## Some more problems related to gravity and moments of inertia

### 1. Planetary Density Structure

Suppose you have a large supply of "planet stuff" and have bid on a contract to build copies of the bodies listed below.

| Parameter | Mercury | Venus | Earth | Mars | Moon | Io | Europa | Ganymede | Callisto |
|---|---|---|---|---|---|---|---|---|---|
| Radius (km) | 2438 | 6051 | 6371 | 3390 | 1737 | 1821 | 1560 | 2634 | 2400 |
| Density (kg $m^{-3}$) | 5430 | 5243 | 5515 | 3934 | 3344 | 3530 | 3020 | 1940 | 1850 |
| Polar moment / $MR^2$ | ????? | ???? | 0.331 | 0.365 | 0.394 | 0.378 | 0.346 | 0.311 | 0.359 |
| rotation period (days) | 58.65 | -243.0 | 1.00 | 1.026 | 27.32 | 1.769 | 3.551 | 7.155 | 16.689 |

Construction grade planet stuff comes in 3 varieties with mean densities of 1000 kg $m^{-3}$, 3000 kg $m^{-3}$, and 8000 kg $m^{-3}$

(a) Suppose that, for preliminary evaluation purposes, you are only required to match the radius and mass (mean density) of each body.

(b) Suppose further that the middle grade stuff is in short supply and you must make do with only the light and heavy grades.

1. what are the radii of the dense inner regions of these bodies?
2. what mass fraction is the dense material?
3. what are the moments of inertia of these bodies?
4. How do your calculated moments of inertia differ from the real moments of inertia of these planetary bodies? Try to explain qualitatively why these differences occur. (Hint: think about real planetary compositions and density distributions versus your "synthetic" planets).

(c) Now attempt to match the density AND moment of inertia values by using all three grades of stuff. You do not need to solve for the radii of the inner regions of the bodies explicitly but you should derive two equations that are polynomials in $r_1$ and $r_2$ (the radii of the innermost and middle shells respectively). Note we cannot explicitly solve for $r_1$ and $r_2$ analytically from these equations.

(d) Finally calculate the surface gravity of the bodies you "designed" in (a). Assume your planets have a perfectly spherical shape.

## 2. Isostatic Compensation

(a) For planets with sufficiently thick crusts it is often a better approximation to distinguish between two crustal layers – an upper and lower crust. If the lower crust has a constant thickness $b_l$ and a density $\rho_{cl}$, and the upper crust has variable thickness with a density $\rho_{cu}$, determine the geoid anomaly associated with isostatically compensated positive topography. (Note that by prescribing constant densities we are investigating an Airy isostacy problem). Use the notation in the class notes for other parameters (topography, crustal thickness etc)

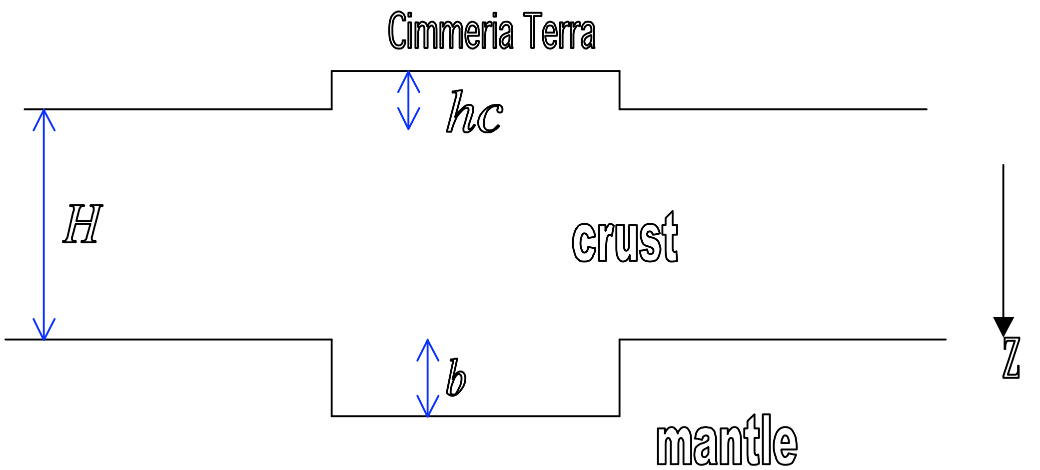
(b) To a first order approximation in topography, $h$, determine the geoid-to-topography ratio, GTR in terms of the depth of compensation, $H$.

(c) For Venus we have the planet-wide estimates of topography and geoid anomaly. These quantities for various significant features on Venus are given below. Using the one-layer crustal model derived in class compute the depth of compensation, H, assuming an Airy isostacy model for all of these features. You will see that an Airy isostacy model is likely not an appropriate model for some of these features – which ones and why? You can assume a crustal density for Venus of 2800 kg $m^{-3}$, and a mantle density of 3300 kg $m^{-3}$. You will need to use your estimate for the mean surface gravity derived in problem 1 (this should be around 8.7 $ms^{-2}$)

| *Feature* | *Geoid Anomaly (m)* | *Topography (km)* |
|---|---|---|
| Ovda Regio | 30 | 4 |
| Beta Regio | 180 | 5 |
| Guinevere Planitia | -50 | - 2 |
| Tellus Regio | 20 | 3 |

### 3) Crustal thickness variations on Mars

In the Cimmeria Terra region assume that the positive topography is supported by a crustal root: i.e., thickened crust. Using the following figure and notation to set up the force balance calculation. Use $\rho_c$ and $\rho_m$ to denote crust and mantle densities respectively.




(a) Derive the expression for the crustal thickening in terms of elevation and crust and mantle densities. Show your work, not just the answer.

(b) Calculate the total crustal thickness in the Cimmeria Terra region. Use the values below for crustal density ($\rho_c$), mantle density ($\rho_m$) and mean crustal thickness (H). You will need to estimate the mean elevation (h) of the region (eyeball it) from the top figure in Figure 2.

$\rho_c$ = 2900 kg m$^{-3}$
$\rho_m$ = 3500 kg m$^{-3}$
H = 50 km

(c) How does your calculation compare with the crustal thickness shown in the lower figure for the Cimmeria Terra region? Give the value or approximate range in crustal thickness from the map for this region.

(d) You will see that in the lower figure of Figure 2, that there is a change in crustal thickness between the northern and southern hemispheres. How well does this follow the boundary shown by the red line (for example: everywhere, everywhere except around "x", very few places, nowhere)? Use the location and feature names in Figure 1 to help you in your answer.

**4) Olympus Mons gravity anomaly. Uncompensated topography?**

The lowermost figure in Figure 2 in the Zuber et al. Paper assumes that the free air anomaly everywhere is due to variations in crustal thickness. In reality some features are not isostatically compensated. Short-wavelength topography may be supported by the strength of the lithosphere.

(a) Assume that the lithosphere beneath Tharsis is very strong and that the load on the lithosphere due to Olympus Mons load is uncompensated. Calculate the Bouguer correction by assuming the infinite sheet approximation holds and that the topography is 21 km (the maximum elevation of Olympus Mons). Use the following values in the Bouguer correction equation:

$G = 6.67 \times 10^{-11}\ m^3\ s^{-2}\ kg^{-1}$
$\rho_c = 2900\ kg\ m^{-3}$

(b) How does your Bouguer correction compare with the free air anomaly? (Write down your estimate of the free air anomaly over Olympus Mons from the center figure).