EOSC 450
Homework 4
Due Weds 11/23

1. In HW3 problem 3 you were faced with taking the Fourier transform of $\phi = 1/r$, where $r^2 = x^2 + y^2 + z^2$. Using a 2D Fourier transform, which removed the dependence on x and y, you found an upward/downward continuation kernel $\phi(\mathbf{k},z)=e^{-2\pi|\mathbf{k}|z}$, where the wavenumber vector $\mathbf{k} = k_x + k_y$. Physically, this parameter relates the potential measured at $z=0$ (a datum, such as sea level or the ground surface) to the potential measured at any altitude z.

Consider now that $\phi(\mathbf{k},z)$ is the disturbing potential arising as a result of an anomaly in the gravity field. Assume that z=0 corresponds with the sea surface and that the measurements are being made again with the GEOSAT satellite at 800 km altitude. Take $\phi_o(\mathbf{k},0)$ to be the potential at z=0. Rewriting the upward continution in terms of the wavelength of the anomaly, $\boldsymbol{\lambda}$, $\phi(\mathbf{k},z)= \phi_o e^{-2\pi z / |\lambda|}$, it is apparent the ratio (z / $|\lambda|$) governs the resolution of the measurements. Explain this comment with pictures and words.

Load geosatd.dat (Download it from the course website if you have deleted it--http://www.eos.ubc.ca/~mjelline/EOSC450%20fall%202005.htm). Use the upward continuation kernel to determine and plot *BOTH* the Fourier transform of the gravity anomaly profile *AND* the gravity anomaly profile itself as observed at 1 km altitude, at 10 km altitude and at 100 km altitude. Now plot it at –10 km and –1000 km depth from sea level. Remember that $\Delta g_{FA} = -\partial/\partial z(\phi(\mathbf{k},z))$.

2. If the lithosphere responds as a thin linearly elastic plate when it is subjected to topographic loads we can find a linear relationship between the gravity anomaly and the topography in the wavenumber domain using Fourier methods we have developed in class. This is the gravity/topography transfer function or admittance:

$$H_a(|k|) = 2\pi G(\rho_c - \rho_w) e^{-2\pi|k|s}\left[1 - e^{-2\pi|k|d}\left\{1 + \frac{D(2\pi|k|)^4}{g(\rho_m - \rho_c)}\right\}^{-1}\right],$$

where the flexural rigidity $D = \left(Eh^3/12(1-\nu^2)\right)$ has units of [N L].

G	-	gravitational constant	6.67×10^{-11} m^3 kg^{-1} s^{-2}
g	-	acceleration of gravity	9.82 m s^{-2}
ρ_w	-	seawater density	1025 kg m^{-3}
ρ_c	-	crustal density	2800 kg m^{-3}
ρ_m	-	mantle density	3330 kg m^{-3}
s	-	mean water depth	(average of topography, must be positive)
d	-	crustal thickness	6000 m
h	-	elastic plate thickness	(0 - 50,000 m)
E	-	Young's modulus	6.5×10^{10} N m^{-2}
ν	-	Poisson's ratio	0.25

a) Plot the admittance function for wave numbers ranging from 0 to 10^{-5} m^{-1} using elastic thicknesses, h, of 0 and 30,000 m. Assume that s = 4000 m. Why does this transfer function approach 0 at high wavenumbers? Why does it approach 0 at low wavenumbers?

b) Use $H_a(|k|)$, h=0, and the topography from the oahu.dat file to calculate a gravity anomaly profile. The procedure is to take the Fourier transform of the topography, multiply it by $H_a(|k|)$ and inverse Fourier transform the result. *NOTE: It will be important to window the data to avoid Gibbs phenomena. Below is a script to help you get going.*

c) Compare this model gravity profile with the observed gravity profile.
- Increase the elastic thickness h until the model gravity matches (visually) the observed gravity profile.
- Plot the transfer function corresponding to this thickness on your plot from part (a).
- How does your estimate of h compare with that which is determined by Watts (1978) (Dwonload the paper from the course website)?
- How old was the lithosphere when this seamount formed. *Hint:* To answer this, see Turcotte and Schubert, *Geodynamics* and learn about the variation of oceanic lithopshere thickness with age.

3. The elastic thickness, h, of the ice shell on Europa is uncertain. One way to estimate h is from the wavelength of topography arising in response to topographic loads applied at the surface.
a) Load the file called NimmoStack.dat (Nimmo, 2003) and plot the profile. This profile is the average of the 6 profiles across a plateau SW of the Cilix impact crater indicated below:

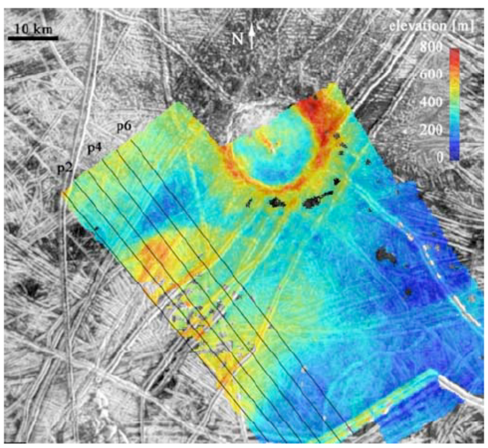
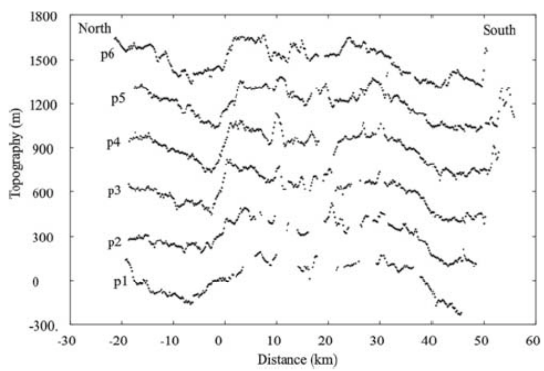


Figure 1. Galileo image of Cilix crater and surroundings with superimposed stereo-derived topography (color). Background image resolution 120 m/pixel. Topography horizontal resolution 500 m, vertical resolution 30–60 m.



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b) In problem 4 of HW 3 you found a transfer function relating the (output) deflection of a thin elastic beam to any (input) load. This is called a flexural response function. Assuming that the ice shell of Europa can be modeled as a thin elastic shell the flexural response function relating the magnitude of the deflection to any load is approximately:

$$\Phi_e(k) = \left[\frac{Dk^4}{(\rho_w - \rho_c)g} + 1 \right]^{-1},$$

where g = 1.3 m s^{-2} , the density of the salt water beneath the ice shell, ρ_w is taken to be 950 kg m^{-3} and the density of the ice crust ρ_c = 950 kg m^{-3}. The flexural rigidity D is defined above. Here, Young's modulus, E = 1 GPa, and Poisson's ratio, ν = 0.3. Plot this function for h = 0, 100 m, 500 m, 1000 m, 3000 m and 6000 m. Discuss your results. Compare the amplitude filtering characteristics of each flexural response function. If you apply the parameters in the table above (now for rocks and not ice) how will the results change? Why?

c) The topographic response to the plateau across which the profiles are taken depends on the shape and magnitude of the loading. Assuming that the plateau has a density that is the same as the ice crust, ρ_c = 950 kg m^{-3}, a height H and length determined from the profiles, and that $g = 1.3$ m s^{-2} find the topographic response to the following situations*:

- A plateau of height H with a sinusoidal shape (half a full wavelength).
- A plateau with a boxcar shape.
- A plateau with a trapezoidal shape.

*To obtain the topographic response you will have to Fourier transform the input load, multiply your spectra by the flexural response function and then inverse Fourier transform the result.