Calculus Practice: EOSC 352

11th September, 2009

1. Calculate the gradient of the following scalar fields:
   (a)
   $$\phi(x, y, z) = \sin(x) + \cos(y),$$
   (b)
   $$\Psi(x, y, z) = \exp(x^2 + y^2 + x^2),$$
   (c)
   $$\Phi(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}},$$
   (d)
   $$T(x, y, z) = T_0 - (x^2 + y^2)/k,$$
   where T_0 and k are constant parameters.

2. Compute the divergence of the following vector fields:
   (a)
   $$\mathbf{q}(x, y, z) = x^2 y\mathbf{i} - y^2 x\mathbf{j} + z\mathbf{k},$$
   (b)
   $$\mathbf{u}(x, y, z) = y\cos(x)\mathbf{i} - x\sin(y)\mathbf{j},$$
   (c)
   $$\mathbf{q}(x, y, z) = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{4\pi(x^2 + y^2 + z^2)^{3/2}}$$

3. Let
   $$\Phi(x, y, z) = (x^2 - y^2)\exp(x)\cos(x) - 2xy\exp(x)\sin(y).$$
   Calculate $\nabla^2\Phi$.

4. Let $\rho = 1 + x^2$. Compute $\int_V \rho\, \mathrm{d}V$, if V is the triangular prism given by $0 < x < 1 - y$, $0 < y < 1$, $0 < z < 1$.

5. Compute the integral $\int_S \mathbf{q} \cdot \hat{\mathbf{n}} \, \mathrm{d}S$, where S is the part of the surface $z = x^2 + y^2$ that lies above the triangle $0 < x < 1 - y$, $0 < y < 1$, $\hat{\mathbf{n}}$ is the upward-pointing unit normal to the surface, and

$$\mathbf{q}(x, y, z) = z\mathbf{i} + y\mathbf{j} - x\mathbf{k}.$$

6. Let

$$\mathbf{q} = x^2 y\mathbf{i} - y^2 x\mathbf{j} + z\mathbf{k},$$

and let S be the surface of the unit sphere, with $\hat{\mathbf{n}}$ its outward-pointing unit normal. Calculate

$$\int_S \mathbf{q} \cdot \hat{\mathbf{n}} \, \mathrm{d}S.$$