimizing a global objective function composed of the model objective function and data misfit. All minimizations are carried out with the Gauss-Newton algorithm and model perturbations at each iteration are obtained by a conjugate gradient least squares method (CGLS), in which only the sensitivity matrix and its transpose multiplying a vector are required. A depth weighting and a sensitivity-based weighting are respectively applied to inversion of surface and down-hole MMR data. The inversion method has been tested with a field data set from Western Australia.

Introduction

The magnetometric resistivity (MMR) method involves measurement of magnetic fields associated with artificially created, non-inductive (DC or pseudo-DC) current flow input to the earth through two electrodes (Edwards and Nabighian, 1991). Historically, for an MMR survey, the wire connecting the two current electrodes is typically laid in a horseshoe array and measurements are made somewhere in between the electrode spread. The observed magnetic field is a combination of contributions from the bipole connecting wire, the 1-D background conductivity earth including the conductive overburden and host rock, and the anomalous field attributed to currents channelled through the conductive targets. The goal of a MMR survey is to extract relative conductivity of the earth from the measurement of the magnetic field.

MMR methods have advantages over conventional electrical resistivity methods. The MMR measurement has greater sensitivity to conductive targets beneath a moderately conductive overburden than does the DC-electrical method. In addition, MMR is only sensitive to the relative conductivity between the targets and their surroundings, not the absolute conductivity values. This makes MMR an attractive technique for detecting poorly conducting targets such as some zinc deposits (Bishop et al., 1997).

Calculation of MMR responses over a conductive structure is usually based upon the modification of a numerical “resistivity” method (Edwards and Nabighian, 1991). This involves two steps. The first is to solve Poisson’s equation for the electric potential by using the standard finite element or finite difference techniques. This is the same procedure implemented in the conventional DC-electrical forward problem. The second step is to calculate the magnetic field through the modified form of the Biot-Savart law in which magnetic field is explicitly expressed as a volume integral of a functional that is proportional to the cross product of gradient of the potential and conductivity throughout the volume (Edwards and Nabighian, 1991).

In this paper we perform forward modelling by solving a mixed sub-problem of electrostatic and magnetostatic problems (Haber, 2000). One first solves an electrostatic problem for a scalar potential, and then solves a magnetostatic equation for magnetic field. The first stage is therefore similar to finite difference modelling mentioned in the previous paragraph, but the second stage is different in that we obtain the magnetic field by solving a differential equation rather than by performing a volume integration. The inverse problem for MMR data is formulated as an optimization problem in which we minimize a model objective function subject to the constraints that the data misfit is achieved to some level. Our model objective function has the flexibility to incorporate extra information, and a Gauss-Newton iterative method is used to obtain the model perturbation at each iteration. The regularization parameter that controls the balance between model norm and misfit is determined through a cooling process. At each iteration we must solve a large matrix system. We use a conjugate gradient least-squares (CGLS) method and hence the majority of computations involve multiplying a sensitivity matrix, or its transpose, by an arbitrary vector. This can be accomplished without explicitly calculating and storing the sensitivity matrix. The code has been extensively tested on synthetic data and here we invert a field data set over a known mineral deposit from Western Australia.

Compution of magnetic field by finite-volumes

In a typical MMR survey, an external current is impressed into the ground through a pair of current electrodes. For MMR modelling we can assume that the exciting source is a direct current (DC). This means the solution for general Maxwell’s equations in the frequency domain can be reduced to a steady-state problem (frequency is zero), which can be written as

\[ \nabla \times \mathbf{E} = 0 , \quad (1a) \]

\[ \nabla \times \mathbf{H} - \sigma \mathbf{E} = \mathbf{J}^e , \quad (1b) \]

\[ \nabla \cdot (\mu \mathbf{H}) = 0 , \quad (1c) \]

where \( \mathbf{E} \) is the electric field intensity in V/m, \( \mathbf{H} \) is the magnetic field intensity in A/m, \( \mathbf{J}^e \) is the external electric current density in A/m², \( \sigma \) and \( \mu \) are the electric conductivity and magnetic permeability, respectively. We define a scalar potential \( \phi \) and a vector potential \( \mathbf{A} \), such that

\[ \mathbf{E} = -\nabla \phi , \quad (2) \]

and

\[ \mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} . \quad (3) \]
In addition, the Coulomb gauge condition is imposed on \( \mathbf{A} \), i.e., \( \nabla \cdot \mathbf{A} = 0 \). Then \( \phi \) and \( \mathbf{A} \) satisfy

\[
\nabla \cdot (\sigma \nabla \phi) = \nabla \cdot \mathbf{J}^*.
\]

\[
\nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} + \sigma \nabla \phi = \mathbf{J}^*.
\] (5)

In order to obtain a numerical solution, the 3-D medium is discretized into a large number of rectangular cells whose conductivity and magnetic permeability values are constant at each cell, using a finite-volume method on a staggered grid (Haber and Ascher, 2001, Haber et al., 2000). In this discretization scheme, \( \mathbf{A} \) and \( \mathbf{J} \) are chosen to be at centers of cell faces, \( \mathbf{H} \) at centers of cell edges, and \( \phi \) at cell centers. With such a discretization scheme, equations (4) and (5) can be assembled as a matrix system for unknowns \( \mathbf{A} \) and \( \phi \) as following

\[
\begin{pmatrix}

\mathbf{A}^{(e)} & \nabla_h \times \nabla_h^{(f)} & - \nabla_h \mathbf{M}_h^{(e)} \nabla_h \times \mathbf{s} & \nabla_h \cdot \mathbf{M}_h^{(e)} \\
0 & \phi \nabla_h \times \mathbf{v}_h^{(f)} & 0 & \nabla_h \cdot \mathbf{v}_h^{(f)} \\
\end{pmatrix} \begin{pmatrix}

\mathbf{j}^* \\
\mathbf{v}_h^{(f)} \\
\end{pmatrix}.
\] (6)

where the matrices \( \mathbf{v}_h^{(e)} \times \) and \( \mathbf{v}_h^{(f)} \times \) are assembled from the discretization of the curl operator, projecting from cell edges to faces and from faces to edges, respectively; matrices \( \mathbf{v}_h \cdot \) and \( \nabla_h \) correspond to the discretization of the div and grad operators. When assembling the discrete first-order partial differential equations, we impose certain boundary conditions: the normal components of \( \mathbf{A} \) and \( \mathbf{J} \) at the outer boundaries are set to 0. The material matrix \( \mathbf{S} \) arises from the discretization of the conductivity, that is a harmonic averaging of values at cell faces. The matrices \( \mathbf{M}_h^{(e)} \) and \( \mathbf{M}_h^{(f)} \) result from an arithmetic averaging of permeability at cell edges and the permeability at the cell centers, respectively. The superscript \( -1 \) represents the inverse of the matrix. The second term \( -\nabla_h \mathbf{M}_h^{(e)} \nabla_h \cdot \) in the first entry serves as a stabilizer, which will make the system positive and definite.

The solution for \( \mathbf{A} \) and \( \phi \) in Eq.(6) is obtained in two steps: first we solve Poisson's equation for \( \phi \), then substitute \( \phi \) into the right hand side of subsystem for \( \mathbf{A} \), and solve for \( \mathbf{A} \). In both steps, we use biconjugate gradient stabilized method (BiCGSTAB), combined with a symmetric successive over relaxation (SSOR) for the solutions. After \( \mathbf{A} \) and \( \phi \) are obtained, the magnetic field \( \mathbf{B} \) can be obtained from

\[
\mathbf{B} = \nabla_h^{(f)} \times \mathbf{A}
\] (7)

**Inversion for conductivity**

Previous section establishes forward modelling of the MMR data \( \mathbf{d} \) from a known conductivity model \( \mathbf{m} \), i.e., \( \mathbf{d} = F[\mathbf{m}] \). \( F \) is the forward operator. For inversion we are given a set of observed data \( \mathbf{d}^{\text{obs}} \) and some information about the errors in the data. Since the inverse problem is non-unique, we can only find a particular solution by generating a model that minimizes a specific model objective function and adequately fits the observed data. This model objective function \( \phi_m \) is chosen by the fact that we often wish to find a model that has minimum structure in the three directions, and at the same time is close to a reference model \( \mathbf{m}_0 \). For the inversion we define the model parameter to be \( m = \ln \sigma \). Because MMR data are only sensitive to relative conductivity, this is a natural choice of model parameter. With the same orthogonal mesh as in the forward modelling, the model objective function has a discrete form

\[
\phi_m(\mathbf{m}) = \left\| W_m(\mathbf{m} - \mathbf{m}_0) \right\|^2.
\] (8)

The data misfit function \( \phi_d \) is the typical \( L_2 \)-norm measure of data misfit

\[
\phi_d = \sum_{i=1}^N \left( \frac{F[\mathbf{m}^e] - d^{\text{obs}}}{\sigma_i} \right)^2 = \left\| W_d (F[\mathbf{m}] - d^{\text{obs}}) \right\|^2,
\] (9)

where \( \sigma_i \) is the estimated standard deviation of the \( i \)-th datum, and \( N \) is the number of the data points.

The inverse problem is formulated as an unconstrained optimization problem, and we find a 3-D earth model that minimizes objective function

\[
\phi = \phi_d + \beta \phi_m,
\] (10)

where \( \beta \) is a trade-off parameter which balances the importance of the model objective function and the data misfit. \( \beta \) is chosen such that the data are adequately fitted, and at the same time, the recovered model is kept as simple as possible.

We construct an iterative procedure by linearizing the forward modelling. At each iteration, the model perturbation can be obtained by solving a standard Gauss-Newton equation

\[
\left( J^T W_d^T J + \beta W_m^T W_m \right) \delta \mathbf{m} = J^T W_d^T \left( F[\mathbf{m}^{(e)}] - d^{\text{obs}} \right),
\] (11)

where \( J \) is the sensitivity matrix whose elements are \( J_{ij} = \frac{\partial d_j}{\partial m_i} \).

The solution to this equation is equivalent to the least-square solution of

\[
W_d J \sqrt{\beta W_m} \delta \mathbf{m} = \left( W_d F[\mathbf{m}^{(e)}] - d^{\text{obs}} \right) - \sqrt{\beta W_m} (\mathbf{m}^{(e)} - \mathbf{m}_0),
\] (12)

and we apply the conjugate gradient least-squares algorithm (CGLS) to solving this system. In the CGLS algorithm, we only require evaluation of the products of the sensitivity matrix and its transpose with a known vector. Roughly speaking, this can be achieved at two extra forward modelling calculations.

**Practical Issues**

In MMR studies we are usually faced with the challenge of trying to find information about the 3-D earth from very few data. In particular, there is often only one source location. Nonuniqueness is extreme and sensible results can be obtained only if additional information can be included. Here we consider two items: additional weighting in the objective function and the importance of estimating a background conductivity.

**Weighting scheme**

The sensitivity functions attain the largest values near the receiver as well as the source electrodes. Correspondingly, the recovered models tend to have structure concentrated near the receivers or source electrodes. As with gravity and magnetics data, there is no inherent depth resolution in the surface MMR data, and without this, the recovered conductivity model tends to concentrate near the surface. To counteract the geometric decay of the kernels, and to distribute the conductivity with depth, a
depth weighting $w(z)$, similar to the one implemented in Li and Oldenburg (1996), takes the form of

$$w(z) = \frac{2z_0}{(z+z_0)^{0.85}},$$

(13)

where $z_0$ is half the thickness of the cell just below the surface, $z$ is the depth to a cell center. This works well for surface data. However, for down-hole data, we use a sensitivity-based weighting (Li and Oldenburg, 2000) to distribute the anomalous material away from current sources. We have found that a weighting function

$$w_j = \begin{cases} \frac{w_j}{\eta} & \text{if } w_j > \eta \\ 1.0 & \text{if } w_j < \eta \end{cases}$$

(14)

usually produces reasonably good results. $w_j$ is determined by

$$w_j = \left( \sum_{i=1}^{N} J_{ij}^2 \right)^{1/4},$$

(15)

where $J_{ij}$ are the elements of the sensitivity matrix. For the purpose of designing a weighting, it suffices to use a simple approximation to the sensitivity, which can be computed by using a current dipole in a homogeneous half-space. The threshold $\eta$ is chosen in practice such that the electrode artifacts can be effectively removed, and the inherent functionality is also kept. We empirically determine $\eta$ by

$$\eta = \frac{(w_j)_{\text{max}}}{15},$$

(16)

where $(w_j)_{\text{max}}$ is the maximum of all weightings.

Background conductivity

Another practical issue is related to the choice of the 1-D background model. For surface MMR data, there are two forms of fundamental ambiguities for recovery of conductivity. Firstly, magnetic field data can determine electrical conductivity only to within a multiplicative constant. Thus for a body buried in a uniform host medium, we can find only the relative conductivity contrast, not the absolute value of conductivity. The second ambiguity arises from the fact that surface MMR cannot distinguish between a homogeneous half-space and a 1-D conductive medium. As a consequence, for a 3-D body in a 1-D layered medium, it is still difficult to get information about the general background 1-D medium, if sources and receivers are at the surface.

In the down-hole MMR case, the second ambiguity theoretically disappears. In practice, however, there are usually just one or two sources and a couple of drill-holes surveyed. In such circumstances our experience is that direct application of a 3-D inversion algorithm does not produce a representative conductivity. Further restriction of model space is required. In particular, the inversion produces much better results when looking for 3-D targets within a known 1-D background. Our approach is to perform a 1-D inversion first to obtain the 1-D background conductivity, and then apply a 3-D inversion to looking for local 3-D targets.

Field example

Surface MMR data were collected at the Mons Cupri deposit of the Pilbara area, Western Australia. As shown in Fig. 1, the general strike of the formation around the Mons Cupri deposit is north-south and dips 30 degrees westward. The ore body is hosted by the Mons Cupri rhyolite fragmental, which is sequentially overlain by the Cistern Formation, Cap shale, Comstock andesite and Whim Creek shale. It has been estimated that the deposit’s resource is 1.5 Mt oxide ore at 1.13 wt% Cu and 1.4 Mt sulfide ore at 1.74 wt% Cu, 1.13 wt% Pb, and 2.48 wt% Zn.

![Figure 1. Geological plan and section view of the Mons Cupri deposit with current electrodes, connecting wires and the MMR survey area.](image)

The current electrodes were set on 1000N at 1500E and 300E, and are aligned with the geologic cross-section. An area of 1000 m by 1000 m over the ore body was surveyed. Eleven survey lines parallel to the strike were 100 m apart, and the station interval inside the central 500 m by 500 m area was 50 m and it was 100 m outside this central area. The observed horizontal magnetic field (South pointing) was reduced to the MMR response in percent. Fig. 2a shows the MMR responses at the 162 stations.

A 3-D model of $2 \times 2 \times 1 \text{ km}^3$ (excluding the air space), without topography, was designed, and discretized into 34 x 34 x 20 cells. The inverse problem was therefore formalized by inverting 162 data to recover the conductivities in these 23,120 cells. The reference model was a uniform half-space of 0.001 S/m. We also assumed that each datum had an error whose standard deviation is equal to 8% of its magnitude plus a base value of 2 anomaly units. The target misfit was set to 162, but the achieved misfit after 13 iterations was 850. Most of the misfit comes from the isolated points such as one at the upper right corner. The predicted data are shown in Fig 2b.

Fig. 3 compares the recovered conductivity model with the geology in the cross-section at 950N. The Lead-Zinc
mineralization consists of two tabular targets separated by about 200 m in depth. The MMR results show a region of high conductivity centered between the two mineralized lenses. This is characteristic of a low resolution image of a complex structure. Overall, we feel that the inversion has been successful in delineating the volume containing the mineralization.

Discussions

We have developed an algorithm to compute the MMR response due to steady current sources in a 3-D environment. We have a practical algorithm to invert both the surface and down-hole MMR data to recover a 3-D conductivity distribution. The work provided here is general, and can be used for any combination of current sources or multiple components of measured magnetic fields. The field example was deemed to be successful in that the low resolution image from the inversion seemed to correspond with the major geological units (in particular the mineralization), however it also highlights the deficiencies in the traditional field approach. Only one source location and one component of magnetic field data were used in the inversion. More source locations, and acquisition of full 3-component data, would greatly improve the results.

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