Inversion of magnetics for UXO discrimination and identification

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Abstract
Inversion of total-field magnetics to recover the dipole moment of an anomaly provides valuable discrimination information. Using the recovered dipole moment it is not possible to reliably classify the item as ordnance/non ordnance. Rather, the items can be placed in a list ordered by the likelihood that they are UXO. The ranking is based on how closely the recovered moment matches the predicted dipoles drawn from a library of ordnance items expected to occur in the area. Error analysis indicates that the recovered dipole moments are well constrained by the inversion.

When using the recovered dipole moment to identify the ordnance type, there is an ambiguity, whereby changing the diameter can be compensated for by changing the length and orientation of the item relative to the Earth’s field. To reliably classify the ordnance, the dimensions need to be constrained, either directly or indirectly by constraining the orientation. Our first attempt to provide this constraint was by modelling ordnance as a spheroid and including the higher order octupole response in the inversion. However, the octupole decays to below the noise floor very rapidly with distance. Additionally the spheroid model is not an accurate approximation of the shape of many projectiles. Our second attempt therefore inverted for the dipole and quadrupole moments of an arbitrary axi-symmetric object. This inversion method provided some orientation constraints but not enough to completely remove the shape ambiguity. We therefore conclude that additional information is required to provide an unambiguous classification of ordnance shape, and suggest as a possible solution, joint inversion with electromagnetic data.

The techniques described in this paper are applied to magnetics data collected in Montana. The identification technique predicted the correct ordnance type 60% of the time.

1 INTRODUCTION
Location of UXOs involves the successful detection and identification of compact metallic objects. Our approach to the discrimination problem is to parameterize the characteristics of the target in some way (e.g. by its location, orientation, shape, size and material properties) and develop a good forward model that allows predicted data to be generated from a given target realization. Inversion can then be used to find the set of target parameters that best fits the data. The target parameters are used to make decisions regarding discrimination (UXO/non-UXO) and identification (what is the ordnance type).

Spheroids have been proposed as an approximate parameterization of ordnance by several authors (McFee, 1989; Altshuler, 1996; Bulter et al., 1998). While the spheroid
does not capture the top-bottom asymmetry of many ordnance items, close agreement between observed anomalies over test stands and spheroid fits have been demonstrated (McFee, 1989; Bulter et al., 1998). Furthermore, the magnetic anomaly from a solid spheroid has been shown to be very similar to a hollow spheroid (Altshuler, 1996).

The response of a spheroid (in fact any compact body) can be decomposed into a series of moments by a multipole expansion (Stratton, 1941). The response of the dipole component dominates the anomalies caused by most buried ordnance due to the rapid falloff with distance of the other components. We have therefore developed an inversion routine (Billings et al., 2002b) to recover the dipole moment and position that best matches an observed anomaly. The recovered dipole moment in then used to rank items according to the likelihood they are UXO. We prefer such a ranking over a definitive statement such as ordnance or non-ordnance because it is not possible to make such a black and white distinction. Discrimination using magnetics is made difficult by the possible presence of remanent magnetization.

The recovered dipole moment can also be used to try to identify the ordnance type. There is some ambiguity in this process because the dipole moment is the product of magnetization with volume. Due to self-demagnetization effects the magnetization can change significantly as the orientation of the body relative to the Earth’s field changes. Thus orientation can be traded with volume to produce the same or similar dipole moment. By recognizing that there are a finite number of ordnance types present in an area (we build up a library), this ambiguity can be reduced significantly. Note that we are less interested in ordnance identification per se than in being able to utilize this information to aid our discrimination efforts (UXO/non-UXO).

We can think of at least two possible methods to constrain the geometry. The first is by trying to provide a direct constraint by including the higher order moments (the octupole) of a spheroid. The second is to provide an indirect constraint by constraining the ordnance orientation. We propose to achieve this by recovering the dipole and quadrupole moments of an arbitrary axially symmetric body.

## 2 Magnetic modelling

The scalar potential, $\phi(r)$ of a compact body occupying the volume $V$ with magnetization $M$ is given by the expression

$$\phi(r) = \frac{1}{4\pi} \int_V M \cdot \nabla \left( \frac{1}{r} \right) dV$$

where $\hat{r}$ is the distance between the source and observation points. A well known and general method (Stratton, 1941) for calculating the magnetic field via this equation is by the multipole method. This proceeds by expanding $\frac{1}{r}$ as a Taylor series about the origin. The magnetic field is then expressed as a sum of moments. The first term in the expansion is the dipole moment which is a 3 element vector whose $i$-th component can be shown to be equal to

$$m^{(d)}_i = \int_V M \cdot \nabla x_i dV = \int_V M_i dV$$

When the magnetization vector is constant the dipole moment is the product of magnetization with volume,

$$m^{(d)} = MV$$
The contribution of the dipole to the magnetic field decays as the 3rd power of distance from the object and dominates the far-field. The next moment is the quadrupole which is a rank 2 tensor with components given by the expression

$$m^{(q)}_{ij} = \int_V \mathbf{M} \cdot \nabla(x_i x_j) dV = \int_V (M_i x_j + M_j x_i) dV$$  \hspace{1cm} (4)$$

The quadrupole decays as the 4th power of distance and is zero for a spheroid due to symmetry. The last moment we consider is the octupole, a rank 3 tensor

$$m^{(o)}_{ijk} = \int_V \mathbf{M} \cdot \nabla(x_i x_j x_k) dV$$  \hspace{1cm} (5)$$

The octupole decays as the 5th power of distance so that its contribution falls to below the noise floor very quickly with increasing distance. Unless the observation point is close to the object, or the data quality is very good, the octupole response will not be observed above the noise floor.

In general the magnetization vector $\mathbf{M}$ has a contribution due to both induced and remanent magnetization. For a spheroid, the induced magnetization is constant and parallel and for large relative permeability can be shown to be

$$M_i = \frac{2 B_i}{\mu_0 \alpha_i}$$  \hspace{1cm} (6)$$

where $B_i$ is the Earth’s magnetic field and $\alpha_i$ is a demagnetization factor that depends only on the shape of the spheroid (and not its size, material properties etc). Equation 6 returns $\mathbf{M}$ in spheroid centered coordinates and assumes $B$ is also spheroid centered. We adopt the convention that $\phi$ is the angle clockwise from North of the projection of the semi-major axis of the spheroid onto a horizontal plane, while $\theta$ is the dip angle (positive upwards) of the axis relative to that plane. To use geographical coordinates we utilize the Euler rotation tensor

$$A = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \theta \sin \psi & \sin \theta \cos \psi & \cos \theta \\ \cos \theta \sin \psi & \cos \theta \cos \psi & -\sin \theta \end{bmatrix}$$  \hspace{1cm} (7)$$

to rotate $\mathbf{B}$ to spheroid centered coordinates and then use its inverse $A^T$ to rotate $\mathbf{M}$ to geographical coordinates,

$$\mathbf{M} = \mu_0^{-1} A^T \mathbf{F} A \mathbf{B}$$  \hspace{1cm} (8)$$

where $\mathbf{F}$ is a diagonal matrix with entries $F_{ii} = 2/\alpha_i$.

### 2.1 Ordnance identification using magnetometry

Keeping in mind the spheroid model and the observation distances and data quality typical of real-world magnetometer surveys, the most information we can usually hope to extract from an anomaly is the dipole moment. In Billings et al. (2002b) we developed a dipole inversion routine that was able to achieve the correct ordnance classification (at Guthrie Road, Montana) about 55% of the time. To expand on the challenges encountered by such a dipole based identification method, we shall initially ignore remanent magnetization and consider only the induced component.

For a given spheroid, the induced dipole moment will be dependent on the angle $\theta$ that the spheroid axis makes with the Earth’s field. There is no azimuthal dependence
Figure 1: (a) Spheroid dimensions that can produce the same dipole as a 105 mm shell at 45° inclination to the Earth’s field. The dimensions and orientation angles corresponding to the labelled points are shown in Table 1; and (b) Orientation of the spheroids shown in (a).

due to the spheroid’s symmetry. By varying the orientation $\theta$, the dipole magnitude and angle with respect to the applied field will change in accordance with Equations (3) and (8).

For a given spheroid at a particular orientation, there are an infinite number of other spheroids that could have produced the same dipole moment (Billings et al., 2002a). For example, Figure 1 and Table 1 show the family of spheroids that can produce the same dipole moment as a 105 mm projectile orientated at 45° to the Earth’s field (magnitude: 0.6 Am$^2$; angle: 34.4° relative to the Earth’s field). Aspect ratios greater than 1 correspond to prolate spheroids, while less than 1 corresponds to oblate spheroids. The reason ambiguity occurs is that we can compensate for a change in aspect ratio by changing the diameter and orientation of the spheroid. For many aspect ratios there are two diameters that can reproduce the moment. The larger diameter involves rotating the spheroid away from the Earth’s field, while the smaller involves a rotation towards the Earth’s field. There are two critical points (0.31 and 2.77) where only one diameter can be used and there are a certain range of aspect ratios (0.32 to 2.76) where no orientation of the spheroid can reproduce the moment.

Table 1: Spheroid dimensions, and their angles relative to the Earth’s field, that produce the same dipole moment as a 105 mm projectile at 45° inclination.

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Aspect ratio</th>
<th>Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>82</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>138</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>138</td>
<td>2.82</td>
</tr>
<tr>
<td>D</td>
<td>327</td>
<td>0.06</td>
</tr>
<tr>
<td>E</td>
<td>327</td>
<td>0.31</td>
</tr>
</tbody>
</table>

In summary, inability to constrain the orientation causes ambiguity. If the orientation could be constrained (and we restrict ourselves to prolate spheroids) then a unique
spheroid could be found for any moment. For example, assume we had some means of
determining that the spheroid orientation was 70°. Then, as illustrated in Figure 1b, the
only spheroid that can reproduce the dipole moment with that particular orientation
has an aspect ratio of 2.9 and a diameter of 150.6 mm.

The analysis just presented ignored remanent magnetization which, if present, will
compound the identification process. When an ordnance hits the ground but does not
explode, it is subjected to a very large shock which is often sufficient to cause demagne-
tization. This shock demagnetization may not be complete, so that a UXO may contain
a contribution due to remanent magnetization. As a partial solution to both ambiguity
and remanent magnetization we suggest the following procedure (Billings et al., 2002a).
For each item in an ordnance library calculate the minimum amount of remanent mag-
netization required to match the recovered dipole moment. The item with the minimum
amount of remanent magnetization is assumed to be the source of the anomaly.

2.2 The Guthrie Road, Montana dataset

To test the inversion methods we consider validated dig sheets from Guthrie Road,
Montana (Youmans and Daehn, 1999). In 1998, G-tek Australia, conducted a magne-
tometer survey of the area using an all-terrain vehicle towing an array of eight cesium
vapor magnetometers (Clark et al., 1999). The system was equipped with a real-time
global positioning system that allowed anomalies to be positioned to within about 20
centimeters.

G-tek found 840 anomalies that were tagged as potential UXO, and these were
excavated in the summer of 1999. We have validated dig sheets for 804 of the anomalies,
which consists of 83 UXO (thirty-four 76 mm projectiles and forty-nine 81 mm mortars),
236 items identified as shrapnel, 347 as metallic debris and 138 attributed to geology
(no metallic item found). Even though it was known a-priori that there were only 76
and 81 mm caliber ordnance present at the site, we make the identification process more
challenging by also including 60 mm mortars and 90, 105 and 155 mm projectiles in our
ordnance library.

For each anomaly, we found the best fitting dipole moment (and location), and
then used the dipole magnitude and orientation relative to the Earth’s field, to predict
the ordnance type. We correctly identified 20 out of the 34 items validated as 76 mm
projectiles (59% success) and 24 out of the 49 items validated as 81 mm mortars (49%
success).

As an attempt to ascertain the cause of the misidentifications we conducted a stan-
dard error analysis of the recovered dipole moments using methods described in Billings
et al. (2002a). The confidence ellipses for the dipole magnitude and angle relative to
the Earth’s field for the anomalies due to 76 mm projectiles and 81 mm mortars at
Guthrie Road, are shown in Figure (2). For each anomaly we have plotted the one-
sigma (68.3%) and two-sigma (95.4%) confidence ellipses about the recovered dipole
moment (assuming Gaussian statistics). Also shown are the trajectories of the dipole
moments for the five different ordnance types (obtained by changing the orientation
of the UXO relative to the Earth’s field). If the dipole magnitude and orientation are
well constrained, then uncertainty should not be a significant cause of misidentification.
For the 76 mm projectiles all except one anomaly have very small uncertainties. The
one outlier was incorrectly identified as a 105 mm projectile. For the 81 mm mortars
all anomalies have very small uncertainties. We therefore conclude that uncertainty in
Figure 2: One sigma (68.3%) and two sigma (95.4%) confidence ellipses, assuming Gaussian statistics, about the recovered dipole moments for (a) 76 mm projectiles; and (b) 81 mm mortars.

The recovered dipole moment is not a significant contributor to misidentification. The uncertainty must be due to one or more of the following: (i) Remanent magnetization; (ii) Modelling deficiencies; and/or (iii) Dipole ambiguity.

2.3 IDENTIFICATION USING THE DIPOLE AND OCTUPOLE

The dipole is unable to constrain the dimension but perhaps we can use the higher order moments of the spheroid. Due to symmetry the next non-zero moment is the octupole. For a spheroid of specified dimension, a method to calculate the octupole response via Equation 5 is given in McFee (1989). Due to the dominance of the dipole, attempting to directly invert for the spheroid dimensions is unlikely to succeed. Rather, the approach we take is to do an inversion for each item in our ordnance library. That is, we fix the diameter and aspect ratio of the spheroid and then solve for the best location \((x, y, z)\), orientation \((\phi, \theta)\) and magnetization vector \(\mathbf{M} = (M_x, M_y, M_z)\). We will thus have predicted data for each of the items in our library (in reality we usually only invert for the best 3 items found by the dipole identification process). To make an identification, we have two choices:

1. Full spheroid best fit method: select the item with the best fitting prediction to the data (lowest residual sum-of-squares);

2. Full spheroid minimum remanent method: select the item that requires the least remanent magnetization by comparing the magnetization \(\mathbf{M}\) returned for each item, with the magnetization predicted by the spheroid model without remanent magnetization.

To illustrate the second method we consider an item that was validated as an 81 mm mortar yet predicted by the dipole identification method to be a 76 mm projectile. The dipole identification process returned remanent magnetizations of 12.4% for the 76...
mm, 15.3% for the 81 mm and 43.2% for the 105 mm (hence the prediction of a 76 mm projectile). We then inverted for the best fitting parameters for each of these spheroids in turn and found the closest magnetization achievable by induced means alone (Table 2). This process revealed that the 81 mm mortar had a remanent magnetization of 12.8% compared to 14.0% for the 76 mm projectile and 43.9% for the 105 mm projectile. Hence the predicted ordnance is an 81 mm mortar which is correct.

When we applied the full spheroid inversion to the Guthrie Road data we almost always achieved a better fit than when using the dipole alone. However, the improvement in fit was generally quite small. Using the full spheroid identification methods outlined above, we achieved the results shown in Figure 3. Clearly, the method based on choosing the best fitting spheroid is unsuccessful at predicting the ordnance type. This means that shape information cannot be obtained using the octupole moment.

The results with the minimum remanent magnetization method are much better than the best fit method. For items validated as 76 mm projectiles there is no difference between the dipole alone and the full spheroid identification results. For the 81 mm mortars there is one extra item (the one used as an example) that was identified correctly using the full spheroid model compared to the dipole alone. The results indicate that there may be a slight (but not statistically significant) advantage in using the
full spheroid model over the dipole approach. However, overall the octupole appears to contribute very little information of use to identification. We speculate that this is due one or more of the following

1. The octupole response rapidly decays to below the noise floor;

2. The relative accuracy of the spatial positioning is inadequate; and

3. The equivalent spheroid is not an accurate approximation to the shape of real ordnance.

2.4 IDENTIFICATION USING THE DIPOLE AND QUADRUPOLE

Thus far, the identification routines we have developed have relied solely on a spheroid model of ordnance. However, this assumption is overly simplistic since most ordnance items do not have that degree of symmetry. In particular, the front and tail ends of ordnance tend to be different: the front is often tapered to provide better aerodynamics and target penetration, while the tail end is often flat (the tails of mortars don’t matter because they are usually made of aluminum). Therefore, a more realistic model of ordnance is an axially symmetric body without front-back symmetry. Such a body has a multipole expansion with both a dipole term and a non-zero quadrupole term (recall that the quadrupole was zero for a spheroid due to symmetry).

Our intent with this new identification method is to try to use the information in the quadrupole to constrain the orientation of the body ($\phi, \theta$). The basis of the idea is that there will be no quadrupole contributions perpendicular to the axis of symmetry but there will be contributions along the symmetry axis. We therefore construct a model with these characteristics and use an inversion procedure to recover a model with the best orientation. Using Equation (4) and assuming the magnetization vector is approximately constant, it can be shown that the quadrupole tensor is

$$
\mathbf{m}^{(q)} = \alpha \begin{bmatrix}
0 & 0 & m_x \\
0 & 0 & m_y \\
m_x & m_y & m_z
\end{bmatrix}
$$

(9)

where $\mathbf{m} = (m_x, m_y, m_z)$ is the dipole moment,

$$
\alpha = \frac{\int_V x_3 dV}{V}
$$

(10)

depends on the asymmetry of the ordnance front to back and $V$ is the volume of the body.

As part of the quadrupole inversion procedure we recover the spatial position, dipole moment, orientation of the spheroid and the asymmetry factor, $\alpha$. We then use the recovered dipole moment to estimate the minimum remanent magnetization required to fit each item in the ordnance library. This time, however, we have a constraint on the orientation of the ordnance, so that we are not free to vary the orientation until the remanence is a minimum; this should reduce the dipole ambiguity.

The quadrupole identification results are shown in Figure 3. There is a slight improvement in the ability to identify compared to the other methods considered in the paper with 23 of the 76 mm (68%) and 27 of the 81 mm ordnance (55%) identified correctly. Thus, overall in 60% of cases we can correctly predict the ordnance type.
3 DISCUSSION

Identification of the ordnance type using the recovered dipole moment is difficult due to non-uniqueness. The dipole moment is the product of volume with the magnetization vector. When the volume changes, self-demagnetization as the orientation of the body is varied, can change the magnetization vector and thus produce an identical or similar dipole moment. However, recognizing that there are only a finite number of ordnance types in an given area, makes identification using the recovered dipole moment feasible. At Guthrie Road correct identification could be achieved about 55% of the time.

A significant cause of misidentification is the inability of the dipole moment to constrain the orientation of the causative body. In an attempt to overcome this problem we developed inversion routines that recover information on the quadrupole and octupole components of the body. The dipole/octupole method we developed uses a spheroid approximation of ordnance to estimate both the dipole and octupole moments. No significant improvement in identification over the dipole method was found. This could be due to one or more of the following: (i) remnant magnetization; (ii) rapid falloff of the octupole component with distance so that it is unable to constrain the orientation; and (iii) the spheroid provides a poor approximation to the octupole moment of real ordnance.

The quadrupole method relaxed the requirement of top-bottom symmetry enforced by the spheroid approximation and attempted to recover the dipole and quadrupole moments of an arbitrary axially symmetric body. The fits to the data were improved and the ability to identify was enhanced slightly. There remains some ambiguity in ordnance shape and we speculate that this could occur for one or more of the following reasons:

1. Noise in the data and inaccuracy in spatial positioning;
2. Remnant magnetization;
3. Some of the ordnance, especially the 81 mm mortars, were deformed on impact so that the assumption of axial-symmetry was violated; and
4. Inadequacy of the model, as we used the spheroid model of ordnance to predict the dipole moments.

4 CONCLUSIONS

We conclude that to provide the required constraints on object dimension, information not contained in the magnetic anomaly is required. A potential source of this extra information is via an electromagnetic (EM) survey. The orientation of a buried object relative to the primary excitation field changes as the EM transmitter coil is moved over the body. This makes the EM anomaly more sensitive to the orientation of the potential UXO than the magnetic anomaly. Joint or cooperative inversion of magnetics and EM may then eliminate shape ambiguity; the EM provides the orientation constraint required by the magnetic dipole identification process.
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References


