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Anisotropic paleosecular variation models: implications for geomagnetic field observables

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Abstract

We present a family of statistical models for paleosecular variation (PSV) of the geomagnetic field that are compatible with paleodirectional and paleointensity variations in lava flows sampling the last 5 Ma, and explore what paleomagnetic observables might be used to discriminate among the various family members. We distinguish statistical models with axial anisotropy, which provide a suitable description for an earth with homogeneous boundary conditions at the core–mantle interface from those with more general anisotropy corresponding to geographically heterogeneous boundary conditions. The models revise and extend earlier ones, which are themselves descendants of CP88, devised by Constable and Parker [Constable, C.G., Parker, R.L., 1988. Statistics of the geomagnetic secular variation for the past 5 m.y. J. Geophys. Res. 93, 11569–11581]. In CP88, secular variation is described by statistical variability of each Gauss coefficient in a spherical harmonic description of the geomagnetic field, with each coefficient treated as a normally distributed random variable: the Gauss coefficients of the non-dipole part of the field exhibit isotropic variability, and the variances are derived from the present field spatial power spectrum. The dipole terms have a special status in CP88, with a non-zero mean for the axial-dipole, and lower variance than predicted from the spatial power spectrum. All non-dipole terms have zero mean except the axial-quadrupole. CP88 is untenable for two reasons: it fails to predict the observed geographic dependence of directional variability in the magnetic field, and it grossly underpredicts the variance in paleointensity data. The new models incorporate large variance in the axial-dipole, and in the non-axial-quadrupole Gauss coefficients, g_2^1 and h_2^1. The resulting variance in paleomagnetic observables depends only on latitude (zonal models), unless the variance in h_2^1 is different from that in g_2^1 (non-zonal models). Non-zonal (longitudinal) variations in PSV, such as the flux lobes seen in the historical magnetic field, are simulated using the non-zonal models. Both the zonal and non-zonal models fit summary statistics of the present dataset. We investigate the influence of persistent non-zonal influences in PSV on various paleomagnetic observables. It is shown that virtual geomagnetic pole (VGP) dispersion is rather insensitive to longitudinal variations in structure of PSV, and that inclination dispersion has the potential to be more informative given the right site distribution.

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There is also the possibility of using paleointensity and geographic variations in the frequency of occurrence of excursional directions to identify appropriate PSV models.

Keywords: Paleosecular variation; Virtual geomagnetic pole; Lava

1. Introduction

The geomagnetic field generated by fluid motions in Earth's electrically conducting liquid outer core exhibits secular variations at time scales ranging from the order of a year to hundreds of millions of years. Recent progress in dynamo simulations suggests that the geodynamo, its secular variation, and reversal rate may be strongly influenced by the nature of, and lateral variations in, core–mantle boundary conditions (Glatzmaier and Roberts, 1995a,b, 1996a,b, 1997; Kuang and Bloxham, 1996, 1997). It is, therefore, of interest to provide a global characterization of the observed geomagnetic secular variation over geological time scales. Any truly Earth-like dynamo simulation should be able to predict variations with similar statistical properties to those recorded paleomagnetically over the past few million years.

Paleosecular variation (PSV) of the geomagnetic field is generally considered to involve changes in the field taking place over time scales ranging from 10^2 to 10^5 years. Two fundamentally different approaches are used in the study of PSV. The first involves the collection of time series of geomagnetic variation at a site; these typically come either from sediment records with high accumulation rates or from archeomagnetic sites that can be accurately dated. Regional or global models of PSV can then be derived by collecting and comparing coeval paleomagnetic time series from sites distributed around the world (see Constable et al., 1995; Daly and LeGoff, 1996; Lund, 1996; Hongre et al., 1998). The second approach, the one considered in this paper, recognizes that it is often impossible to collect a time series of observations. This is generally the case for studies of paleosecular variation from lava flows (PSVL). Repeated volcanic eruptions at a location provide spot records of the geomagnetic field at the time of cooling of the flow, but the exact timing of, and interval between eruptions cannot be known. The solution is to collect many samples from each location covering a sufficient time interval that they provide a statistically meaningful sample of the geomagnetic field and the variability to be expected due to secular variation. Such records can then be used to generate a global statistical description of the PSV. This global description can provide important constraints for the new generation of dynamical simulations of the geodynamo, which cannot be expected to reproduce the exact temporal evolution of the geomagnetic field because of their sensitivity to initial conditions.

Many PSV models have been generated over the past few decades. They are mostly based exclusively on observations of the direction of the paleomagnetic field, and have been recently reviewed, for example, by Johnson and Constable (1996) or Merrill et al. (1996). This paper reconsiders PSV models based on giant Gaussian processes of the type initially proposed by Constable and Parker (1988), hereinafter called CP88. Such models may be used to make predictions about a number of paleomagnetic observables, such as statistical distributions of field directions and paleointensity data, and dispersion in either virtual geomagnetic pole (VGP) position or local geomagnetic field directions due to PSV. We illustrate the inadequacies of CP88 in the light of current datasets and show how it may be modified for compatibility with recent compilations of paleodirectional and paleointensity observations. Modifications to CP88 have been suggested by others (Kono and Tanaka, 1995; Hulot and Gallet, 1996; Kono and Hiroi, 1996; Quidelleur and Courtillot, 1996), but until now, no model of this type has been proposed that simultaneously satisfies both directional and paleointensity datasets. Camps and Prévot (1996) have proposed a model incorporating intensity variations, but this relies on correlations between dipole and non-dipole contributions to the secular variation. The new model presented here differs from CP88 in that variations in the individual Gauss coefficients depend on spherical harmonic order as well as degree, so that these variations can no longer be considered isotropic, but reflect the special structure imposed on the field through the coordinate system defined by Earth's rotation axis. The lack of isotropy in the Gauss coefficients influences the latitudinal variations in the expected distributions of geomagnetic observables, but need not introduce any longitudinal effects. The possible influence of longitudinally varying PSV structure is investigated through a more general form of anisotropy in secular variation of the Gauss coefficients. This is viewed as a kind of proxy for the effect of spatially heterogeneous core–mantle boundary conditions; the consequences for potential paleomagnetic observables are considered.

2. The giant Gaussian process: an isotropic source of PSV

Constable and Parker (1988) proposed that properties of the present field be used as a guide in constructing statistical models for PSV. They parametrized their model in terms of a statistical description for the Gauss coefficients found in the conventional spherical harmonic representation of the geomagnetic field generated in Earth's core. The magnetic field $\vec{B}$, at time t and position specified by radius, r, co-latitude, θ, and longitude, ϕ, is written as the gradient of a harmonic potential, Ψ, where:

$$\Psi(r,\theta,\phi,t) = a\sum_{l=1}^{\infty}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+1}\left(g_l^m(t)\cos m\phi + h_l^m(t)\sin m\phi\right)P_l^m(\cos\theta). \tag{1}$$

P_l^m are partially normalized Schmidt functions describing the latitudinal variation of the field. At any time t, the Gauss coefficients, g_l^m and h_l^m, determine the size of spatially varying components of the field that increase in complexity with spherical harmonic degree l. A secular variation model based on observations of the modern field would consist of a functional description of each $g_l^m(t)$ and $h_l^m(t)$, but this is not feasible for the paleofield: therefore, we seek a statistical description of the temporal variation in each g_l^m and h_l^m. Since we do not know the exact t associated with an observation, we consider each observation of the field to be a statistical sample from a random variable whose distribution depends on that of the random variables g_l^m, h_l^m. Implicit in this description is the idea that the temporal sampling can also be considered a random variable, and ideally, that the temporal sampling is uniform in the statistical sense. CP88 prescribed a PSV model with the following properties determined by the distributions of the g_l^m and h_l^m.

(1) All Gauss coefficients with degrees $l = 1$–12 are isotropic in their variability. By isotropic, we mean that the statistical variations of Gauss coefficients about their mean value do not depend on the orientation of the coordinate system in which the Gauss coefficients are defined. Apart from the mean values, the statistical distributions for g_l^m and h_l^m vary only with spherical harmonic degree, l, not with the order m. Note that this does not imply that the orthogonal components of the magnetic field are isotropic in their variability at any point on Earth's surface (Constable and Parker, 1988; Table 1).

(2) PSV is described by a Gaussian statistical variability in the spherical harmonic coefficients, with their variance derived from the present field spectrum. The present spectrum is white (or flat) for degrees $l = 2$–12 at the core–mantle boundary giving a value of $(\sigma_l)^2 = [((c/a)^{2l}\alpha^2)/((l+1)(2l+1))]$ the variance at Earth's surface as a function of l. With the exception of g_1^0 and g_2^0, the average value for each spherical harmonic coefficient is zero.

(3) The dipole terms are special, reflecting its dominance in the present field spectrum even at the core–mantle boundary. The average value for the axial-dipole is $\bar{g}_1^0 = -30$ μT and the variance in the dipole terms is smaller than predicted by the rule given in (2) for $l = 2$–12. For CP88, $\sigma_1 = 3$ μT.

(4) $\bar{g}_2^0 = 0.06\,\bar{g}_1^0$ to allow for the persistent far-sided effect in VGPs first documented by Wilson (1970). The combination of a persistent g_2^0 term whose spherical harmonic representation is symmetric about the equator, with $\bar{g}_1^0$, which is anti-symmetric means that many magnetic observables have different properties in the northern and southern hemispheres.

The isotropic variations in the Gauss coefficients for the geomagnetic potential suggest a secular variation model in which there is no preferred directional dependence, but this isotropy does not extend to the actual physical measurements of the geomagnetic field given by the gradient of the potential. CP88

Table 1
Parameters for candidate PSV models

	CP88	CJ98	CJ98.nz	QC96
Axial bias ⇒ TAF				
Dipole $\bar{g}_1^0$	-30.0 μT	-30.0 μT	-30.0 μT	-30.0 μT
Quadrupole $\bar{g}_2^0$	-1.8 μT	-1.5 μT	-1.5 μT	-1.2 μT
Variance ⇒ PSV				
Isotropic part	$\alpha = 27.7$ μT	$\alpha = 15.0$ μT	$\alpha = 15.0$ μT	$\alpha = 27.7$ μT
$(\sigma_l)^2 =$	$[((c/a)^{2l}\alpha^2)/((l+1)(2l+1))]$	$[((c/a)^{2l}\alpha^2)/((l+1)(2l+1))]$	$[((c/a)^{2l}\alpha^2)/((l+1)(2l+1))]$	$[((c/a)^{2l}\alpha^2)/((l+1)(2l+1))]$
Anisotropic part				
$\sigma_1^0 =$	$0.5\sigma_1 = 3$ μT	$3.5\sigma_1 = 11.72$ μT	$3.5\sigma_1 = 11.72$ μT	3.0 μT
$\sigma_1^1 =$	$0.5\sigma_1 = 3$ μT	$0.5\sigma_1 = 1.67$ μT	$0.5\sigma_1 = 1.67$ μT	3.0 μT
$\sigma_2^0 =$	$\sigma_2 = 2.14$ μT	$\sigma_2 = 1.16$ μT	$\sigma_2 = 1.16$ μT	1.3 μT
For g, $\sigma_2^1 =$	$\sigma_2 = 2.14$ μT	$3.5\sigma_2 = 4.06$ μT	$\sigma_2 = 1.16$ μT	4.3 μT
For h, $\sigma_2^1 =$	$\sigma_2 = 2.14$ μT	$3.5\sigma_2 = 4.06$ μT	$7.0\sigma_2 = 8.12$ μT	4.3 μT
$\sigma_2^2 =$	$\sigma_2 = 2.14$ μT	$\sigma_2 = 1.16$ μT	$\sigma_2 = 1.16$ μT	1.3 μT

showed that the variance in the local vertical component of the magnetic field is substantially larger than that in the horizontal components. The mean values of both the north and vertical components of the magnetic field vary with latitude. The fact that the mean values of some of the zonal coefficients are non-zero suggest the possibility of an axial anisotropy in the secular variation. In the simplest form of axial anisotropy, only the g_l^0 term has a variance distinct from those of the other g_l^m and h_l^m. However, axial symmetry in the secular variation is preserved, provided differences in the secular variation are restricted to different order Gauss coefficients within each degree, and that is the definition for axial anisotropy that we adopt here. Such an axial anisotropy would be manifest by variances that depend on the value of m within each spherical harmonic degree l. More general anisotropy would be manifest by g_l^m and h_l^m having different variances for the same values of l and m, implying preferred orientations in variation along one or more directions perpendicular to the rotation axis.

CP88 provided a minimalist description of the PSV during stable polarity times. The model contains no temporal covariance, and each Gauss coefficient is statistically independent of all the others. We note that in principle, it might be possible to estimate temporal covariances from long time series of paleomagnetic observations (as done by Hulot and Le Mouël, 1994 for the historical field). Four parameters associated with CP88 can be adjusted to fit the observations (see Table 1); α determines the variance $(\sigma_l)^2$ for $l = 2$–12, additionally, there are the mean values of the axial-dipole and axial-quadrupole contributions to the field and the variance $(\sigma_1)^2$ in the dipole contributions to the field. These parameters reflect constraints on the model imposed by the observations; for CP88, the values of the average axial-quadrupole term, and the dipole terms' variance were chosen to fit PSVL observations of the direction of the geomagnetic field over the past 5 Ma as compiled by Lee (1983). $\bar{g}_1^0$ was arbitrarily set to 30 μT, and the variance of the non-dipole terms was derived from the white spectral fit for the non-dipole present field. One advantage of this model over earlier descriptions is that it supplies a complete (albeit over-simplified) description in terms of the spherical harmonic coefficients that allows either direct calculation or simulation of the expected distributions for any geomagnetic observable. Thus, it can be used to make predictions about field intensity distributions, for example, and these may be tested against the distributions of available observations. Although its only parameters are the means and variances of distributions of Gauss coefficients, CP88 provides a complete statistical description for PSV.

Before turning to the paleointensity observations, it should be noted that CP88 already exhibits inadequacies when predictions from it are compared with more recent compilations of directional data from PSVL (Quidelleur et al., 1994; Johnson and Constable, 1996). Johnson and Constable (1996) (JC96) completed a recompilation of all the 0–5 Ma PSVL data published up to the end of 1992, and exercised more stringent selection criteria than previously imposed. They compared predictions from CP88 with these data, and discovered firstly that predicted VGP latitude distributions did not agree well with the observations (see their Fig. 3). Too few low latitude poles were predicted; perhaps more strikingly, the observations exhibit different distributions for normal and reverse polarity data, suggesting that the time-averaged field (TAF) may be different during the Matuyama reversed interval from the normal polarity TAF over the past 5 Ma. In subsequent modelling of the TAF with PSVL data, Johnson and Constable (1995, 1997) show that although different models are obtained for the TAF for normal and reverse polarity observations, they cannot be shown to be required by the current distribution of observations. Others have also discussed whether normal and reverse polarity TAF models are discernibly different (see Merrill et al., 1996). If this observation proves to reflect genuine field behavior (rather than rock magnetic recording errors), it may indicate the need for averaging over more than one polarity interval in order to estimate the TAF.

A second area in which CP88 performs poorly is in predicting geographic variations in PSV. One of the few robust observations derived from PSV studies is that root-mean-square angular deviation of VGPs about the geographic axis increases with latitude. Many PSV models have been based entirely on fitting the structure of latitudinal variation in this dispersion (see Merrill et al., 1996 for a recent review). Fig. 1 shows the predictions for CP88 compared with the observations. The fit is poor, basically because all the structure in the CP88 simulations derives from the structure of the TAF underlying the model; the non-zero average zonal quadrupole generates a decrease in dispersion as one moves from southerly latitudes to northerly ones, while the observations show the lowest values in latitude bands around the equator. Some recent studies (Gubbins and Kelly, 1993; Johnson and Constable, 1995, 1997; Kelly and Gubbins, 1996) suggest a more complex spatial structure to the TAF, however, the dominant term in these spherical harmonic TAF models is typically the axial-quadrupole term. Thus, the introduction of other reasonable non-zero-mean Gauss coefficients while retaining the variance structure of CP88 still results in a poor fit to the PSVL dispersion data. Johnson and Constable (1996) noted that the fit could be improved by decreasing the variance in the PSV and allowing for much larger orientation errors in the paleomagnetic observations. A local Fisherian distribution about the mean direction for such uncertainties translates into a latitude dependence in VGP dispersions. The required orientation errors are of the order of 8°. These seem too large

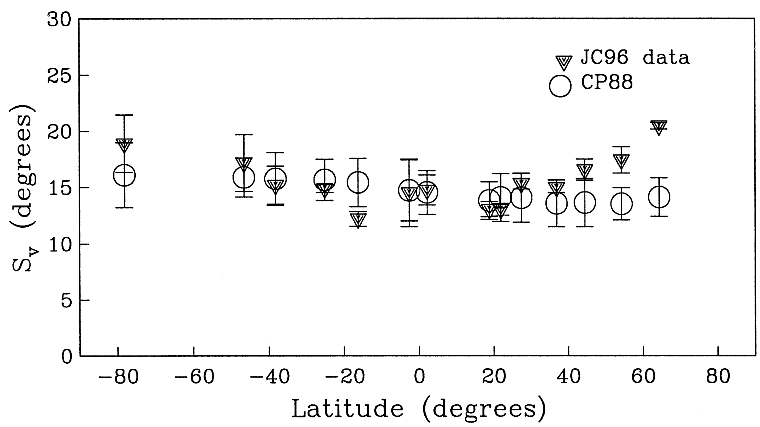


Fig. 1. Variation with site latitude in S_V, VGP dispersion about the geographic pole for JC96 dataset, and simulations from CP88.

unless they are attributed to systematic biases like inadequate estimation of local horizontal, or strong local magnetic anomalies, since statistical orientation errors should average to about 1° or less when five or six samples are taken from each flow.

Kono and Tanaka (1995) showed that an isotropic model like CP88 cannot generate significant variations in VGP dispersion with latitude, but that extra variance in Gauss coefficients of degree two and order one might generate the observed variation in VGP dispersion with latitude. This idea was developed further by Hulot and Gallet (1996) who suggested pre-eminence of order one terms up to about spherical harmonic degree four, emphasizing the need for anisotropy in the field variations, and/or cross-correlations among the Gauss coefficients. Quidelleur and Courtillot (1996) carried out extensive simulations for available data sites with a variety of different parameters for the means and variances of the Gauss coefficients, and concluded that the simplest modification to CP88 compatible with observed VGP dispersion required a standard deviation for the g_2^1 and h_2^1 terms about three times larger than for the rest of the quadrupole terms. Parameters for their preferred model, C1, (here designated as QC96) are in Table 1. They also showed that the inclusion of plausible TAF models containing terms in addition to the axial-quadrupole has a rather minimal effect on the latitudinal structure in VGP dispersion, a result that we confirmed for our recently published TAF model, LSN1 (Johnson and Constable, 1997).

The introduction of larger variance for order one Gauss coefficients destroys the isotropy of the PSV model, but the anisotropy introduced is only with respect to Earth's rotation axis. It seems plausible that the geodynamo should be strongly influenced by Earth's rotation, and by the presence and relative size of the inner core within the outer core, and that this could be reflected in the spectrum of the PSV. As we shall see, paleointensity data require even stronger bias of this kind, but in the variations of the axial-dipole contribution.

3. Paleointensity data

Observations of geomagnetic paleointensity are time-consuming and difficult to make. Although both sedimentary and igneous rocks can provide information about paleofield intensity, sedimentary records can be difficult to evaluate, because of the lack of internal consistency checks (see Tauxe, 1993 for a review of the subject). We restrict our considerations here to those derived by either the Thellier or Shaw method (Thellier and Thellier, 1959; Shaw, 1974) from samples whose remanence is supposed to be a TRM. A compilation of such observations has been published by Tanaka et al. (1995) (here designated as TKU) and includes data derived from rock units older than 0.02 Ma. Of the 1122 observations, 216 have age estimates less than 5 Ma and come from field states that are non-transitional (for compatibility with the directional dataset we take this to mean the documented VGP latitude lies between 55°S and 55°N). Their geographic and temporal distributions are indicated in Fig. 2. Although the number and geographic distribution of these data are inferior, they may be considered to be representative of the same kind of field behavior as described by the PSVL directional data compiled by JC96. Any candidate PSV model should be compatible with the geomagnetic field behavior exhibited by these intensity observations.

TKU summarize the properties of the entire dataset and indicate that the subset whose ages span 0.02–10 Ma are compatible with a dominantly, but not exclusively dipolar field geometry, with average dipole moment of 8.2×10^{22} A m^2. The standard deviation about this mean value is large and it is important to assess what fraction of this variability is due to experimental error in estimating the magnitude of the geomagnetic field, rather than due to actual temporal and spatial variability in the field itself. Senanayake et al. (1982) undertook a detailed study of the sources of error in paleointensity measurements they made using both Shaw and Thellier methods on basalts less than 5-Ma old. They present convincing arguments based on a statistical analysis of their observations that the standard error in their data due to rock magnetic sources is about 10% of the observed field magnitude, and does not depend on which method is used. This is typical of the size of error quoted in the TKU database (as found in the original publications), but one should keep in mind that this may not take into account systematic biases or undetected alteration taking place during the paleointensity experi-

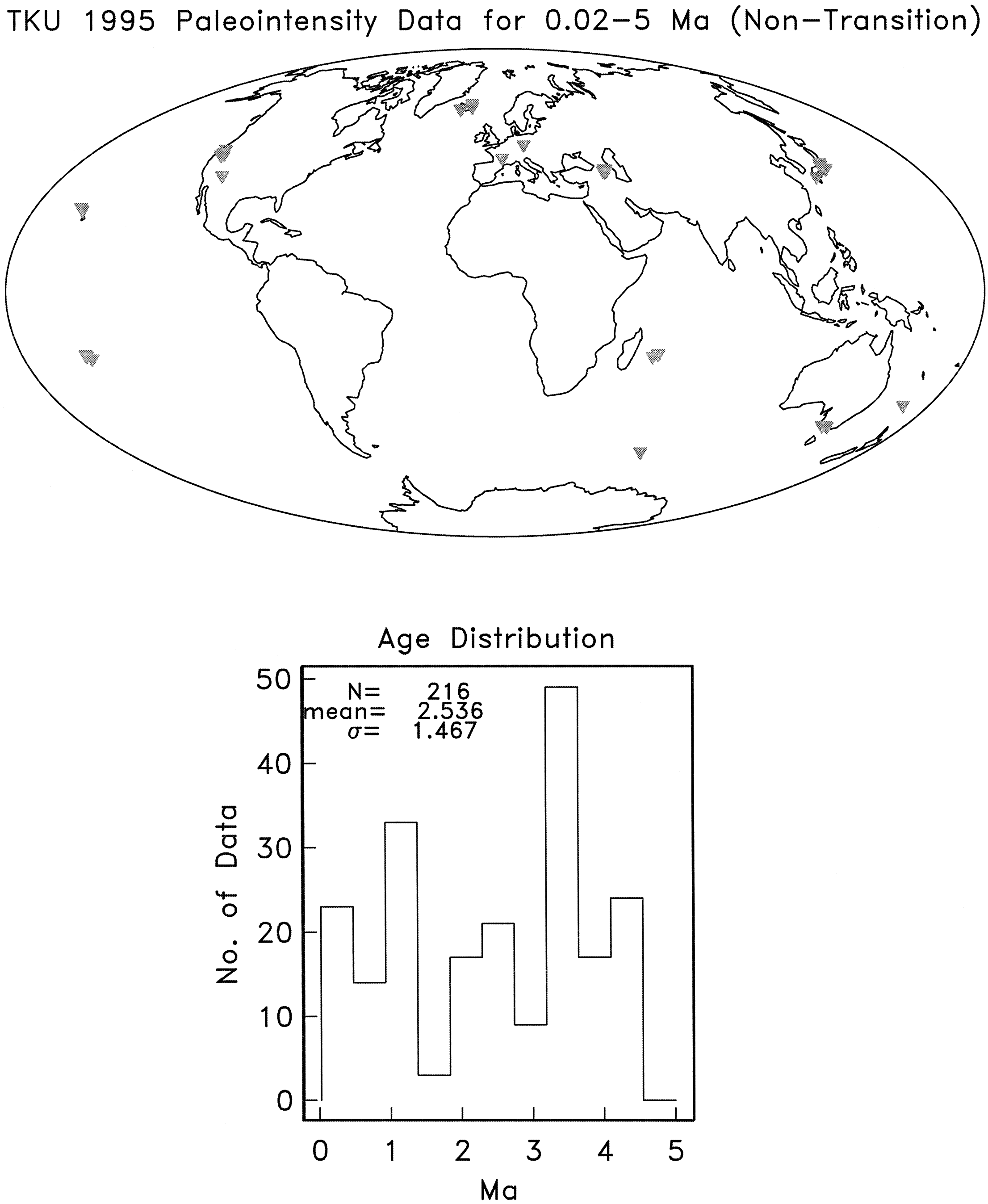


Fig. 2. Geographic and temporal distributions of 0.02–5 Ma Shaw and Thellier method paleointensity observations in TKU dataset.

ment. TKU also note that the standard deviation in virtual dipole moments (VDMs) estimated by the Shaw method is substantially larger (factor of 1.7) than for the Thellier method, but the origin of this difference is not obvious. It might reflect larger rock magnetic errors, but could also be a consequence of biases in temporal and spatial distribution of the samples for the different techniques. There has been considerable controversy over the degree of reliability of paleointensity observations (see, e.g., Merrill, 1987; Walton, 1987, 1988a,b). In the absence of better information, we have conservatively doubled the error provided by the paleointensity practitioners and used 20% as a plausible estimate of uncertainty due to experimental errors.

4. CP88 predictions and axially anisotropic PSV

Figs. 3 and 4 compare paleointensity data for 0–5 Ma to simulations made from CP88 for both local field intensity and VDMs. The same spatial distribu-

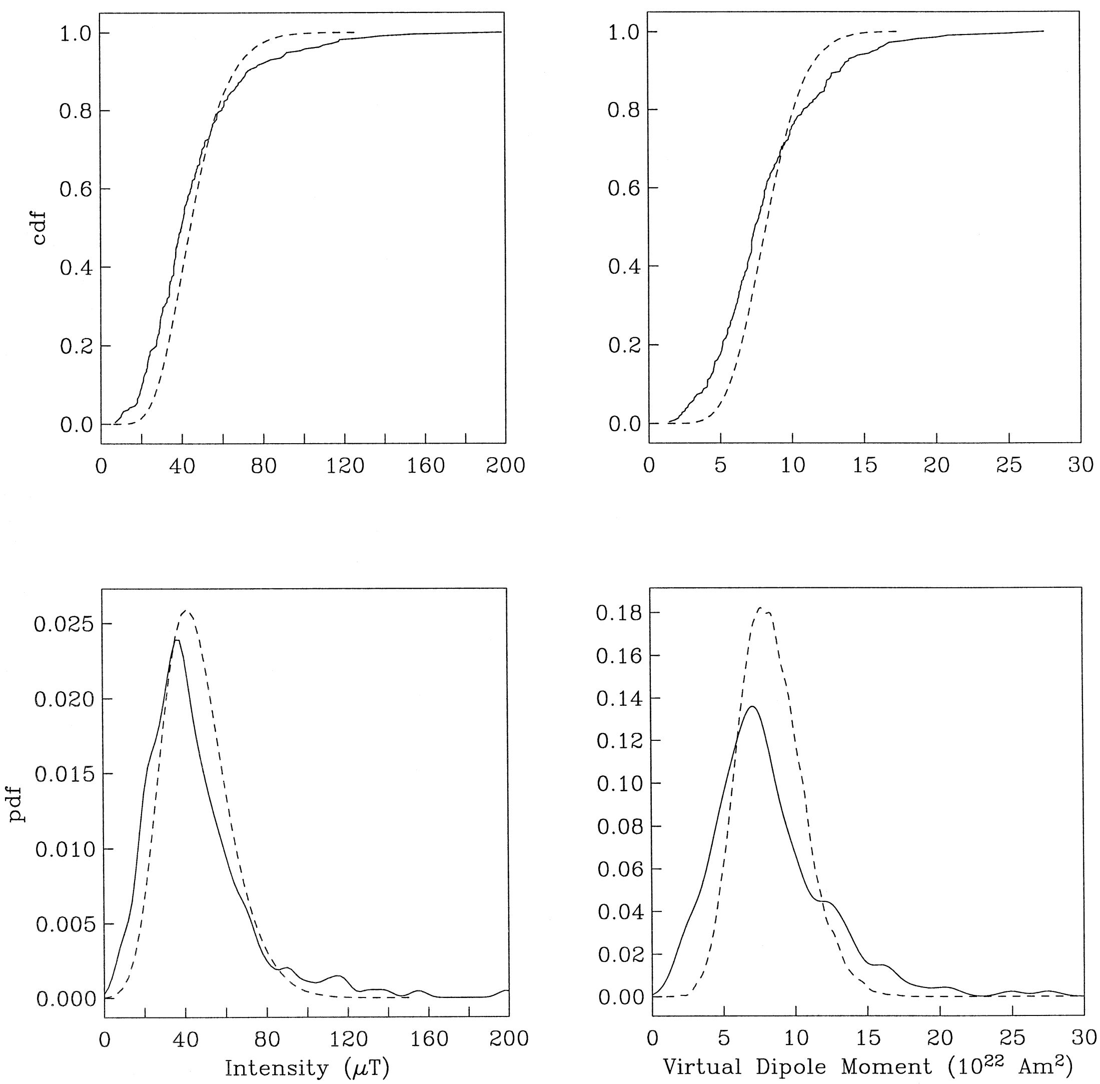


Fig. 3. Cumulative distribution functions predicted by CP88 (dashed line, upper plots) at same sites as TKU data for local field intensity and VDM. Solid line gives empirical distribution function of TKU data. Lower figures compare estimates of probability density function derived from CP88 (dashed) and TKU data (solid).

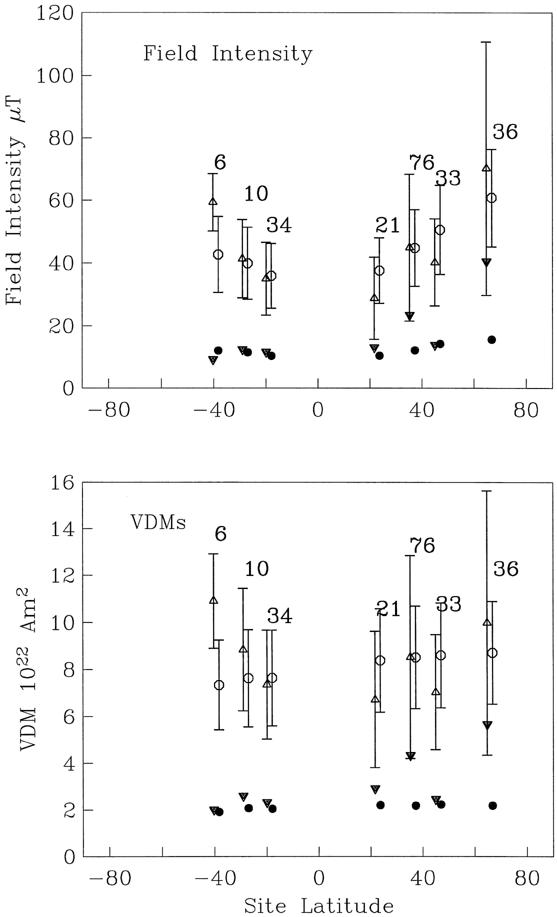


Fig. 4. Average (open triangles) and standard deviation (error bars and also plotted separately as closed triangles) of 0–5 Ma data from TKU paleointensity database, and comparisons with simulations from CP88 plus 20% rock magnetic noise (circles). (a) Field intensity as a function of site latitude, (b) VDM as a function of site latitude. Note the increase in standard deviation with increasing intensity. Numbers of data from TKU are indicated for each latitude band. CP88 predictions are offset slightly in latitude for viewing clarity.

tion and numbers of observations as in Fig. 2 are used for each of 100 simulations. Rock magnetic noise is represented in the simulations by Gaussian noise with a standard deviation of 20% of the predicted intensity. Fig. 3 compares the cumulative distribution functions and the probability density functions of local field intensities and VDMs for the CP88 simulation (dashed lines) with the TKU dataset (solid lines). The fit for the cumulative distribution functions is poor: CP88 has very few low values for both intensity and VDM: at about 20 μT or 5×10^{22} A m^2 it rises sharply and then levels off by about 80 μT or 15×10^{22} A m^2. In contrast, the TKU observations of both local field intensity and VDMs are considerably more variable. The probability density function estimates also show that while the averages are about right, the variance in the distribution of observations is substantially larger than in the CP88 simulations. It is not clear that any significance should be attached to the subsidiary bumps away from the main peak in the TKU density estimate because of the rather poor geographic distribution of observations. In Fig. 4, the averages and standard deviations are shown as a function of site latitude. Again, we see that while the average values are about right, the variance in intensity and VDM predicted by CP88 is too small. Even with our conservative error estimates, the global standard deviation expected from CP88 is only 26.4% of the average value for the VDMs; it is 48.4% in the observations. No reasonable model of experimental uncertainty can account for this discrepancy. One might think that since the VGP latitude observations also have a higher dispersion than predicted by CP88, that the problem could be resolved simply by increasing the variance α in the statistical model. This is not so. Kono and Hiroi (1996) demonstrated that a model of this kind does improve the fit to the intensity data. However, the increase in variance required to satisfy the paleointensity observations produces too large a dispersion for the 0–5 Ma directional data, and still does not provide the form of latitudinal variation required by the directional data (Constable, 1994). One alternative is that the discrepancies between model predictions and data in Figs. 1 and 3 are indicative of fundamental flaws in the CP88 model. A clue to their origin lies in an earlier compilation of about 1200 archeointensity data most of which come from the time interval 0–12 ka (McElhinny and Senanayake, 1982). From these observations, McElhinny and Senanayake estimate the standard deviation due to variability in the dipole moment at about 18% over the last 10,000 years, a number that agrees quite well with that expected from CP88 when we use an uncertainty estimate of 10% as they did. We infer from this that the CP88 model may be reasonable over time spans often regarded as typical for

PSV, namely 10^2 to 10^4 years, but that over the 0–5 Ma time interval it breaks down. We also speculate that the way in which it breaks down is that the axial-dipole term in the spherical harmonic expansion has much larger variability, and a longer time constant than the rest of the secular variation. A test of this hypothesis is indicated in Fig. 5a, where we compare 0–5 Ma PSVL data (JC96) with simulations from CJ98, a new model (see Table 1) in which the axial-dipole has significantly enhanced variance, and guided by the discussion in Section 3, we also allow extra variance in the order one quadrupole terms. The concentration of power in the axial-dipole term allows large intensity fluctuations without generating an enormous increase in directional variability. The non-axial-quadrupole terms generate the observed increase in VGP dispersion with latitude and as noted earlier by QC96 the fit to the observations is vastly improved over that for CP88. A major difference between CJ98 and QC96 is that the new model

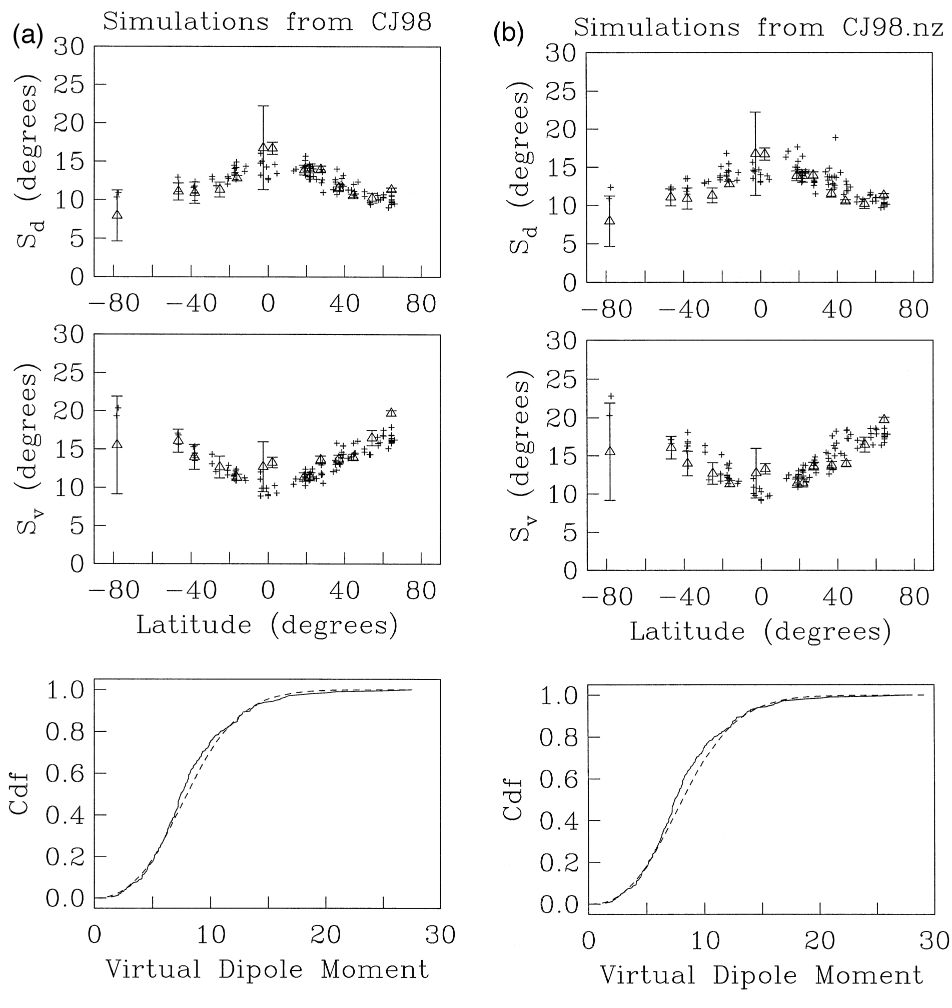


Fig. 5. (a) Comparisons of PSV model CJ98 with 0–5 Ma paleointensity data of Tanaka et al. (1995) and with 0–5 Ma PSVL data from Johnson and Constable (1996). Bottom plot shows the cumulative distribution function for VDM for the data (solid) and model (dashed). The middle and upper plots show VGP and directional dispersion, S_V and S_d, respectively, as a function of site latitude for the JC96 data (open triangles) and CJ98 model (pluses). The data are averaged in latitude bands, and the mean dispersions along with one standard error are shown. Model results are the average of 10 simulations at each data site.

provides an excellent fit to the paleointensity data as evidenced by the lowermost panel in Fig. 5a, in which the predicted sample distribution function for VDMs is compared with the TKU data. At this point, we have chosen to just use the cumulative distribution function for the VDM as a diagnostic for fit to the paleointensity observations: we prefer not to use probability density functions as in the lower part of Fig. 3 because they are smoothed representations of the data, rather than the direct observations that go into the upper part. Subdivision by site latitude as in Fig. 4, provides very small number of observations in some latitude bins: these are unlikely to be statistically representative of the PSV. Thus, we consider the essential properties of the paleointensity data to be well summarized by the cdf as shown in Fig. 5a.

5. Discussion: axial anisotropy vs. general anisotropy

Fig. 5a suggests that it is time to revise our approach to modeling PSV of the geomagnetic field, and allow greater freedom than is available in the CP88 model. Parametrization for the PSV in terms of statistical distribution for the Gauss coefficients is still desirable, because this allows us to predict distributions for typical paleomagnetic observables. Model CJ98 of Table 1 is one candidate that will fit the current observations, but it is far from the only one: it seems appropriate at this point to consider other viable alternatives. CJ98 introduces a dependence on coordinate system into the PSV, resulting in secular variation that varies with latitude, but not with longitude. The variations in the Gauss coefficients are axially anisotropic, that is, for any specific values of l and m the g_l^m and h_l^m have the same variances. As noted earlier by Hulot and Gallet (1996), we find that we could include small amounts of excess variance in Gauss coefficients of either or both of degree 3, order 1, and degree 4, order 1, and still maintain an adequate fit to the VGP dispersions and paleointensity data. These $m = 1$ terms only affect the latitude dependence of PSV, provided that the variance is the same for both g_l^m and h_l^m. We consider CJ98 to be a viable model for an Earth with homogeneous CMB properties: under these conditions the geodynamo might be expected to produce a time-averaged magnetic field that is zonal, and PSV whose properties only depend on latitude, not longitude. This is an axially anisotropic model for the Gauss coefficients generating the PSV.

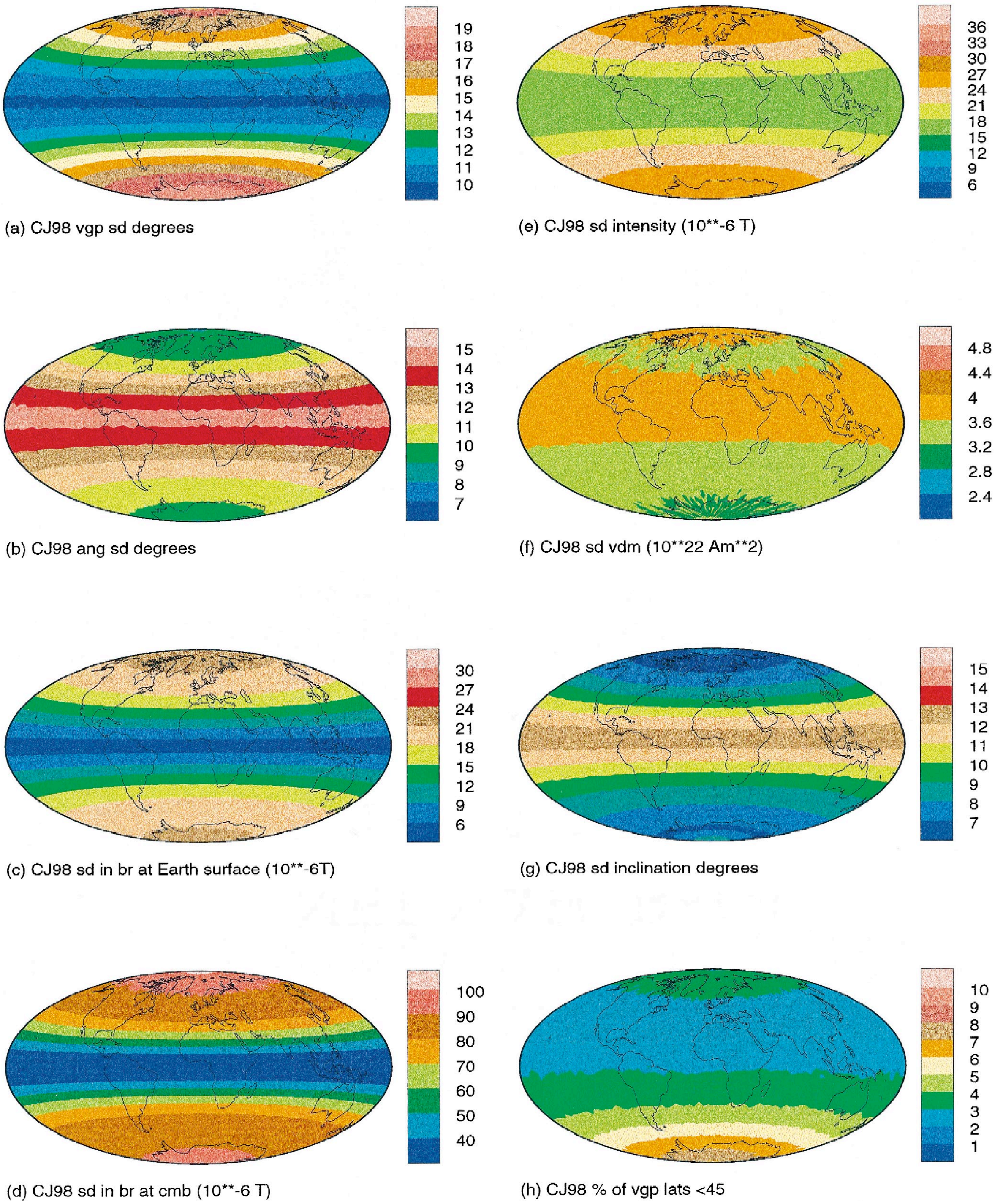
A more controversial issue is whether there are differences in secular variation between the Pacific and Atlantic hemispheres, which would require a more general form for the statistical anisotropy in the Gauss coefficients. Historical records of geomagnetic secular variation indicate approximately symmetric pairs of lobes of enhanced magnetic flux in both northern and southern hemispheres over the Americas and Eastern Asia, rather vigorous secular variation within the region bounded by these longitudes, and comparatively subdued activity in the Pacific region. There has been considerable discussion of whether this is a long-standing feature of the field that might reflect the influence of geographically heterogeneous core–mantle boundary conditions (Runcorn, 1992; McElhinny et al., 1996; Johnson and Constable, 1997, 1998; Carlut and Courtillot, 1998). A related issue is the equally controversial suggestion that the path followed by VGPs during geomagnetic reversals also tends to fall near the longitudinal swaths defined by these flux lobes (Clement, 1991; Laj et al., 1991; Valet et al., 1992; Prévot and Camps, 1993; Love, 1998). If hemispheric differences are a long-standing feature of the field, then they might be reflected in either a bias in the TAF and/or a difference in the variance in the PSV between Pacific and Atlantic hemispheres. Recent models published by Johnson and Constable (1997, 1998) suggest persistent similarities in the TAF over time scales ranging from hundreds to millions of years, but the biases involved are small and at present, the available data appear inadequate to achieve a satisfactory resolution to these questions. Here, we explore by simulation the consequences of allowing PSV variation to have large geographical variations of the kind that might be envisaged if the historical secular variation scenario persisted for the past few million years. The statistical PSV models for the Gauss coefficients that we test are not just axially anisotropic, they can, in addition, have an azimuthal anisotropy about the geographic axis with Gauss coefficients g_l^m and h_l^m in some cases having different statistical distributions for the same l and m. Such models can represent arbitrary anisotropy in the statistical properties for the Gauss coefficients and will no longer correspond to geomagnetic observables with the same statistical properties on small circles about the rotation axis, but can generate arbitrarily complex geographical variations. As we shall see, this allows the selection of suitable paleomagnetic observables to detect any large scale heterogeneity in the geomagnetic field. We find that the VGP dispersion conventionally used as a measure of PSV is perhaps the least sensitive parameter for detecting such geographical heterogeneity.

We start from an investigation of the latitudinal dependence of dispersion (VGP dispersion and local angular dispersion in directions) and of the VDM cumulative distribution functions for our revised PSV model CJ98, which has enhanced variance in g_2^1 and h_2^1 relative to other quadrupole terms (Fig. 5a). As we noted earlier, these provide a robust summary of the characteristics of the lava flow data, and are adequately modeled by CJ98. Now we investigate whether the data are compatible with a more complex heterogeneity in the statistical model. In Fig. 5b, we show the same paleomagnetic observables (the variation of dispersion with latitude and the VDM cumulative distribution function) for a similar model, but one that allows for significant low order (and, therefore, large scale) lateral heterogeneity in the PSV. In this second model, the model parameters are identical to CJ98, except that the excess variance in the order one quadrupole terms is all in the h_2^1 term rather than being divided equally with g_2^1. This version is designated CJ98.nz (see Table 1). The overall fit to the local directional and VGP dispersion data is very similar for both models. Fits to the VDM distribution for models CJ98 and CJ98.nz are essentially indistinguishable. We also investigated a similar model in which all the excess variance is in g_2^1 rather than h_2^1. The fit to the observations was marginally poorer than for CJ98, but the differences cannot be considered significant. Thus, we see that the available distribution of sites does not allow the inference that general anisotropy in the statistical variations of the Gauss coefficients is required to fit the observations.

Despite the null result concerning fit to the statistics of Fig. 5, the generally anisotropic models are worth investigating further in order to determine whether there are other geomagnetic observables that could be used to discriminate among various axially anisotropic and simple generally anisotropic models. We examine differences between CJ98 and CJ98.nz by comparing geographical variations in some of the diagnostics associated with PSV. We simulated 100,000 independent temporal samples of the geomagnetic field predicted by CJ98 at a grid of sites, and then calculated the standard deviation in a variety of geomagnetic parameters related to PSV for each site (Fig. 6a–g). For compatibility with our datasets, we excluded those simulated data with VGP latitudes less than 55° for a normal polarity field configuration. In principle, all of these parameters can be measured or estimated from paleomagnetic observations and their geographic variation used as a means of assessing the viability of the model. In practice, there are large geographic regions from which we have very little data, or inadequate temporal sampling especially for intensity data. The most commonly used measures for PSV are either the rms angular deviation of VGP from the geographic axis with deviations restricted to less than some maximum angle, to avoid inclusion of transitional or excursional data (Fig. 6a) or the angular standard deviation of the local field vector about its average value (Fig. 6b). We show variations in the radial component of the magnetic field at both Earth's surface and the core–mantle boundary for comparison with variations seen in historical field models (Fig. 6c,d). Expected standard deviation in paleointensity, VDMs and inclination about the local average field value provide additional diagnostics. The number of excursional directions expected is also

Fig. 6. Geographic variation in various dispersion parameters predicted by 100,000 simulations from CJ98 at each location on the geographic grid. (a) VGP dispersion about the geographic pole as a function of position, (b) angular standard deviation in local magnetic field vector about its average direction, (c) standard deviation in radial magnetic field at Earth's surface, (d) standard deviation in radial magnetic field at the core–mantle boundary, (e) standard deviation in local field intensity at Earth's surface, (f) standard deviation in VDM, (g) standard deviation in inclination about mean direction,(h) percentage of time that VGP latitudes are expected to be less than 45°.

plotted Fig. 6h, and we see that more excursional data are predicted at high southerly latitudes, than elsewhere, and by this measure, the field is most stable at low to mid-latitudes in the northern hemi-



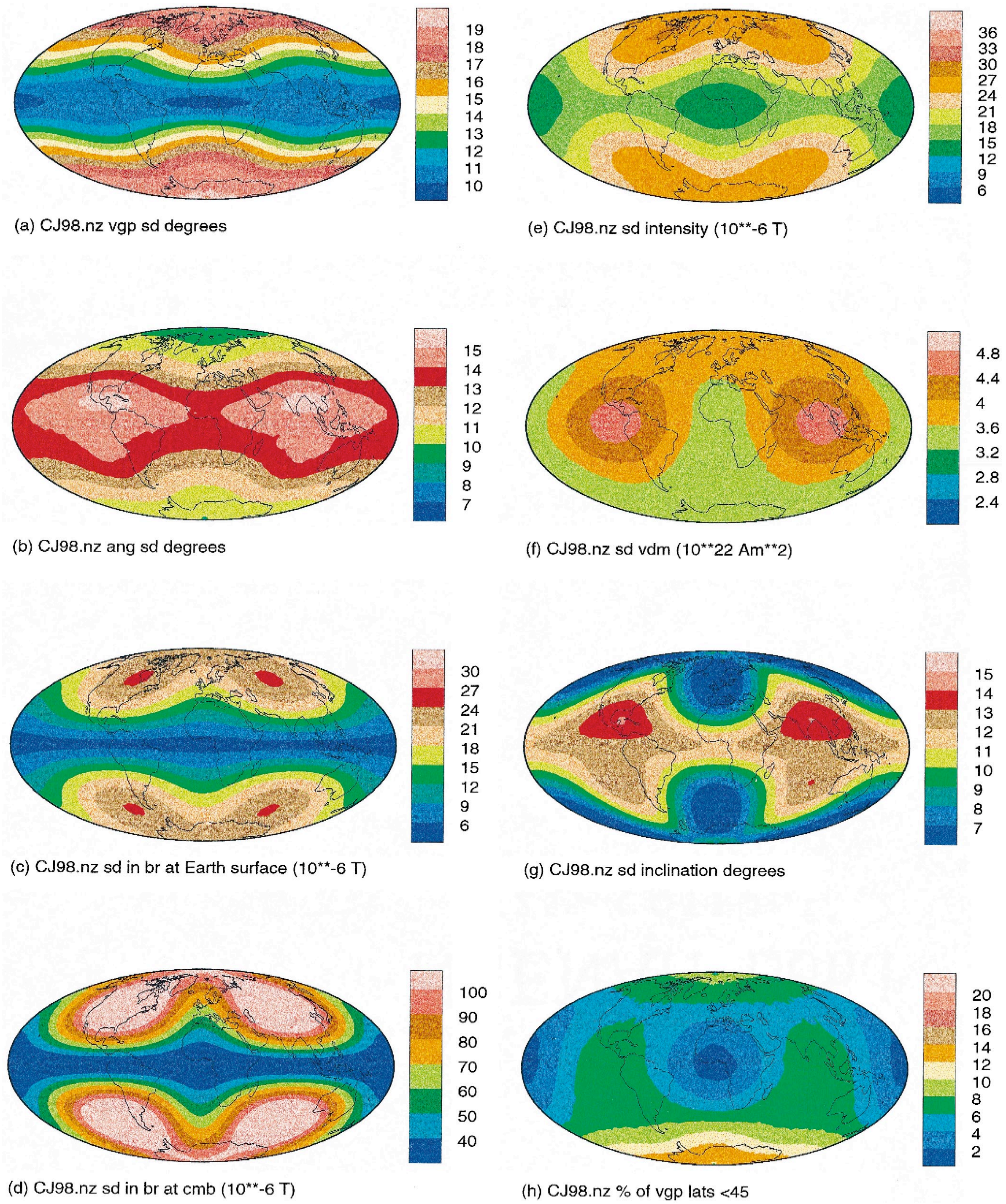


Fig. 7. Same as Fig. 6, but predictions are from CJ98.nz.

sphere. None of the standard deviations in paleomagnetic observables are symmetric about the equator, a direct consequence of the non-zero average value for the statistics of the axial-quadrupole term, g_2^0.

In Fig. 7, we show a simulation from CJ98.nz. We see that CJ98.nz has large variations in pairs of lobes in the radial magnetic field at Earth's surface and at the core–mantle boundary. These are somewhat similar to the flux lobes seen in the present and historical field, providing the motivation for investigating such a model, though we emphasize here that we cannot know from the historical field that the variation in these lobes will be large over geological time scales. Despite these large variations, there is comparatively little variation in the VGP dispersion around a line of latitude (Fig. 7a). The transformation to VGP coordinates has served to boost the importance of variations in the dipole part of the field at the expense of the potentially more interesting non-dipole contribution to the PSV. Thus, despite the large contrast in the properties of the secular variation, we only see a peak-to-peak variation of about 3° in VGP dispersion, even with a dataset of 1000 samples at each site — far better than we are likely to obtain in practice. One would need very accurate observations at carefully selected sites in order to detect the difference in VGP dispersion at a given latitude. Figs. 6 and 7 explain why CJ98.nz fits the variation in actual VGP dispersion with latitude about as well as CJ98 (Fig. 5a and b). McElhinny et al. (1996) recently argued against the existence of unusually low secular variation in the Pacific region, on the basis of comparisons between average VGP dispersions in the Pacific and Atlantic hemispheres. However, our simulations suggest that it is unlikely they would have been unable to discriminate between models CJ98 and CJ98.nz using VGP dispersion as the diagnostic. Thus, the existence of the so-called Pacific Dipole Window cannot be ruled out using lateral variations in VGP dispersion. In fact, one might argue that if large lateral variations in VGP dispersion do exist in the dataset, they are most likely to arise from inadequate temporal sampling of the geomagnetic field — meaning that the dipole variation has been inadequately represented.

Angular standard deviation about the mean field direction (Fig. 7b) is also rather insensitive to the lateral heterogeneity imposed by CJ98.nz. However, the standard deviation in inclination varies strongly with position (Fig. 7g). There is a predicted range of about 6° between the minimum and maximum values around a given mid-latitude small circle: this is a signal that is potentially detectable given the right data. Fig. 6e,f and Fig. 7e,f show that it may also be possible to derive useful information from paleointensity data and VDMs, although at present, the dataset has inadequate spatial and temporal coverage. CJ98.nz and CJ98 are both compatible with the statistical distribution of paleointensity observations.

The remaining panels in Figs. 6 and 7h show the percentage of VGP latitudes in the simulations that fall below 45°. Genuine paleomagnetic observations with this property are often considered excursional or transitional, depending on subsequent temporal evolution of the field. We see that the probability of occurrence of excursions defined in this way also varies with position — another signal that is potentially amenable to measurement. Excursions are more probable in CJ98.nz, a result that is compatible with observations derived from dynamical dynamo simulations, that suggest laterally varying thermal boundary conditions are more likely to generate reversals than uniform ones (Glatzmaier and Roberts, 1997).

6. Conclusions

We have presented a family of PSV models that satisfy summary statistics from both paleodirectional and paleointensity data for the geomagnetic field. In an earlier model, CP88, the secular variation is considered part of an isotropic Gaussian process, with a static bias supplied by a non-zero mean in the axial-dipole and axial-quadrupole terms. However, the data require that the variance in the statistical distribution of Gauss coefficients have anisotropy with respect to Earth's rotation axis. For CJ98, this takes the form of excess variance relative to the other $l=1$ and $l=2$ terms in the g_1^0, axial-dipole contribution, and the h_2^1 and g_2^1, non-axial-quadrupole variations, respectively. Small extra contributions in higher degree g_l^1 and h_l^1 are also compatible with the observations.

Provided the variance is partitioned equally between the g_l^m and h_l^m terms for any given l and m, this kind of model will not exhibit any longitudinal dependence in the various paleomagnetic observables. All the anisotropy can be considered to be with respect to Earth's rotation axis. However, we showed that the summary data considered here are fit equally well by a model in which all the excess quadrupole variance is in the h_2^1 term and the g_2^1 part is considered part of the isotropic process. We treat this longitudinally varying PSV model as an analog of the kind of behavior that might be expected from lateral heterogeneity at the core–mantle boundary influencing the geodynamo, and use it to look for detectable paleomagnetic signatures of such longitudinally varying boundary conditions. VGP dispersion is rather insensitive to such longitudinal variations, but inclination dispersion could be more informative given the right site distributions. In the model used for illustration mid-latitude sites around a small circle show variations of as much as 6° in inclination dispersion. This case probably represents an end-member among possible scenarios for the geomagnetic field. Relative frequency of occurrence of excursional directions in different locations may also be an indicator for longitudinal heterogeneity in PSV, reflecting influence of core–mantle boundary conditions.

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References

Camps, P., Prévot, M., 1996. A statistical model of the fluctuations in the geomagnetic field from paleosecular variation to reversal. Science 273, 776–779.

Carlut, J., Courtillot, V., 1998. How complex is the time-averaged geomagnetic field over the last 5 million years?. Geophys. J. Int. 134, 527–544.

Clement, B.M., 1991. Geographic distribution of transitional VGPs: evidence for non-zonal equatorial symmetry during the Matuyama–Brunhes geomagnetic reversal. Earth Planet. Sci. Lett. 104, 48–58.

Constable, C.G., 1994. The time-averaged geomagnetic field and paleosecular variation models: possible constraints from paleointensity observations. EOS Trans. Am. Geophys. Union 75 (44), 193.

Constable, C.G., Parker, R.L., 1988. Statistics of the geomagnetic secular variation for the past 5 m.y. J. Geophys. Res. 93, 11569–11581.

Constable, C.G., Lund, S.P., Johnson, C.L., 1995. Global geomagnetic field models for 0–3000 BP. EOS Trans. Am. Geophys. Union 76 (44), F164.

Daly, L., LeGoff, M., 1996. An updated and homogeneous world secular variation database: 1. Smoothing of the archaeomagnetic results. Phys. Earth Planet. Inter. 93, 159–190.

Glatzmaier, G.A., Roberts, P.H., 1995a. A three-dimensional self-consistent computer simulation of a geomagnetic field reversal. Nature 377, 203–208.

Glatzmaier, G.A., Roberts, P.H., 1995b. A three-dimensional convective dynamo solution with rotating and finitely conducting inner core and mantle. Phys. Earth Planet. Inter. 91, 63–75.

Glatzmaier, G.A., Roberts, P.H., 1996a. Rotation and magnetism of Earth's inner core. Science 274, 1887–1891.

Glatzmaier, G.A., Roberts, P.H., 1996b. On the magnetic sounding of planetary interiors. Phys. Earth Planet. Inter. 98, 207–220.

Glatzmaier, G.A., Roberts, P.H., 1997. Simulating the geodynamo. Contemp. Phys. 38, 269–288.

Gubbins, D., Kelly, P., 1993. Persistent patterns in the geomagnetic field over the past 2.5 Myr. Nature 365, 829–832.

Hongre, L., Hulot, G., Khoklov, A., 1998. An analysis of the geomagnetic field over the past 2000 years. Phys. Earth Planet. Inter. 106, 311–336.

Hulot, G., Gallet, Y., 1996. On the interpretation of virtual geomagnetic pole (VGP) scatter curves. Phys. Earth Planet. Inter. 95, 37–53.

Hulot, G., Le Mouël, J.L., 1994. A statistical approach to Earth's main magnetic field. Phys. Earth Planet. Inter. 82, 167–183.

Johnson, C.L., Constable, C.G., 1995. The time-averaged geomagnetic field as recorded by lava flows over the past 5 Myr. Geophys. J. Int. 122, 489–519.

Johnson, C.L., Constable, C.G., 1996. Paleosecular variation recorded by lava flows over the last 5 Myr. Philos. Trans. R. Soc. London, Ser. A 354, 89–141.

Johnson, C.L., Constable, C.G., 1997. The time-averaged geomagnetic field: global and regional biases for 0–5 Ma. Geophys. J. Int. 131, 643–666.

Johnson, C.L., Constable, C.G., 1998. Persistently anomalous Pacific geomagnetic fields. Geophys. Res. Lett. 25, 1011–1014.

Kelly, P., Gubbins, D., 1996. The geomagnetic field over the past 5 Myr. Geophys. J. Int. 128, 315–330.

Kono, M., Hiroi, O., 1996. Paleosecular variation of field intensities and dipole moments. Earth Planet. Sci. Lett. 139, 251–262.

Kono, M., Tanaka, H., 1995. Mapping the Gauss coefficients to the pole and the models of paleosecular variation. J. Geomagn. Geoelectr. 47, 115–130.

Kuang, W., Bloxham, J., 1996. Numerical investigations of the geodynamo. EOS Trans. AGU 77 (22), 141.

Kuang, W., Bloxham, J., 1997. An Earth-like numerical dynamo. Nature 389, 371–374.

Laj, C., Mazaud, A., Weeks, R., Fuller, M., Herrero-Bervera, E., 1991. Geomagnetic reversal paths. Nature 351, 447.

Lee, S., 1983. A study of the time-averaged paleomagnetic field for the last 195 million years. PhD Thesis, Australian National University.

Love, J.J., 1998. Paleomagnetic volcanic data and geometric regularity of reversals and excursions. J. Geophys. Res. 103, 12435–12452.

Lund, S.P., 1996. A comparison of Holocene paleomagnetic secular variation records from North America. J. Geophys. Res. 101, 8007–8024.

McElhinny, M.W., Senanayake, W.E., 1982. Variations in the geomagnetic dipole: 1. The past 50,000 years. J. Geomagn. Geoelectr. 34, 39–51.

McElhinny, M.W., McFadden, P.L., Merrill, R.T., 1996. The myth of the Pacific dipole window. Earth Planet. Sci. Lett. 143, 13–22.

Merrill, R.T., 1987. Use and abuse of intensity data. Nature 329, 197–198.

Merrill, R.T., McElhinny, M.W., McFadden, P.L., 1996. The Magnetic Field of the Earth: Paleomagnetism, the Core and the Deep Mantle. Academic Press.

Prévot, M., Camps, P., 1993. Absence of preferred longitude sectors for poles from volcanic records of geomagnetic reversals. Nature 366, 53–56.

Quidelleur, X., Courtillot, V., 1996. On low degree spherical harmonic models of paleosecular variation. Phys. Earth Planet. Inter. 95, 55–77.

Quidelleur, X., Valet, J.-P., Courtillot, V., Hulot, G., 1994. Long-term geometry of the geomagnetic field for the last 5 million years; an updated secular variation database from volcanic sequences. Geophys. Res. Lett. 21, 1639–1642.

Runcorn, S.K., 1992. Polar path in geomagnetic reversals. Nature 356, 654–656.

Senanayake, W.E., McElhinny, M.W., McFadden, P.L., 1982. Comparisons between the Thelliers' and Shaw's palaeointensity methods using basalts less than 5 million years old. J. Geomagn. Geoelectr. 34, 141–161.

Shaw, J., 1974. A new method of determining the magnitude of the paleomagnetic field, application to five historic lavas and five archeological sample. Geophys. J. R. Astron. Soc. 39, 133–141.

Tanaka, H., Kono, M., Uchimura, H., 1995. Some global features of paleointensity in geological time. Geophys. J. Int. 120, 97–102.

Tauxe, L., 1993. Sedimentary records of relative paleointensity of the geomagnetic field: theory and practice. Rev. Geophys. 31, 319–354.

Thellier, E., Thellier, O., 1959. Sur l'intensité du champ magnétique terrestre dans la passé historique et géologique. Ann. Geophys. 15, 285–378.

Valet, J.P., Tucholka, P., Courtillot, V., Meynadier, L., 1992. Paleomagnetic constraints on the geometry of the geomagnetic field during reversals. Nature 356, 400–407.

Walton, D., 1987. Improving the accuracy of geomagnetic intensity measurements. Nature 328, 789–791.

Walton, D., 1988a. Comment on ''Determination of the intensity of the Earth's magnetic field during archaeological times: reliability of the Thellier technique''. Rev. Geophys. 26, 13–14.

Walton, D., 1988b. The lack of reproducibility in experimentally determined intensities of the Earth's magnetic field. Rev. Geophys. 26, 15–22.

Wilson, R.L., 1970. Permanent aspects of the Earth's non-dipole magnetic field over Upper Tertiary times. Geophys. J. R. Astron. Soc. 19, 417–439.