EOSC 352 - Continuum Dynamics
Equation Summary

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} h\,\mathrm{d}V = \int_{V(t)} \frac{\partial h}{\partial t}\,\mathrm{d}V + \int_{S(t)} h\mathbf{u}\cdot\hat{\mathbf{n}}\,\mathrm{d}S$$

$$\int_V \nabla\cdot\mathbf{q}\,\mathrm{d}V = \int_S \mathbf{q}\cdot\hat{\mathbf{n}}\,\mathrm{d}S$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} h\,\mathrm{d}V = -\int_{S(t)} \mathbf{q}\cdot\hat{\mathbf{n}}\,\mathrm{d}S + \int_{V(t)} a\,\mathrm{d}V$$

$$\nabla\cdot\mathbf{q} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}$$

$$\nabla T = \mathbf{i}\frac{\partial T}{\partial x} + \mathbf{j}\frac{\partial T}{\partial y} + \mathbf{k}\frac{\partial T}{\partial z}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

$$\frac{\partial h}{\partial t} + \nabla\cdot(h\mathbf{u}) + \nabla\cdot\mathbf{q} = a$$

$$h = \rho c_p T$$

$$\mathbf{q} = -k\nabla T$$

$$\rho c_p\frac{\partial T}{\partial t} + \rho c_p\mathbf{u}\cdot\nabla T - \nabla\cdot k\nabla T = a$$

$$\nabla\cdot\nabla T = \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$\hat{\mathbf{n}} = \pm\frac{\mathbf{k} - \frac{\partial h}{\partial x}\mathbf{i} - \frac{\partial h}{\partial y}\mathbf{j}}{\sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}}$$

$$\mathrm{d}S = \sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}\,\mathrm{d}x\,\mathrm{d}y$$

$$\int \mathbf{q}\cdot\hat{\mathbf{n}}\,\mathrm{d}S = \int\int q_z - q_x\frac{\partial h}{\partial x} - q_y\frac{\partial h}{\partial y}\,\mathrm{d}y\,\mathrm{d}x$$

$$A = \pi r^2, \qquad A = 2\pi r h, \qquad A = 4\pi r^2$$

$$V = \pi r^2 h, \qquad V = \frac{4}{3}\pi r^3$$

$$\frac{\partial}{\partial t}\mathrm{Re}\left[T_0\exp(i\omega t+\lambda x)\right]=\mathrm{Re}\left[\frac{\partial}{\partial t}T_0\exp(i\omega t+\lambda x)\right]$$

$$\frac{\partial}{\partial x}\mathrm{Re}\left[T_0\exp(i\omega t+\lambda x)\right]=\mathrm{Re}\left[\frac{\partial}{\partial x}T_0\exp(i\omega t+\lambda x)\right]$$

$$\mathrm{Re}[\exp(a+ib)]=\exp(a)[\cos(b)+i\sin(b)]$$

$$a+ib=r\exp(i\theta)\Rightarrow\tan(\theta)=b/a,\qquad r=\sqrt{a^2+b^2}$$

$$\sqrt{i}=\pm\frac{1+i}{\sqrt{2}}.$$

$$f(x)=f(x_0)+f'(x_0)(x-x_0)+f''(x_0)(x-x_0)^2/2!+f'''(x_0)(x-x_0)^3/3!+\ldots$$

$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x\exp\left(-y^2\right)\,\mathrm{d}y$$

$$n\xi^{n-1}\theta(\xi)+\xi^n\theta'(\xi)=(\xi^n\theta(\xi))'$$

$$\mathbf{a}=(a_1,a_2,a_3)$$

$$\mathbf{c}=\mathbf{a}+\mathbf{b}\Leftrightarrow c_i=a_i+b_i$$

$$\mathbf{b}=\lambda\mathbf{a}\Leftrightarrow b_i=\lambda a_i$$

$$\mathbf{a}\cdot\mathbf{b}=\sum_{i=1}^{3}a_ib_i=a_ib_i$$

$$a_{ij}b_j=\sum_{j=1}^{3}a_{ij}b_j=a_{i1}b_1+a_{i2}b_2+a_{i3}b_3$$

$$\delta_{ij}=\begin{cases}1 & \text{if } i=j\\ 0 & \text{otherwise}\end{cases}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)}h\,\mathrm{d}V=\int_{V(t)}\frac{\partial h}{\partial t}\,\mathrm{d}V+\int_{S(t)}hu_jn_j\,\mathrm{d}S$$

$$F_i=\int_{S(t)}\sigma_{ij}n_j\,\mathrm{d}S$$

$$F_i=\int_{V(t)}f_i\,\mathrm{d}V$$

$$p_i=\int_{V(t)}\rho u_i\,\mathrm{d}V$$

$$\frac{\mathrm{d}p_i}{\mathrm{d}t}=\int_{S(t)}\sigma_{ij}n_j\,\mathrm{d}S+\int_{V(t)}f_i\,\mathrm{d}V$$

$$L_{ij}=\int_{V(t)}\rho(x_iu_j-x_ju_i)\,\mathrm{d}V$$

$$\frac{\mathrm{d}L_{ij}}{\mathrm{d}t}=\int_{S(t)}(x_i\sigma_{jk}-x_j\sigma_{ik})n_k\,\mathrm{d}S+\int_{V(t)}x_if_j-x_jf_i\,\mathrm{d}V$$