EOSC 250 - Geophysical Fields and Fluxes
Calculus Quiz Practice

time: 45 minutes. Do as many problems as you can.

Compute $\mathrm{d}f/\mathrm{d}x$ for the following functions (where a, b, k and n are constants):

1. $f(x) = \cos(ax+b)$: $\quad f'(x) = -a\sin(ax+b)$ (chain rule)
2. $f(x) = x\sin(x)$: $\quad f'(x) = \sin(x) + x\cos(x)$ (product rule)
3. $f(x) = \sin(x)\cos(x)$: $\quad f'(x) = \cos^2(x) - \sin 2(x) = \cos(2x)$ (product rule)
4. $f(x) = \exp(x^2)$: $\quad f'(x) = 2x\exp(x^2)$ (chain rule)
5. $f(x) = x\log(x)$: $\quad f'(x) = \log(x) + 1$ (product rule)
6. $f(x) = x\exp(kx)$: $\quad f'(x) = \exp(kx) + kx\exp(kx)$ (product rule)
7. $f(x) = [\log(kx)]^n$: $\quad f'(x) = n[\log(kx)]^{n-1}/x$ (chain rule)

Note: log here (and anywhere in this course) is log with base $e = 2.71\ldots$

Compute the following indefinite integrals (where n and k are constants):

1. $\int \sin(ax+b)\mathrm{d}x = -\frac{1}{a}\cos(ax+b) + C$ (change variables to $u = ax+b$)
2. $\int x\cos(x)\mathrm{d}x = x\sin(x) + \cos(x) + C$ (integration by parts)
3. $\int \exp(kx)\mathrm{d}x = \frac{1}{k}\exp(kx)$
4. $\int \frac{1}{a-x}\mathrm{d}x = -\log(a-x) + C$ (change variables to $u = a-x$)
5. $\int (ax+b)\exp(x)\mathrm{d}x = (ax+b-a)\exp(x) + C$ (integration by parts)
6. $\int \sin^n(x)\cos(x)\mathrm{d}x = \frac{1}{n+1}\sin^{n+1}(x) + C$ (change variables to $u = \sin(x)$
7. $\int x\cos(x) + \sin(x)\mathrm{d}x = x\sin(x) + C$ (integration by parts)
8. (more difficult) $\int -\log[\sin(x)] \times \cos(x)\mathrm{d}x = \sin(x)\,(1 - \log[\sin(x)]) + C$ (change variables to $u = \sin(x)$)

Compute the following definite integrals (where a, b, n and k are constants):

1. $\int_0^1 \exp(x)\mathrm{d}x = \exp(1) - 1$
2. $\int_0^1 \cos(\pi x)\mathrm{d}x = 0$

3. $\int_a^b \cos^n(kx)\sin(kx)\mathrm{d}x = \frac{cos^{n+1}(ka)-\cos^{n+1}(kb)}{k(n+1)}$ (change variables to $u = \cos(kx)$)

4. (more difficult) $\int_a^b \cos^2(x)\mathrm{d}x = \int_a^b \frac{1}{2}(\cos(2x)+1)\,\mathrm{d}x = \frac{\sin(2b)-\sin(2a)}{4} + b - a$ (trigonometric identity $\cos(2x) = 2\cos^2(x) - 1$)

Given $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, compute

1. $|\mathbf{a}| = 5$

2. $\mathbf{a} - 3\mathbf{b} = -3\mathbf{k} + \mathbf{j} - 3\mathbf{k}$

3. $\mathbf{a} \cdot \mathbf{b} = 10$

4. $\mathbf{a}\times(\mathbf{a}+\mathbf{b}) = \mathbf{a}\times\mathbf{b} = (3\times1-4\times2)\mathbf{k}+(4\times1-0\times1)\mathbf{i}+(0\times2-1\times3)\mathbf{j} = 4\mathbf{i}-3\mathbf{j}-5\mathbf{k}$

5. The component of $\mathbf{b}$ parallel to $\mathbf{a}$ is $\mathbf{b}\cdot\mathbf{a}/|\mathbf{a}| = 10/5 = 2$.