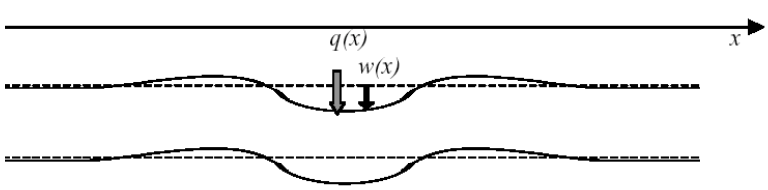
EOSC 450 HW#3
More experience with Fourier Transforms…
Due in class for discussion Weds 10/19

1) Download the file called geosatd.dat from the course website. Also download the .m file that is meant to help you get started. This dataset has 6 columns: time, latitude, longitude, geoid height (meters), free-air gravity anomaly (milligals), and the uncertainty in the gravity anomaly. We have not talked about the geoid yet explicitly. The geoid is an equipotential surface (for real!) with a datum that is sea level. Anomalies are measured in terms of height (i.e changes in gravitational potential energy) relative to sea level. On a map, locate where this profile has been taken. Make 2 plots on one page that show gravity anomaly and geoid anomaly as a function of latitude.
2) Take the derivative of the geoid anomaly using the first difference algorithm in Matlab (use help <diff> if you need it at the command line) and the definition of the derivative. Plot the result as a function of latitude. The Fourier transform of the derivative of a function is $i2\pi k$ times the Fourier transform of the original function (property 5 in the review notes). k is the linear wavenumber (units of 1/wavelength). Show that this relationship also holds for a discrete time series. Compute and plot on the same page the derivative by multiplication in the Fourier domain. Apply a phase shift to the FFT so it will be aligned with the first difference derivative. In a third plot on the same page show the difference between the two calculated derivatives. Discuss your results.
3) Our work so far has shown that a gravitational field can be represented as being proportional to grad(ϕ), where $\phi=1/r$ and $r^2 = x^2 + y^2 + z^2$. Find the 2D Fourier Transform of ϕ. Leave your result in the wave number domain, $\phi(\mathbf{k},z)$. Let's assume that the Geosat spacecraft measured gravity at an altitude of 800 km. What would the power spectrum for the gravity anomaly look like if the data were measured 200 km above the Earth's surface (i.e. from 600 km altitude)? How about 1000 km below the Earth's surface?
4) An important problem in understanding the geodynamics of the Earth and other planets are the support mechanisms for topographically interesting features (e.g. mountains, volcanoes, trenches….). One issue is the extent to which such features are supported as a result of the elastic properties of planetary lithospheres. One way to analyze this problem is to look at the deformation around the feature. That is, the wavelength of the bending in the lithosphere can be diagnostic of the thickness of the elastic part of the lithosphere. We will look at this in some detail quite soon. As an appetizer, consider the load of a seamount on the Pacific plate. The simplest description of the problem is the deflection, w(x), of an infinitely long rigid "beam" of lithosphere with elastic thickness, d, in response to a point load, q(x). The elastic thickness is *less than* the total oceanic lithospheric thickness h. Why is this (recall the "brittle ductile transition" you likely heard about in a class somewhere…)? Why is it ok to assume an infinite length and a point load here?



The Model:

$$Dw^{iv}(x) + \Delta\rho g w = q(x)$$

with the given approximations becomes:

$$Dw^{iv}(x) + (\rho_{mantle} - \rho_{water}) g w(x) = V_o \delta(x)$$

where

the 1st term is the resistance to bending

the 2nd is the stabilizing buoyancy force (restoring force)

the 3rd term is the point force.

D is the flexural rigidity ($\propto$ E, elastic modulus (10^{10} Pa), and $\propto$ d^3) [N m]

V_o is an applied stress [N/m^2]

Let's simplify the equation a bit further to bring out the physics of what is going on. Divide through by V_o. Next, let's take advantage of an approximately constant lithospheric thickness h = 200 km. This way we can scale all our deflections to this thickness. To do this, let w' = w/h and x'=x/h, where the primed variables are dimensionless (scaled) lengths. Substitute these expressions for w and x and apply the chain rule to get a new 4th order ode:

$$\left(D/V_o h^3\right) w'^{(iv)}(x) + \left(\frac{(\rho_{mantle} - \rho_{water})gh}{V_o}\right) w'(x) = \delta(x)$$

This procedure is called non-dimensionalization and scaling. Notice that a consequence of what we have done is the terms in parentheses indicate explicit relationships between a driving and a retarding stress: (Flexural strength / Imposed stress) and (Restoring buoyancy / imposed stress). This procedure allows you to relate the main driving/retarding forces to individual terms in the ode. The main thing to pay attention to is whether ($D / V_o h^3$) is large or small compared to ($\Delta\rho gh / V_o$).

Take the Fourier transform of both sides of this equation and find the transfer function relating the deflection response to the input force as a function of wavenumber. Plot *the magnitude or modulus* of the transfer function as a function of wavenumber (kh) for arbitrary (V_o), ($\Delta\rho gh$) and ($D/h^3 \approx Ed^3/h^3$), conditions. Note that d is usually less than or equal to about 1/2h. $\Delta\rho$ is around 3000. Now vary d between, say, 1 km, 10 km, 50 km and 100 km. What do you learn? Is the amplitude of the flexural response the same for all wavelengths as you increase or decrease d? Asked differently, describe the amplitude filtering characteristics of your solutions for different d. For each d, for example, describe the situation when the wavelength of the deflection is much smaller than, comparable to, or larger than h.