EOSC 250 - Geophysical Fields and Fluxes
Equation Summary

$$A = \pi r^2$$

$$A = 2\pi r h$$

$$A = 4\pi r^2$$

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$V = \pi r^2 h$$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{1}{3} \times \text{base} \times \text{height}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(Iy) = I(x)\frac{\mathrm{d}y}{\mathrm{d}x} + I(x)f(x)y = I(x)g(x), \qquad \frac{\mathrm{d}I}{\mathrm{d}x} = I(x)f(x)$$

$$M = \int_V \rho \,\mathrm{d}V$$

$$E = \int_V e \,\mathrm{d}V$$

$$\hat{\mathbf{n}} = \pm \frac{\mathbf{k} - \frac{\partial h}{\partial x}\mathbf{i} - \frac{\partial h}{\partial y}\mathbf{j}}{\sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}}$$

$$\mathrm{d}S = \sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}\,\mathrm{d}x\,\mathrm{d}y$$

$$\int \mathbf{q}\cdot\hat{\mathbf{n}}\,\mathrm{d}S = \int\int q_z - q_x\frac{\partial h}{\partial x} - q_y\frac{\partial h}{\partial y}\,\mathrm{d}y\,\mathrm{d}x$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho\,\mathrm{d}V = -\int_S \rho\mathbf{v}\cdot\hat{\mathbf{n}}\,\mathrm{d}S$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V e\,\mathrm{d}V = -\int_S e\mathbf{v}\cdot\hat{\mathbf{n}}\,\mathrm{d}S - \int_S \mathbf{q}_c\cdot\hat{\mathbf{n}}\,\mathrm{d}S + \int_V a\,\mathrm{d}V$$

$$\nabla\cdot\mathbf{q} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}, \qquad \mathbf{q} = q_x\mathbf{i} + q_y\mathbf{j} + q_z\mathbf{k}$$

$$\int_S \mathbf{q}\cdot\hat{\mathbf{n}}\,\mathrm{d}S = \int_V \nabla\cdot\mathbf{q}\,\mathrm{d}V$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\frac{\partial e}{\partial t} + \nabla\cdot(e\mathbf{v}) + \nabla\cdot\mathbf{q}_c = a$$

$$\mathbf{q}_c = -k\nabla T, \qquad e = \rho c T$$

$$\nabla T = \mathbf{i}\frac{\partial T}{\partial x} + \mathbf{j}\frac{\partial T}{\partial y} + \mathbf{k}\frac{\partial T}{\partial z}$$

$$\rho c\frac{\partial T}{\partial t} + \rho c\mathbf{v}\cdot\nabla T - \nabla\cdot(k\nabla T) = a$$

$$-\nabla\cdot(k\nabla T) = a$$

$$\nabla\cdot\nabla T = \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$-k\nabla^2 T = a$$

$$-\frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right) = a(x),$$

$$-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(rk\frac{\mathrm{d}T}{\mathrm{d}r}\right) = a(r)$$

$$-\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2k\frac{\mathrm{d}T}{\mathrm{d}r}\right) = a(r)$$

$$\frac{\mathrm{d}q}{\mathrm{d}x} = a(x) \qquad q(x) = -k\frac{\mathrm{d}T}{\mathrm{d}x}$$

$$\frac{1}{r}\frac{\mathrm{d}(rq)}{\mathrm{d}r} = a(r), \qquad q(r) = -k\frac{\mathrm{d}T}{\mathrm{d}r}$$

$$\frac{1}{r^2}\frac{\mathrm{d}(r^2q)}{\mathrm{d}r} = a(r) \qquad q(r) = -k\frac{\mathrm{d}T}{\mathrm{d}r}$$

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \qquad r = |\mathbf{r}| = \sqrt{x^2+y^2+z^2}, \qquad \hat{\mathbf{r}} = \frac{\mathbf{r}}{r}.$$

$$\nabla T(r) = \frac{\mathrm{d}T}{\mathrm{d}r}\hat{\mathbf{r}}$$

$$\nabla\cdot[q(r)\hat{\mathbf{r}}] = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2 q(r)\right].$$

$$\mathbf{q}(\mathbf{r}) = \frac{Q_0}{4\pi r^2}\hat{\mathbf{r}}.$$

$$T(\mathbf{r}) = \frac{Q_0}{4\pi k r}$$

$$\mathbf{q}(\mathbf{r}) = \frac{Q_0}{4\pi|\mathbf{r}-\mathbf{r}_0|^2}\frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|}$$

$$T(\mathbf{r}) = T_\infty + \frac{Q_0}{4\pi k|\mathbf{r}-\mathbf{r}_0|}$$

$$\mathbf{q}(\mathbf{r}) = \sum_i \frac{Q_i}{4\pi|\mathbf{r}-\mathbf{r}_i|^2}\frac{(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|}$$

$$T(\mathbf{r}) = T_\infty + \sum_i \frac{Q_i}{4\pi k|\mathbf{r}-\mathbf{r}_i|}$$

$$\int_C \mathbf{f}\cdot \mathrm{d}\mathbf{r} = \int_{t_1}^{t_2}\left[f_x(x(t),y(t),z(t))\frac{\mathrm{d}x}{\mathrm{d}t} + f_y(x(t),y(t),z(t))\frac{\mathrm{d}y}{\mathrm{d}t} + f_z(x(t),y(t),z(t))\frac{\mathrm{d}z}{\mathrm{d}t}\right]\,\mathrm{d}t$$

$$\nabla\times\mathbf{f} = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)\mathbf{k}$$

$$\nabla\times\mathbf{f} = \begin{vmatrix} \mathbf{j} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$$

$$\int_S (\nabla\times\mathbf{f})\cdot\hat{\mathbf{n}}\,\mathrm{d}S = \int_C \mathbf{f}\cdot\,\mathrm{d}\mathbf{r}$$

$$\phi = -\int_C \mathbf{f}\cdot\,\mathrm{d}\mathbf{r}$$

$$\phi(\mathbf{r}_B) - \phi(\mathbf{r}_A) = \int_C \nabla\phi\cdot\,\mathrm{d}\mathbf{r}$$

$$\mathbf{f} = -\nabla\phi$$

$$\mathbf{f} = -\frac{Gm}{r^2}\hat{\mathbf{r}}$$

$$\phi = -\frac{Gm}{r}$$

$$\nabla^2\phi = 4\pi G\rho, \qquad \mathbf{g} = -\nabla\phi$$

$$-\epsilon\nabla^2\phi = \rho_c, \qquad \mathbf{E} = -\nabla\phi$$

$$\nabla(fg) = (\nabla f)g + f(\nabla g)$$

$$\nabla f(g) = \frac{\mathrm{d}f}{\mathrm{d}g}\nabla g$$

$$\nabla \cdot (\phi \mathbf{f}) = \phi \nabla \cdot \mathbf{f} + \mathbf{f} \cdot \nabla \phi$$

$$\nabla \times (\phi \mathbf{f}) = (\nabla \phi) \times \mathbf{f} + \phi \nabla \times \mathbf{f}$$

$$\nabla \times \nabla \phi = \mathbf{0}$$

$$\nabla \cdot (\nabla \times \mathbf{f}) = 0$$