

THE Hf-W ISOTOPIC SYSTEM AND THE ORIGIN OF THE EARTH AND MOON

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Background concepts presented by

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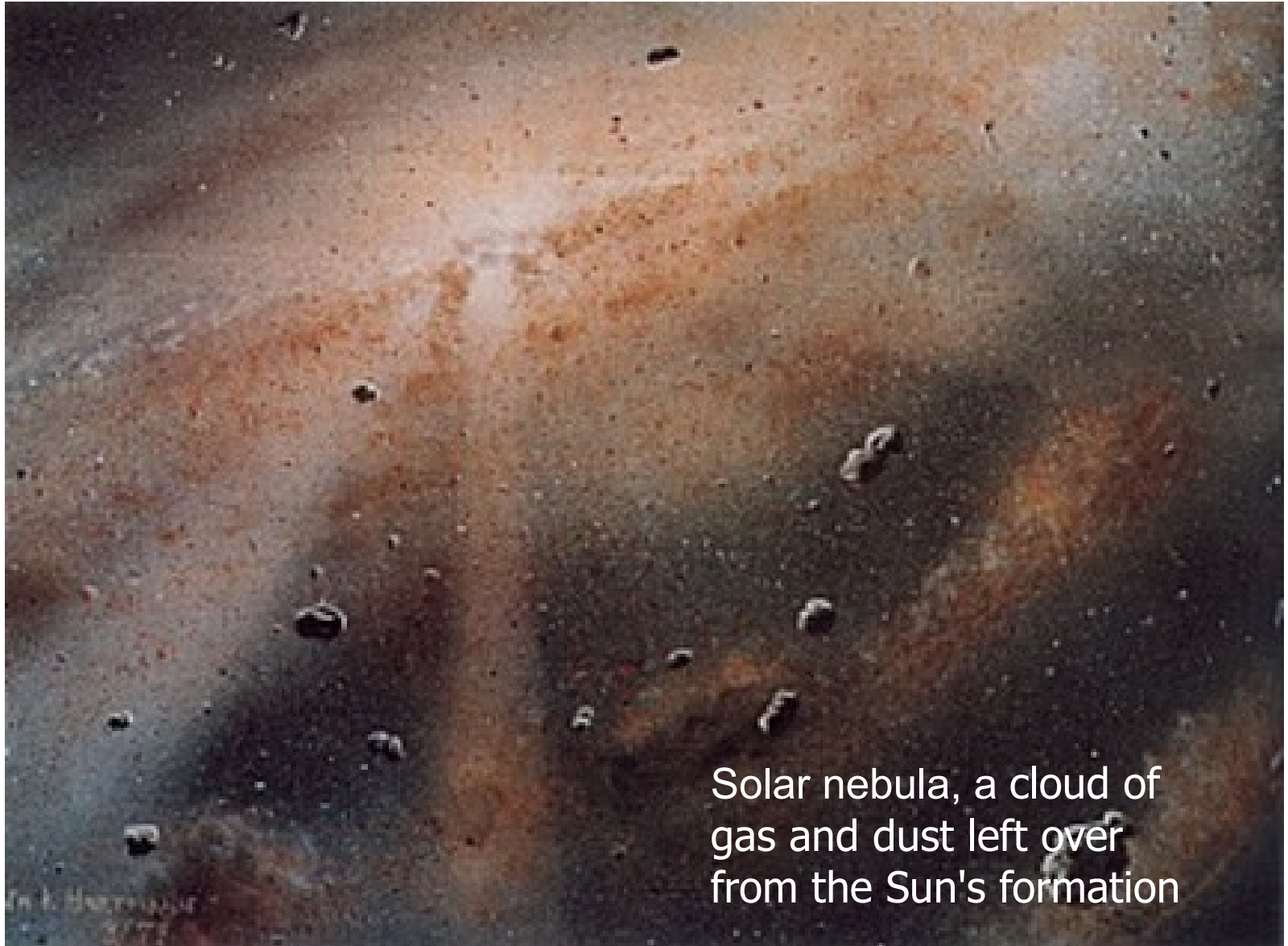
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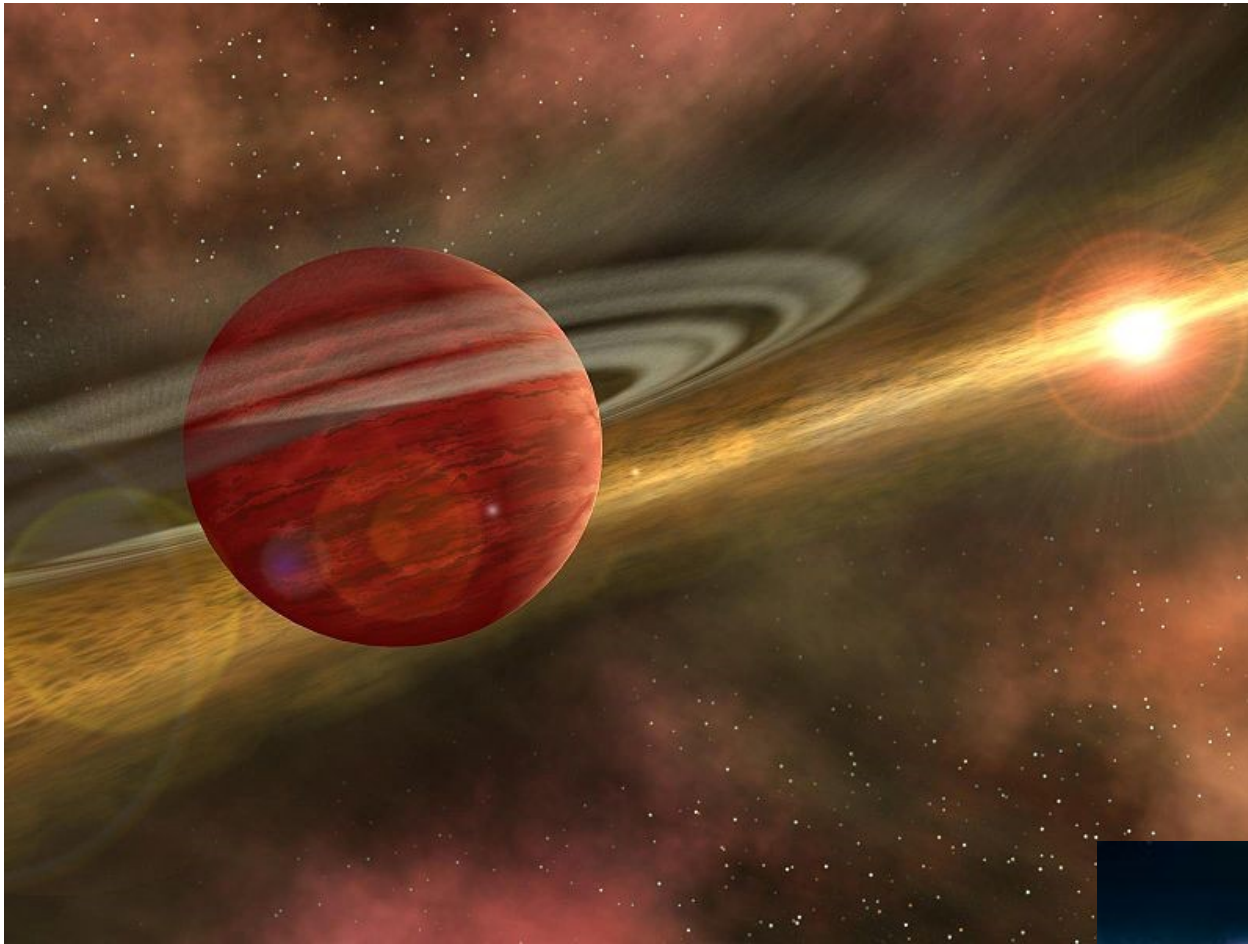
Question:

Can **Hf-W** system be used as a chronometer for the Earth formation and core differentiation?

1. Planet-building and core formation process
2. Radiometric dating using **Hf-W** system
3. Chondritic meteorites
4. Two-stage **Hf-W** fractionation model
5. Continuous **Hf-W** core formation model
6. **W** isotope model constraints from Moon, Mars, Vesta and meteorites

1. Planet-building and core formation process





Planetary embryos

“The Earth’s last collision”



Chemical complexities:

Rate of accretion vs. Rate of nebular cooling

A) $R_{\text{ACCR}} > R_{\text{COOL}}$

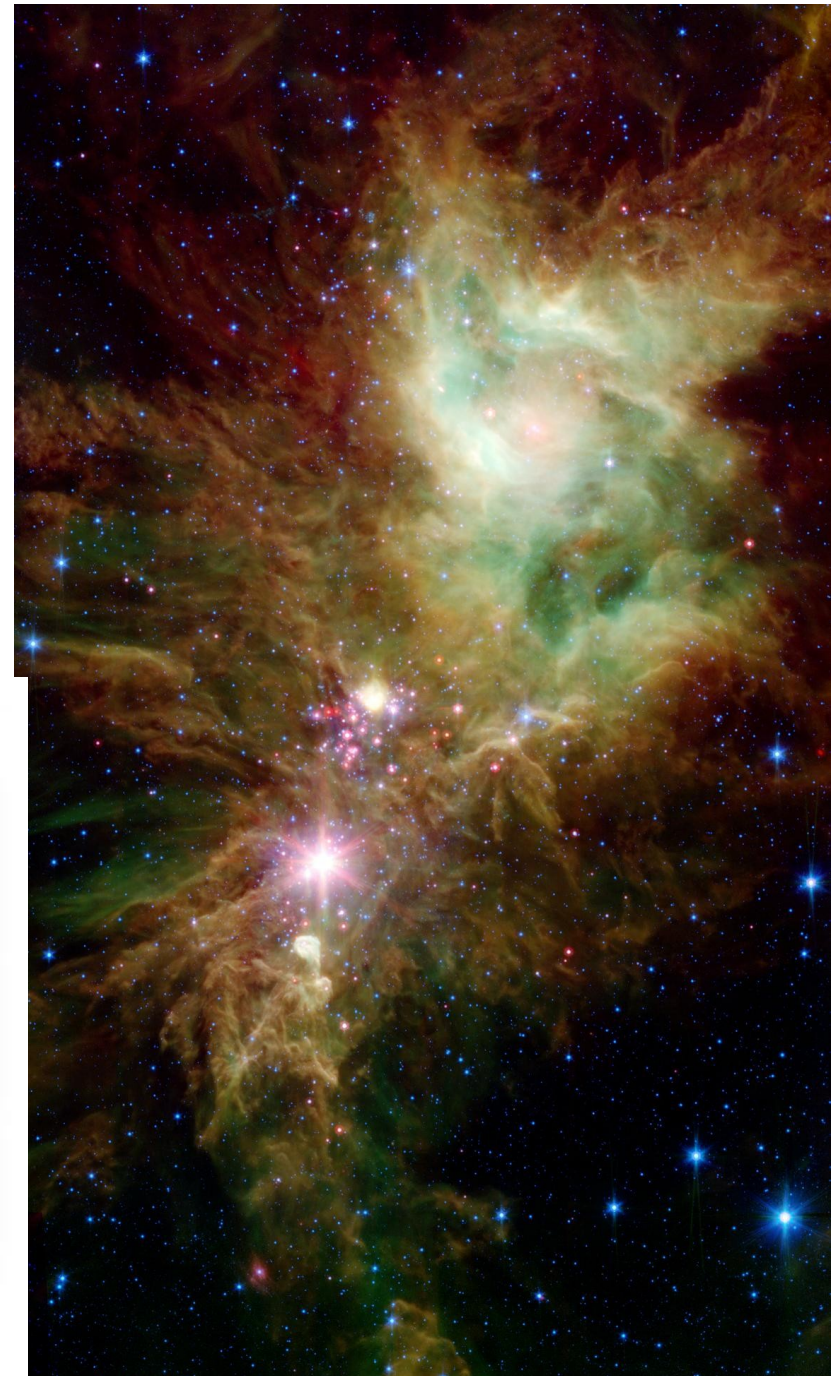
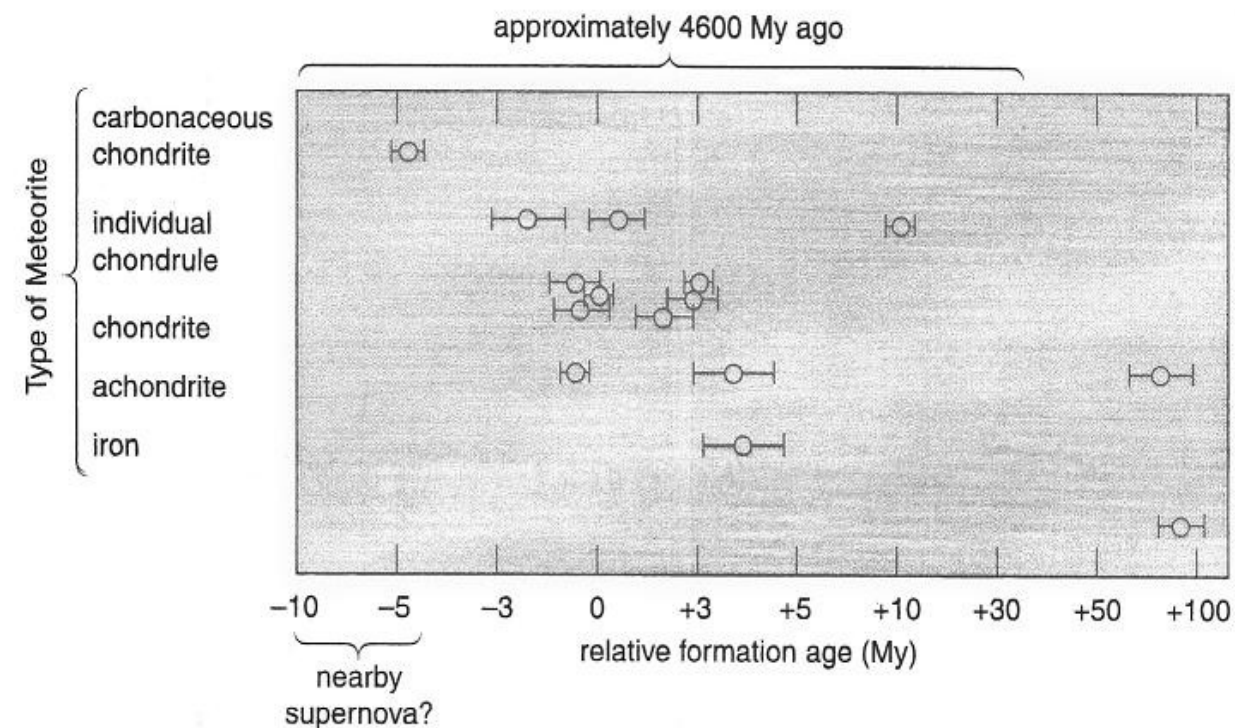
- Refractories (e.g. **W**) accrete first, and
- Planets start with partially formed iron cores

B) $R_{\text{ACCR}} < R_{\text{COOL}}$

- Planetary matter is well mixed

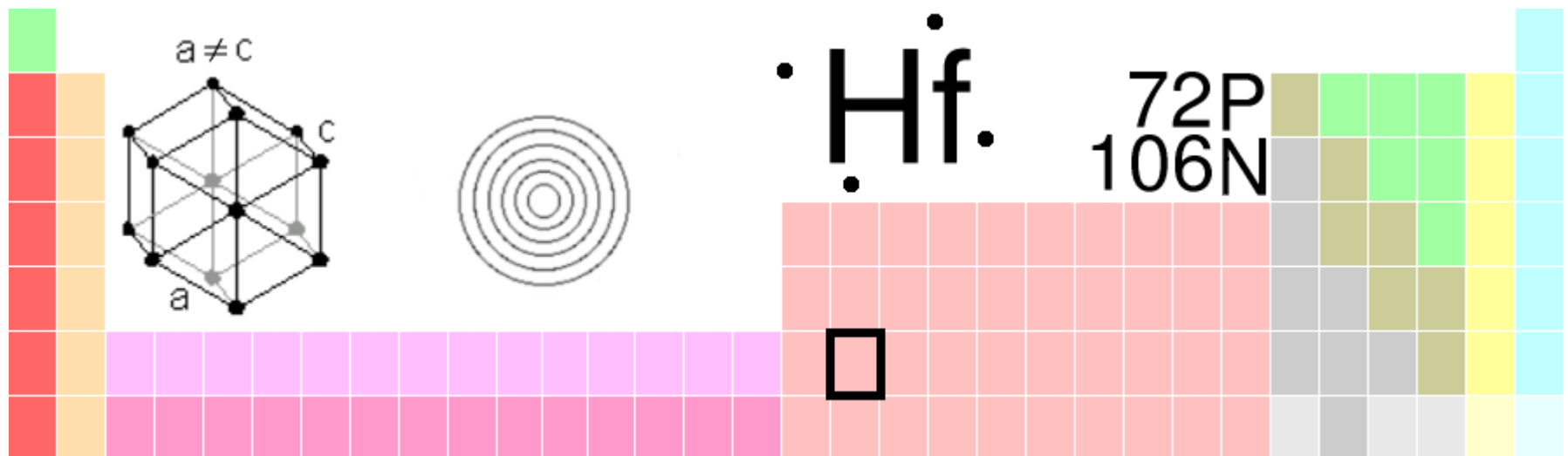
Source of unstable radioactive elements?

^{129}I	$T_{1/2} = 17 \text{ Myr}$
^{26}Al	$T_{1/2} = 0.72 \text{ Myr}$
^{182}Hf	$T_{1/2} = 9 \text{ Myr}$



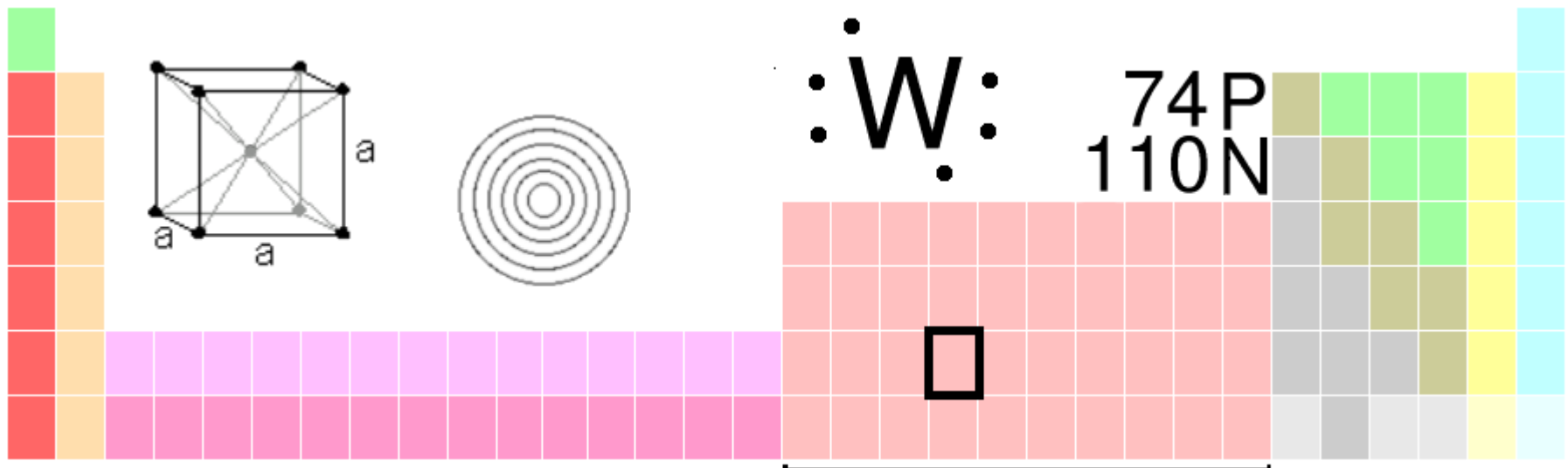
Hafnium

- Hafnium has many isotopes most of which are stable
- The most common is Hf-180 making up 35% of the natural abundance
- Hf-182 beta decays to W-182 with a half life of 9 Myr
- Lithophilic

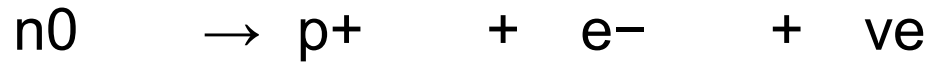


Tungsten

- W-183 and W-182 have a natural abundance of 14.3% and 26.5% respectively
- Both are stable and W-182 is the daughter product of the Hf-182 decay.
- Siderophilic

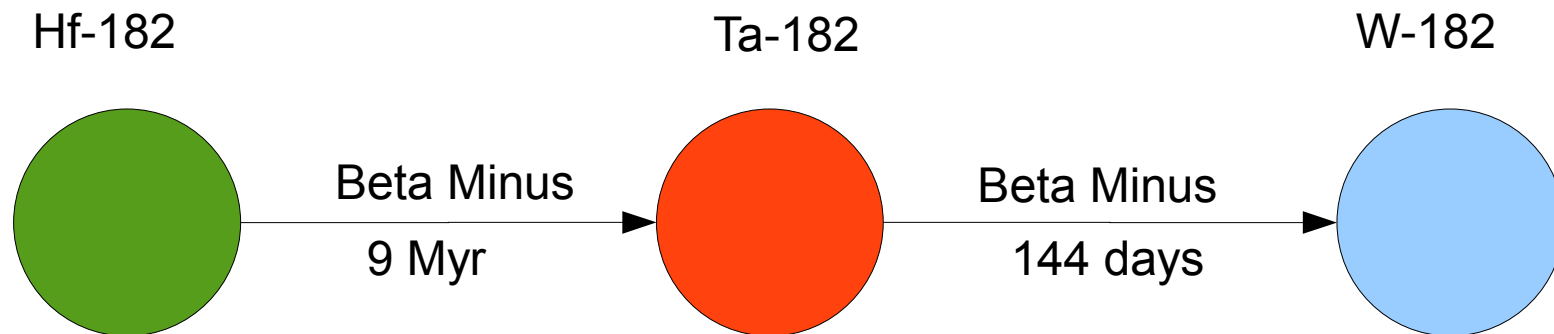


Beta Decay



- A neutron decays to a proton, electron and an anti-neutrino.
- The number of nucleons is unchanged, but the atomic number increases by 1

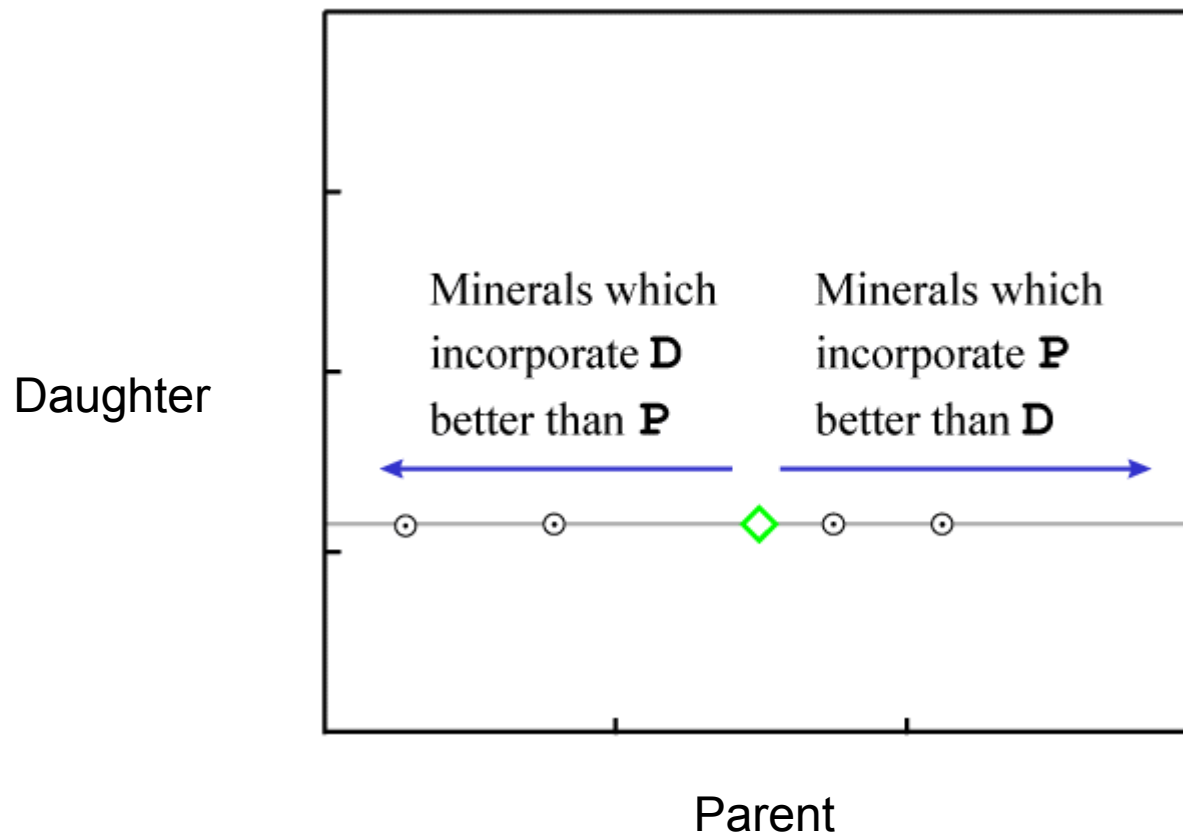
The Hf-W System



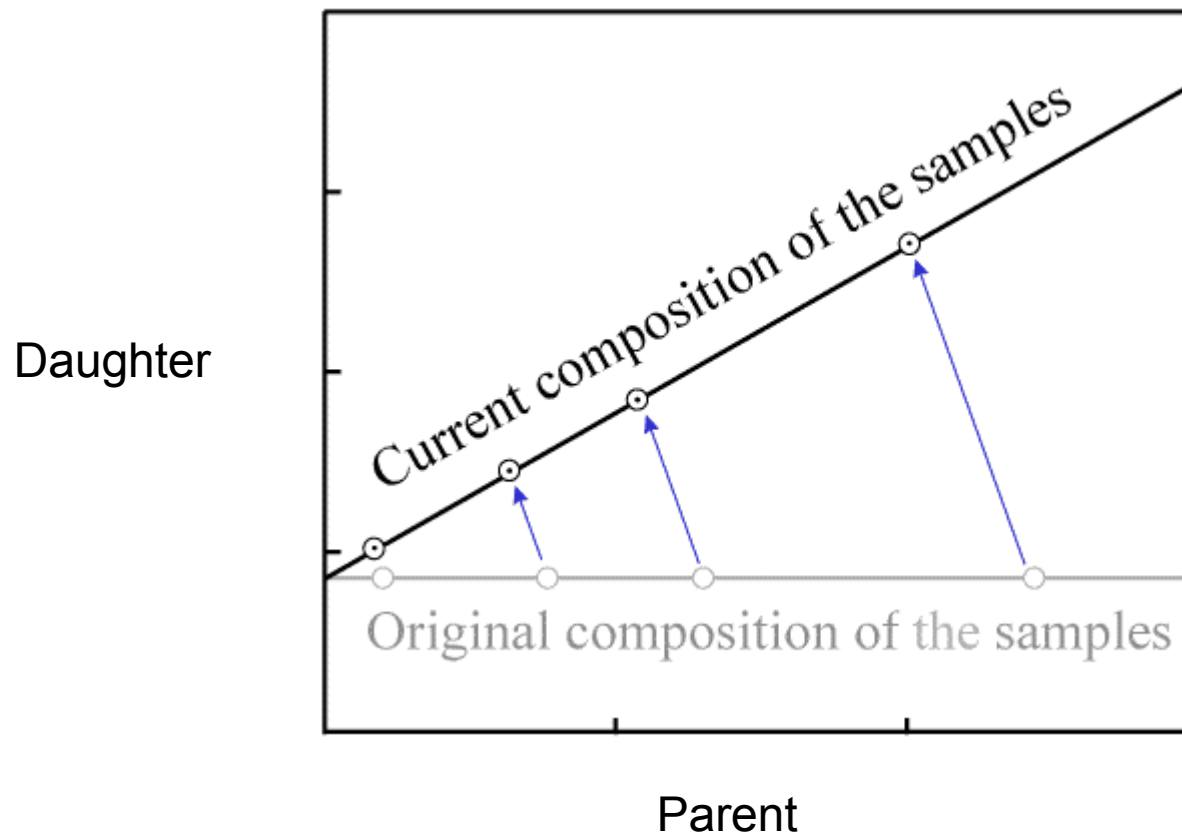
$$^{182}\text{Hf}(t) = ^{182}\text{Hf}_0 e^{-\lambda t}$$

How Does Geochronometry Work?

- As minerals form from a reservoir, they have different amounts of parent isotopes, but the same amount of daughter isotopes.
- As time moves forward the parent decays and the concentrations shift .



- After a sufficient number of half lives decay stops and the concentrations become fixed.
- It is important that we have stable isotopes of both the parent and daughter product.



$$\left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^T = \left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^0 + \left(\frac{{}^{182}\text{Hf}}{{}^{183}\text{W}}\right)^0 (1 - e^{-\lambda t})$$

Present
Amount

Original
Amount

Contribution
from Decay

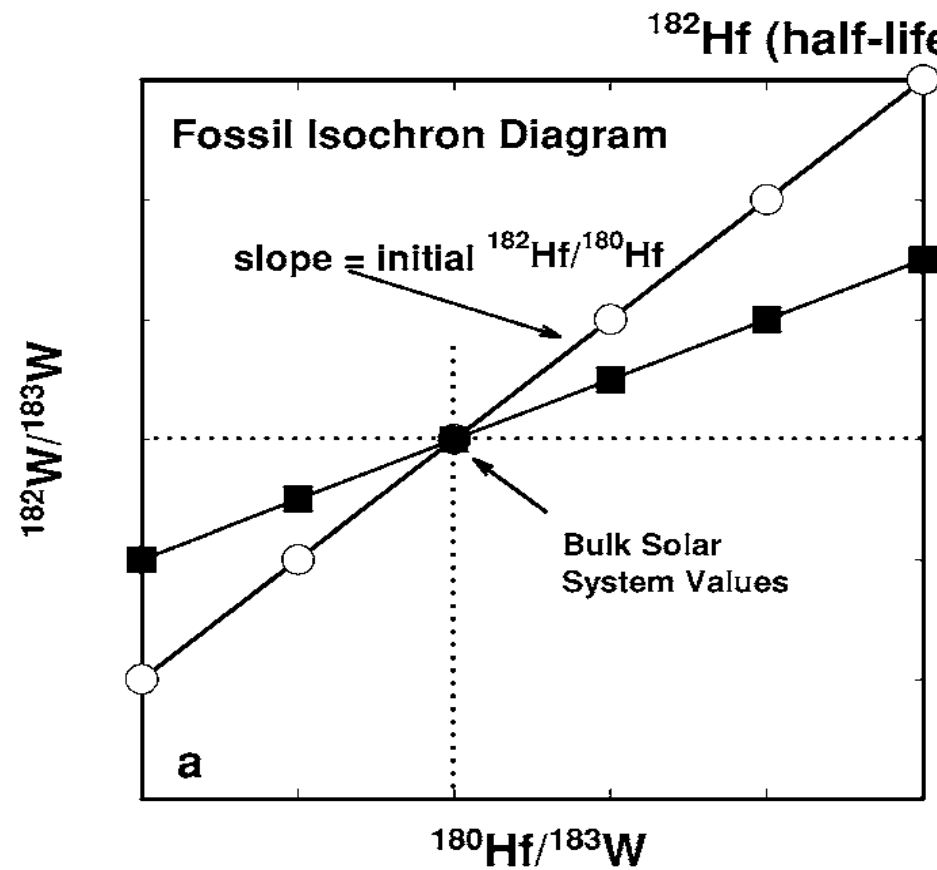
$$\left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^T = \left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^0 + \left(\frac{{}^{182}\text{Hf}}{{}^{183}\text{W}}\right)^T (e^{\lambda t} - 1)$$

$$\left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^T = \left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^0 + \left(\frac{{}^{180}\text{Hf}}{{}^{183}\text{W}}\right)^T \left(\frac{{}^{182}\text{Hf}}{{}^{180}\text{Hf}}\right)^T (e^{\lambda t} - 1)$$

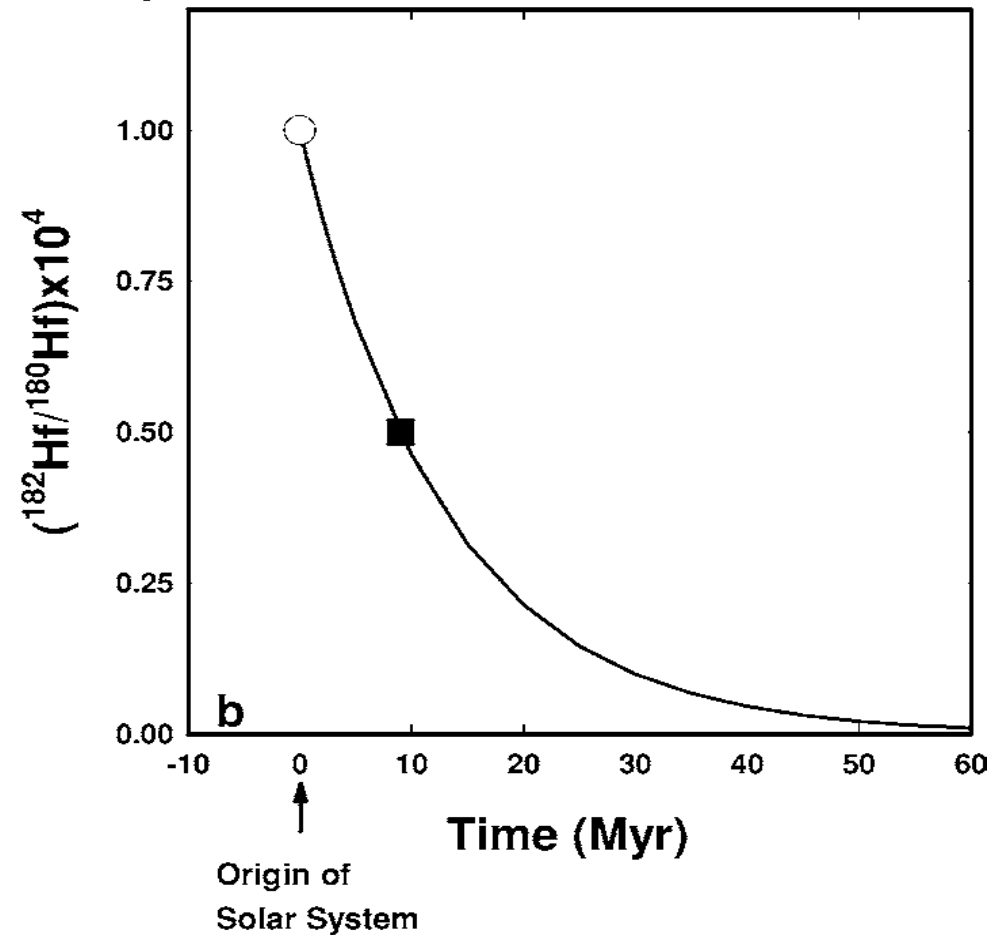
$$\left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^T = \left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^0 + \left(\frac{{}^{180}\text{Hf}}{{}^{183}\text{W}}\right)^T \left(\frac{{}^{182}\text{Hf}}{{}^{180}\text{Hf}}\right)^0 (1 - e^{-\lambda t})$$

$$\left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^T = \left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}}\right)^0 + \left(\frac{{}^{180}\text{Hf}}{{}^{183}\text{W}}\right)^T \left(\frac{{}^{182}\text{Hf}}{{}^{180}\text{Hf}}\right)^0$$


Measure stable isotope concentrations, at the present time T
 Derive past values of parent and daughter isotopes from these

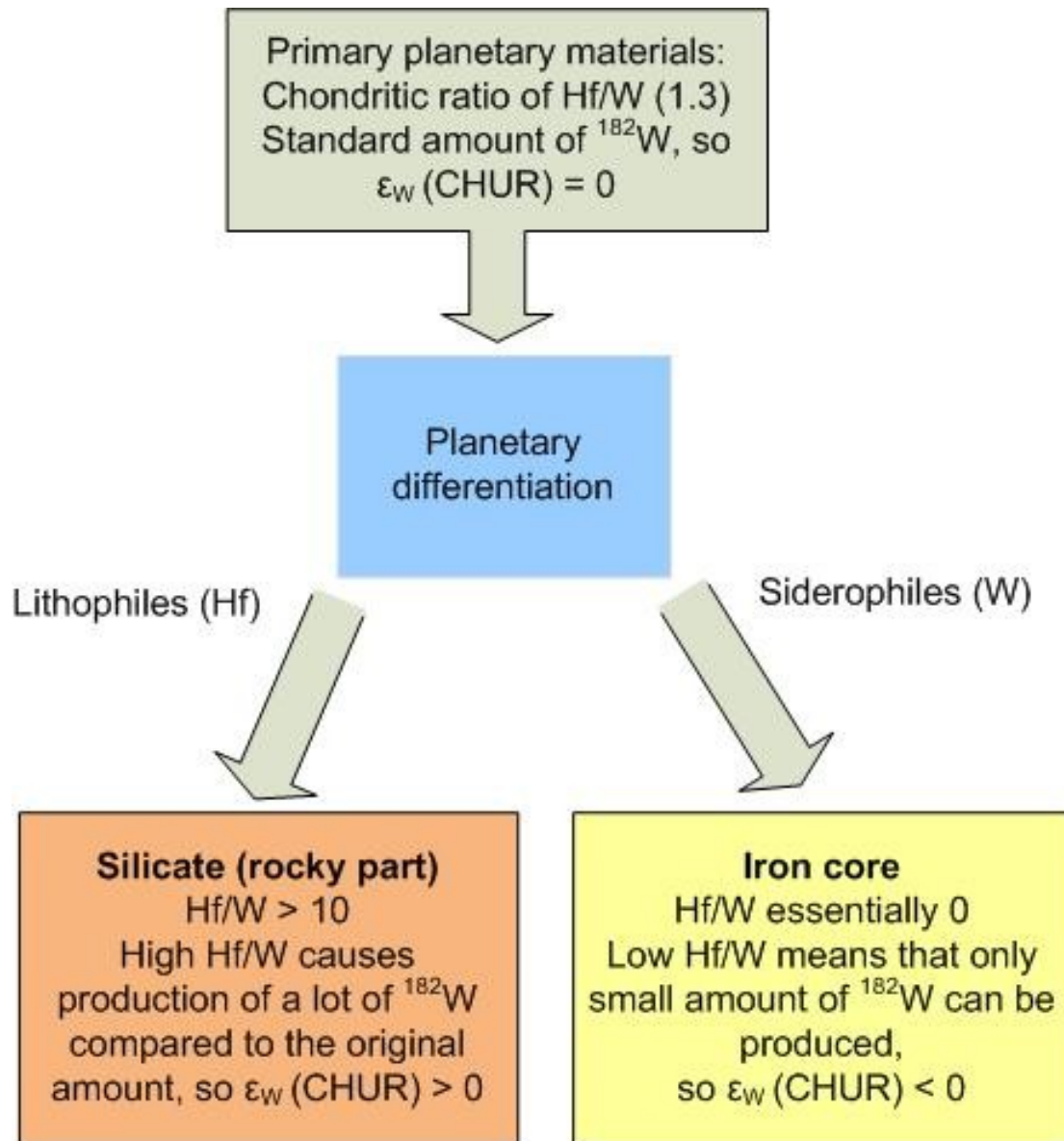


**Stable Reference
Isotopes: ^{183}W , ^{180}Hf**



$$\left(\frac{^{182}\text{W}}{^{183}\text{W}}\right)^T = \underbrace{\left(\frac{^{182}\text{W}}{^{183}\text{W}}\right)^0}_{\text{Intercept}} + \left(\frac{^{180}\text{Hf}}{^{183}\text{W}}\right)^T \underbrace{\left(\frac{^{182}\text{Hf}}{^{180}\text{Hf}}\right)^0}_{\text{Slope}}$$

Planetary differentiation



Bulk Silicate Earth (**BSE**)

$$\varepsilon_{W(BSE)}(0) = \left[\frac{(^{182}\text{W}/^{183}\text{W})^0}{(^{182}\text{W}/^{183}\text{W})_{BSE}^0} - 1 \right] \times 10^4$$

CHondritic Unfractionated Reservoir (**CHUR**)

$$\begin{aligned} \varepsilon_{W(CHUR)}(0) &= \left[\frac{(^{182}\text{W}/^{183}\text{W})^0}{(^{182}\text{W}/^{183}\text{W})_{CHUR}^0} - 1 \right] \times 10^4 \\ &= \varepsilon_{W(BSE)}(0) + 1.9 \end{aligned}$$

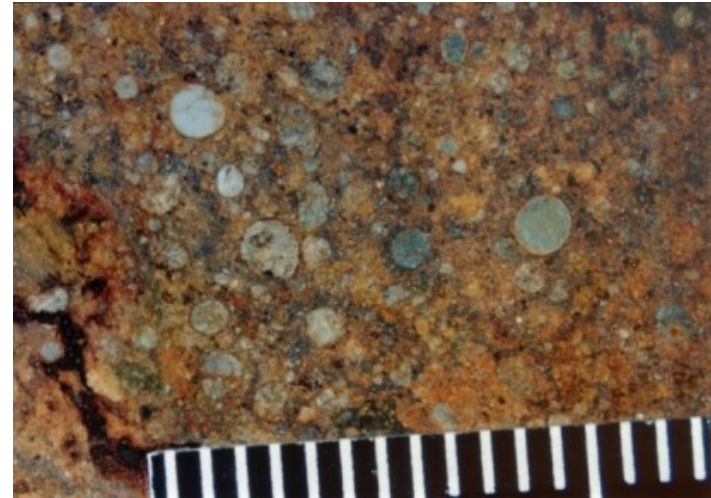
Meteorites

Differentiation

- **Chondrites**
 - Stony meteorites with “chondrules” (Gr. “seed”)
 - Building blocks of the planetary system
- **Achondrites**
 - Stony meteorites w/o “chondrules”
- **Stone Irons**
 - Resemble where the Earth core meets the mantle
- **Irons**
 - Resemble the outer core of the Earth



Chondrites



mm

Chondrules

Iron meteorites



IIAB



IVAB

Ages

A) Solidification age

B) Formation age

Many ancient meteorites have S.A. = F.A.

The age of solar system ~ 4.566 Gy

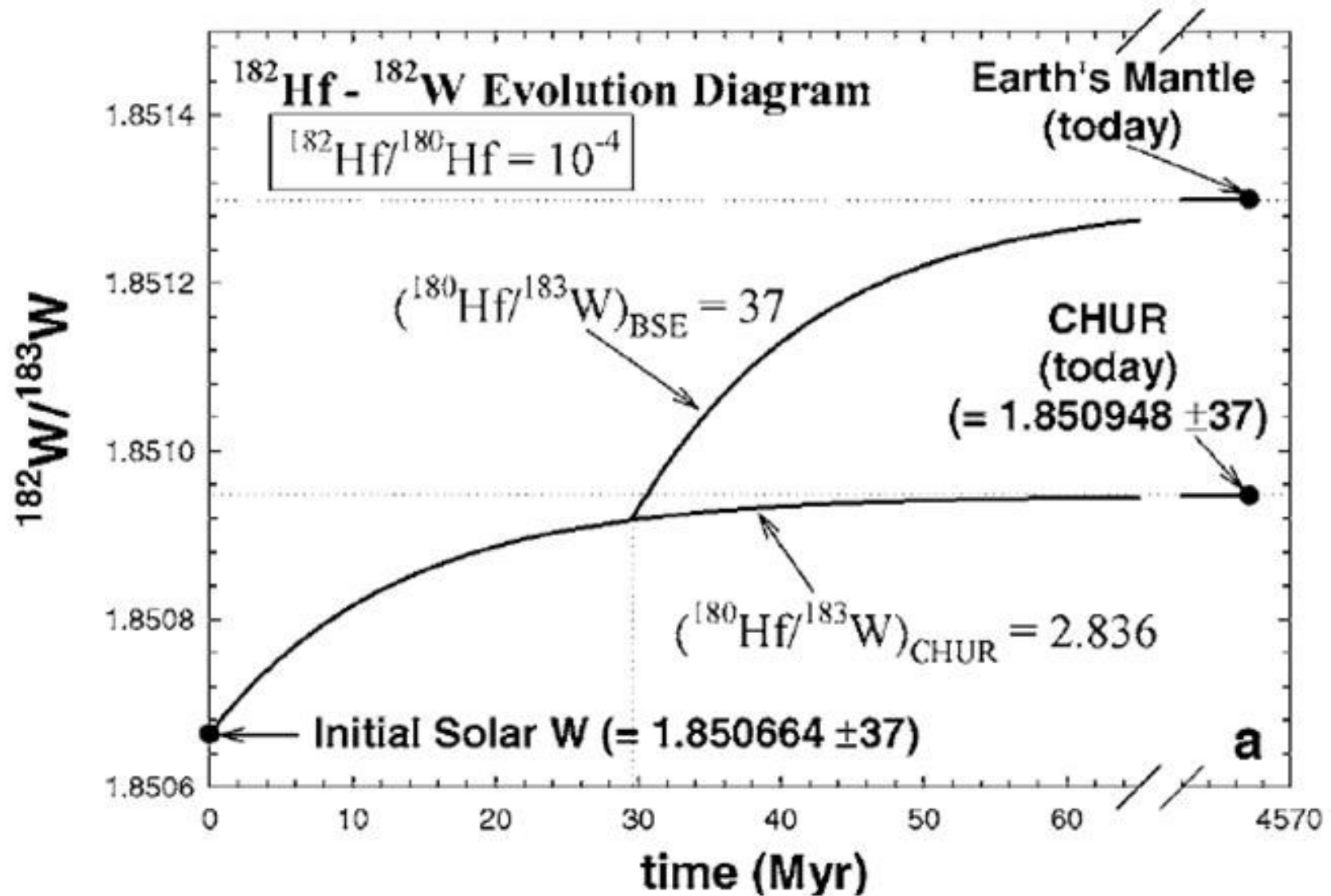
Two-stage **Hf-W** fractionation model

$$\varepsilon_{W(CHUR)j}^*(t) = q_W \left(\frac{{}^{182}\text{Hf}}{{}^{180}\text{Hf}} \right)_{T_0} f_j^{\text{Hf}/\text{W}} [e^{-\lambda t_{2stage}} - e^{-\lambda t}]$$

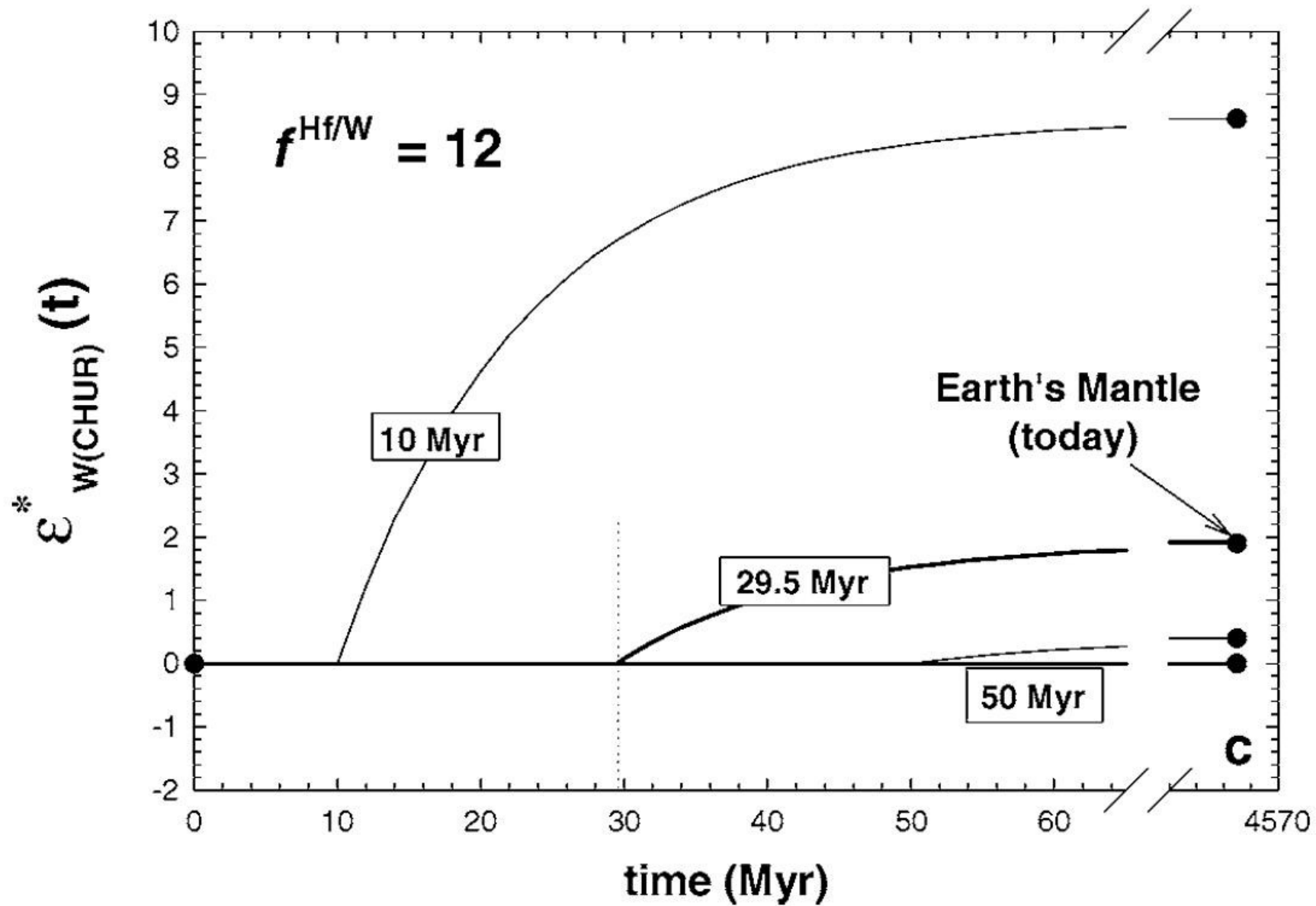
$$t_{2stage} = T_0 - T_{cf}$$

$$q_W = 10^4 ({}^{180}\text{Hf}/{}^{182}\text{W})_{CHUR}^0 = 1.55 \times 10^4$$

$$t_{2stage} = \frac{1}{\lambda} \ln \left[\frac{\left(\frac{{}^{182}\text{Hf}}{{}^{180}\text{Hf}} \right)_{T_0} \left[\left(\frac{{}^{180}\text{Hf}}{{}^{183}\text{W}} \right)_j - \left(\frac{{}^{180}\text{Hf}}{{}^{183}\text{W}} \right)_{CHUR} \right]}{\left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}} \right)_j^0 - \left(\frac{{}^{182}\text{W}}{{}^{183}\text{W}} \right)_{CHUR}^0} \right]$$

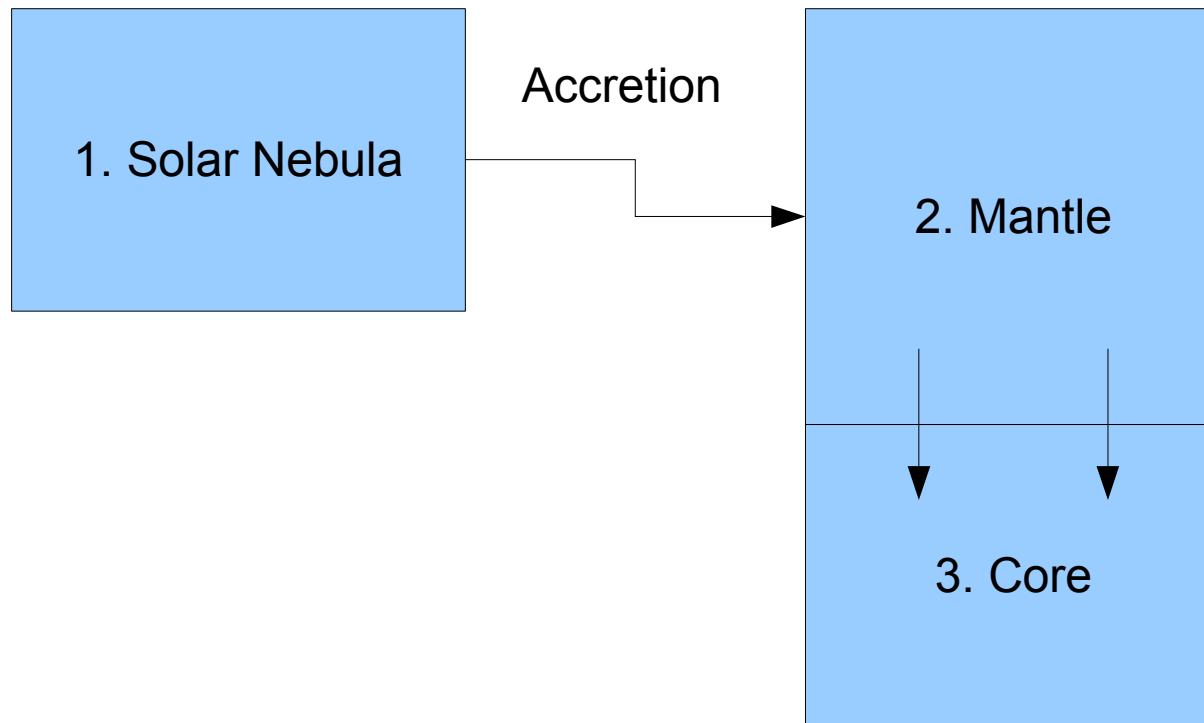


$^{182}\text{Hf} - ^{182}\text{W}$ evolution in early Solar System



^{182}Hf - ^{182}W evolution relative to $\epsilon_w(\text{CHUR})$

Continuous Model



Fractionation

$$f_i^{Hf/W} = \frac{(^{180}\text{Hf} / ^{183}\text{W})_j}{(^{180}\text{Hf} / ^{183}\text{W})_{CHUR}} - 1$$

A measure of how separated Hf and W have become

Transport of Stable Species

$$d_{i23} = \frac{c_{i3}}{C_{i2}} \quad \gamma = \frac{\dot{M}_{23}}{\dot{M}_{12}}$$

$$\frac{C_{i2}}{C_{i1}} = \frac{1}{(d_{i23} - 1)\gamma + 1}$$

$$\frac{C_{i3}}{C_{i1}} = \frac{d_{i23}}{(d_{i23} - 1)\gamma + 1}$$

Transport of Parents

$$f_2^{r/s} = \frac{\gamma(d_{s23} - d_{r23})}{(d_{r23} - 1)\gamma + 1}$$

$$f_3^{r/s} = \frac{(\gamma - 1)(d_{s23} - d_{r23})}{d_{s23}((d_{r23} - 1)\gamma + 1)}$$

Since $d_{r23} = 0$, we have:

$$f_2^{r/s} = \gamma \frac{d_{s23}}{1 - \gamma}$$

$$f_3^{r/s} = -1$$

Transport of Daughter Isotope

For the specific case of Hf-182 we have that concentrations of daughter products don't vary much over earth's history, and the system is extinct. These give simplifications which lead to the forms:

$$\epsilon_{W(CHUR)} = 10^4 \frac{C_{^{182}\text{Hf}_1}(0)}{C_{^{182}\text{W}_1}(0)} \lambda f_2^{H/W} I(\lambda, t)$$

$$I(\lambda, t) = \int_0^t \left(\frac{M_2(x)}{M_2(t)} \right)^{1+f_2^{r/s}} e^{-\lambda x} dx$$

$$\epsilon_{d3} = \frac{-\epsilon_{d2}}{f_2^{r/s}}$$

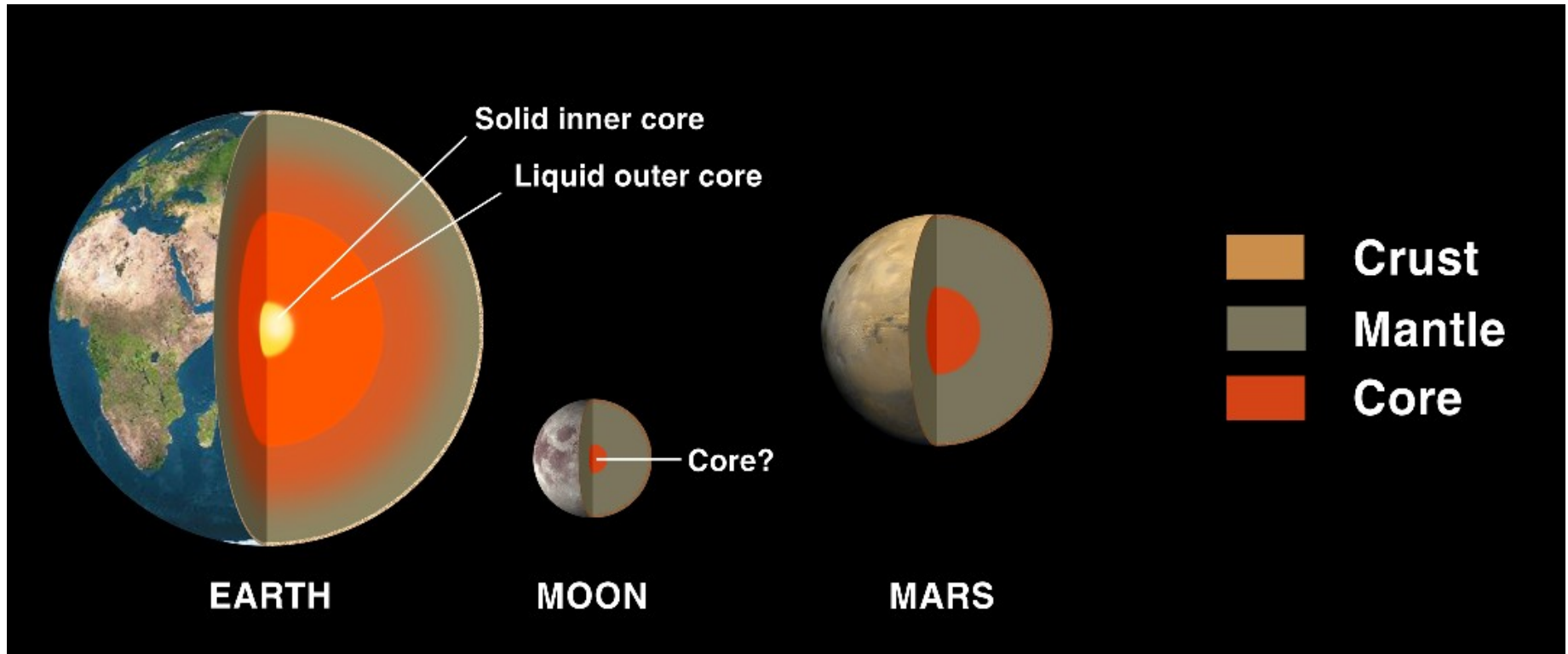
W isotope model constraints

TABLE 2 The isotopic composition of tungsten and chemical fractionations of various planetary mantles and cores

	Mantle $\epsilon_{\text{W(BSE)}} (0)$	Mantle $\epsilon_{\text{W(CHUR)}} (0)$	Core $\epsilon_{\text{W(BSE)}} (0)$	Core $\epsilon_{\text{W(CHUR)}} (0)$	Mantle $f^{\text{Hf/W}}$	$t_{2\text{stage}}$ (Myr)	Core mass fraction (γ)	$D_{\text{W}}^{\text{met/sil}}$
Earth	$\equiv 0$	1.9 ± 0.2	-2.06 ± 0.05	-0.16 ± 0.05	12 ± 2	29.6	0.325	24.9
Moon	1.3 ± 0.4	3.2 ± 0.4	-2.08 ± 0.05	-0.18 ± 0.05	18 ± 2	28.1	$<0.02, 0.325$	$>882, 37$
IAB	19.1	21	-3.30 ± 0.15	-1.40 ± 0.15	(15)	<2.8	0.2	60
IIAB	22.1	24	-3.50 ± 0.36	-1.60 ± 0.36	(15)	<2.9	0.2	60
IIIAB	29.5	31.4	-3.99 ± 0.60	-2.09 ± 0.60	(15)	<0.5	0.2	60
IVA	30.8	32.7	-4.08 ± 0.69	-2.18 ± 0.69	(15)	<0.5	0.2	60
IVB	29.2	31.1	-3.97 ± 0.42	-2.07 ± 0.42	(15)	~ 0	0.06	235
Vesta	17 ± 2	19 ± 2	-3.17 ± 0.34	-1.27 ± 0.34	15 ± 2	2.6	0.2	60
Mars—S source	0.5 ± 0.5	2.4 ± 0.5	-3.10 ± 0.52	-1.20 ± 0.52	2.0 ± 0.8	3.3	0.2	8
Mars—NC source	2.5 ± 0.5	4.4 ± 0.5	-2.63 ± 0.25	-0.73 ± 0.25	6 ± 1	9.7	0.2	24

Values in italics are primary data (see text for sources), other values are inferred from these. Numbers in parentheses are best guesses.

Cores of terrestrial planets



Earth

$$t_{2stage} = 28.1 \text{ Myr}$$

Moon

$$t_{2stage} = 29.5 \text{ Myr}$$

Mars

$$t_{2stage} = 3.3/9.7 \text{ Myr}$$

Iron meteorites

$$t_{2stage} < 2.8 \text{ Myr}$$

Asteroid *Vesta*

- Eucrites (“easily discerned”)
- Vesta’s mantle $t_{2stage} = 2.6 \text{ Myr}$

