EOSC 450 Problem Set 1: A Math Review (Due: Wednesday, September 21)

1. Vectors...

1) Find vector $\bar{c} = \bar{a} \times \bar{b}$ where $\bar{a} = (0, 2, 5)$ $\bar{b} = (2, -4, 0)$
Sketch what you have done. What physical measure does the cross product give?

Now take the dot product of the same two vectors. What physical measure does the dot product give?

2) The expression
$\bar{a} \times (\bar{b} \times \bar{c})$
is the "vector triple product" and is something we run into a lot.

a) Prove: $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$

b) Prove: $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$

c) Is $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$ (show your work) ?

3) If $\bar{f}$ is a vector function with continuous second derivatives show that

$\nabla \times (\nabla \times \bar{f}) = \nabla(\nabla \cdot \bar{f}) - \nabla^2 \bar{f}$

4) Remind yourself and me whether the gradient, divergence and curl of a vector returns a scalar or a vector. What do div, grad and curl mean physically? What are the math definitions? Use sketches to help.

5) Show that the vector A$\mathbf{i}$ + B$\mathbf{j}$ + C$\mathbf{k}$ is normal to the plane whose equation is $Ax + By + Cz = D$. Hint: Show that it is perpendicular to the vector joining any two points $(x1, y1, z1)$ and (x2,y2,z2) lying in the plane.

2. Fourier Series and MATLAB

1) Find the Fourier series of the function $f(x) = e^x$ on $-\pi < x < \pi$. Use MATLAB to plot the function and the Fourier series representation using the number of terms that gives a "nice" fit. Start with one term. Show each choice. How do you decide on "nice".

2) Do the same thing for the function $f(x) = |x|$ on the same interval.

3) Once again for a square wave defined by $f(x) = -1$ when $x\epsilon(-\pi, 0)$ and $f(x) = 1$ when $x\epsilon(0, \pi)$.

4) Do the terms in a Fourier series form an orthogonal basis? Why? What is an orthogonal function and why is this property particularly useful when dealing with Fourier coefficients?

3. Heatflow, Laplace's equation, separation of variables and Fourier series

A sheet of metal coincides with the square in the xy plane whose vertices are the points (0,0), (1,0), (1,1) and (0,1). The two (upper and lower) faces of the sheet are insulated. The sheet is sufficiently thin that heat flow within it may be regarded as two dimensional. Said differently, a 2D approximation is permitted because the thickness of the sheet, $h << 1$. The edges parallel to the x-axis are insulated. The left-hand edge is maintained at the constant temperature 0. If the temperature distribution $T(1, y) = f(y)$ is maintained along the right hand edge, find the steady-state temperature distribution throughout the sheet. Plot your solution with MATLAB.

Hints:
1) Steady-state implies that there is no time-dependence in this problem.
2) Use the technique of "Separation of Variables" to solve this linear PDE.