Identification of UXO by regularized inversion for Surface Magnetic Charges

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Outline

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   - Properties of charges

2. Inversion of surface magnetic charges
   - Regularization
   - Total magnetic charges and regularization

3. Application to test stand and field data
   - TEM test stand
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Model developed by Shubitidze et al., 2005. Assume surface charges on spheroid around ordnance

\[ H^{sc}(r) = \int_{s} G(r,r') \, \sigma_{m}(r') \, dr' \]

Define normalization of charges by primary field

\[ \sigma_{m}(r') = \Omega(r') \, H^{pr}(r').n \]

Solve \([Z] [\Omega] = [H^{data}]\)

Goal: Total magnetic charge

\[ Q = \int_{s} \Omega(s') \, ds' \]

Expectation: total magnetic charge can discriminate different types of ordnance
Example

SMC for a 40 mm projectile (test stand data)

1. Charges distributed in patches

Result:
- excellent fit to data
- extreme variations in charge distribution

Questions:
- numeric: how does Q vary with resolution?
- model stability: does Q remain the same when depth & orientation change, or when noise level rises? is Q a reliable discriminant for a UXO?
Example

SMC for a 40 mm projectile (test stand data)

1. Charges distributed in patches

2. Charges distributed in rings

**Pro:** excellent prediction

**Cons:**
- total magnetic charge $Q$ depends on resolution
- extreme variations of charge distribution
- not shown, $Q$ varies with depth and orientation
Properties of surface magnetic charges

Questions:
- what controls the distribution of charges $\Omega$?

- does the normalization of surface charges by primary field
  \[ \sigma_m(r') = \Omega(r') \ [H^{pr}(r').n] \]
  ensure that normalized surface charges $\Omega$ depend on ordnance only?

Analysis:
Gauss says
\[ \sigma_m(r') = 2 \mu_0 H^{sc}(r').n \]
therefore
\[ 2 \mu_0 \ [H^{sc}(r').n] = \Omega(r') \ [H^{pr}(r').n] \]
defines the normalized charge $\Omega$.

Let's see how $[H^{sc}(r').n]$ and $[H^{pr}(r').n]$ relate...
Properties of charges

Normalized surface charge distribution for different orientations

Primary field: 1x1 m transmitter loop
Target: cylinder
Secondary field: predicted by Method of Auxiliary Sources (MAS), exact solution

Compare:
- ratio $H^{sc}.n/H^{pr}.n$
- analytic solution by Method of Moment
Properties of charges

Normalized surface charge distribution for different orientations

Result: normalized surface charges depend on excitation
Each excitation (position, orientation) generates a charge distribution. Inversion of data from multiple excitations (position, orientation) requires averaging of charge distributions.
**Inversion: regularization**

**Original problem:** invert for \( \mathbf{m} = \mathbf{\Omega} \)

\[ \mathbf{Z} \mathbf{m} = \mathbf{d} \]

given data \( \mathbf{d} = \mathbf{H}^{\text{data}} \) at surface. This is an ill-posed problem.

**Regularized problem:** incorporate knowledge as constraints:

\[
\text{minimize} \quad ||\mathbf{W}_d (\mathbf{Z} \mathbf{m} - \mathbf{d})||^2 + \lambda \ ||\mathbf{W}_m (\mathbf{m} - \mathbf{m}_o)||^2
\]

\( \text{data misfit} \quad \text{model misfit} \)

\( (\mathbf{W}_d \) data weighting matrix, \( \mathbf{W}_m \) regularization operator, \( \lambda \) regularization parameter and \( \mathbf{m}_o \) reference model)\)

**Transfer of knowledge:**

- \( \mathbf{W}_m \) : first derivative operator to impose smoothness

- \( \lambda \) and \( \mathbf{m}_o \) : charge distribution is an **average** of the ensemble of charge distributions that fit the ensemble of **excitations**. Use uniform charge distribution as reference model.
Inversion: effect of regularization

\[ \text{minimize } || W_d (Z m - d) ||^2 + \lambda || W_m (m - m_o) ||^2 = \Phi_d + \lambda \Phi_m \]

Effect of regularization:

1. reduces amplitude of variation in charge distribution. Limit case is uniform charge distribution
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1. reduces amplitude of variation in charge distribution. Limit case is uniform charge distribution

2. provides controlled balance between data misfit $\Phi_d$ and model smoothness $\Phi_m$
**Inversion: effect of regularization**

\[
\text{minimize} \quad \| W_d (Z m - d) \|^2 + \lambda \| W_m (m - m_0) \|^2 = \Phi_d + \lambda \Phi_m
\]

Effect of regularization:

1. reduces amplitude of variation in charge distribution

2. balances data misfit $\Phi_d$ and model smoothness $\Phi_m$

3. induces changes in total magnetic charge $Q$

**Solution:** choose model such that misfit is small and $Q$ is similar to $Q_{\text{uniform}}$
Application to test stand data

Data collected at USACE-ERDC Vicksburg, MS

Sensors: Geonics EM-63 (26 time channels) and Geophex GEM3 (10 frequencies)
Application to test stand data: TEM

EM-63 data: total magnetic charge $Q$ as a function of time for horizontal, vertical and oblique (45°) orientations, at shallow and deep positions.

Result: 1. $Q$ is stable when position of target varies
2. Each target has different $Q$
3. Magnitude of $Q$ scales with size of target
4. Time decay reflects differences in material
Total magnetic charge $Q$ as a function of time.
15 standard items, from 20 mm to 155 mm

Results:
1. $Q$ is stable
2. Each target has different $Q$
3. Magnitude of $Q$ scales with size of target
4. Time decay reflects differences in material
5. Same applies for FEM data
Field data collected at Sky Research test plot in Ashland, OR, with EM-63 sensor mounted on cart (30 cm clearance) over M42, 40 mm and 60 mm projectiles.

Legend:
- thick lines: $Q$ inferred from field data
- thin lines: $Q$ from test stand data

Result: total magnetic charge from field data falls within range of value from test stand data

Conclusion: identification of buried ordnance in the field is possible
Conclusion

1. Understanding magnetic charges: recovered charge distribution reflects averaging of multiple excitations

2. Method of Surface Magnetic Charges requires regularization

3. Each type of ordnance has a characteristic total magnetic charge that reflects its geometry and material

4. SMC is a viable model for inversion of EMI field data
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... more stuff ...
Application to test stand data: FEM

13 standard items, from 20 mm to 155 mm

Results:

1. Total magnetic charge is stable
2. Each target has different total magnetic charge
3. Magnitude of total magnetic charge scales with size of target
4. Time decay reflects differences in material
Inversion: examples

Regularization curve and total magnetic charge for a 40 mm projectile

Total magnetic charge in a low Signal to Noise environment for 40, 60, 90 mm and M42 projectiles

Result: regularized inversion ensures stability of total magnetic for changes in position and orientation, even with high noise level