Shear Wave Velocities in Metamorphic Rocks at Pressures to 10 Kilobars[1]

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Shear wave velocities are reported at pressures to 10 kb for a variety of metamorphic rocks. The velocities were found to vary with propagation and displacement directions. Anisotropies at high pressures correlate with preferred mineral orientation. Calculated shear velocities from modal analyses are within a few per cent of the mean observed velocities. Elastic constants calculated from the shear wave velocities and previously reported compressional wave velocities are given for isotropic elastic theory. It is suggested that the elastic properties of micaceous rocks are similar to materials with hexagonal symmetry.

INTRODUCTION

In the present paper shear velocities are reported for a number of metamorphic rocks. In addition to presenting new data on the elasticity of rocks which may be important constituents of the crust, the measurements permit conclusions to be reached regarding the effects of mineral orientation and mineral composition on the velocity of shear waves. The shear velocities coupled with previously reported compressional wave velocities [*Christensen*, 1965] allow the calculation of seismologically important elastic constants of metamorphic rocks at pressures to 10 kb.

The velocities are obtained by measuring the transit time of plane shear waves in cylinders of rocks. The experimental details and accuracy of this method are discussed by *Simmons* [1964]. The shear waves are generated and received by ac-cut quartz transducers with natural resonance frequencies of 3 Mc/s. Copper foil, 0.013 cm in thickness, cemented to one side of the quartz transducers with silver conducting epoxy considerably extends the life of the fragile quartz disks. The sides of the transducers containing the copper foil are placed in contact with the electrodes. A thin coating of Dow resin 276-V9 is used as a coupling medium between the rock specimen and transducers.

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DATA

Shear velocities for the metamorphic rocks are given in Table 1. Descriptions of the rocks along with compressional wave velocities are given by *Christensen* [1965]. The rock samples are identical with those used in compressional wave velocity measurements.

The directional dependence of shear velocity has been examined in detail. Whereas anisotropy of compressional wave velocities is related only to the direction of propagation, an additional variable, the displacement direction, is also important in the measurement of shear velocities. Thus in Table 1 velocities are reported for three propagation directions oriented at right angles to one another along with various displacement directions which depend upon the symmetry of the rock.

Velocities recorded with increasing pressure are usually slightly lower than those obtained with decreasing pressure. This difference is only a few parts per thousand at pressures above 4 kb but increases significantly at low pressures. As with compressional wave velocities, the shear velocity hysteresis is high in the schist specimens, low in the slate, quartzite, and feldspathic mica quartzite, and intermediate in the gneiss and amphibolite specimens. *Birch and Bancroft* [1940], using resonance torsional vibrations of long cylindrical samples to measure shear velocities, also found the largest hysteresis to be characteristic of schist specimens. The velocities reported in Table 1 are mean values at fixed pressures.

TABLE 1. Shear Wave Velocities (km/sec) in Metamorphic Rocks
X, *Y*, Propagation or displacement parallel to foliation, banding, or cleavage.
Z, Propagation or displacement normal to foliation, banding, or cleavage.
(First letter signifies propagation direction; second, displacement direction.)

Rock	Orientation	Pressure, kb										
		0.1	0.2	0.4	0.6	0.8	1.0	2.0	4.0	6.0	8.0	10.0
Garnet schist	*XZ*	2.6	2.7	2.9	2.92	2.94	2.96	3.02	3.06	3.09	3.14	3.18
Thomaston, Conn.	*XY*	3.4	3.6	3.7	3.71	3.75	3.77	3.82	3.85	3.88	3.90	3.91
	YZ	2.8	2.9	3.0	3.08	3.10	3.12	3.14	3.20	3.25	3.28	3.30
	YX	3.7	3.7	3.8	3.81	3.85	3.86	3.88	3.91	3.92	3.94	3.96
	ZX, *ZY*	2.6	2.8	2.9	2.94	2.98	3.01	3.04	3.07	3.11	3.14	3.17
Mean		3.0	3.1	3.3	3.29	3.32	3.35	3.38	3.42	3.45	3.48	3.51
Staurolite-garnet schist	*XZ*	2.5	2.6	2.7	2.75	2.77	2.79	2.83	2.89	2.95	2.99	3.04
Litchfield, Conn.	*XY*	3.8	3.9	3.9	3.97	3.99	4.00	4.04	4.08	4.12	4.14	4.17
	YZ	2.6	2.8	2.9	2.92	2.93	2.95	3.00	3.02	3.05	3.08	3.10
	YX	3.5	3.7	3.8	3.83	3.88	3.90	3.95	3.98	4.00	4.03	4.04
	ZX, *ZY*	2.5	2.6	2.8	2.89	2.91	2.93	2.98	3.06	3.09	3.16	3.21
Mean		3.0	3.1	3.2	3.27	3.29	3.32	3.36	3.41	3.44	3.48	3.51
Kyanite schist 1	*XZ*	2.6	2.7	2.8	2.95	3.05	3.07	3.09	3.15	3.21	3.27	3.32
Torrington, Conn.	*XY*	2.9	3.1	3.3	3.28	3.31	3.32	3.40	3.48	3.51	3.56	3.60
	YZ	2.7	2.8	2.8	2.91	2.96	3.00	3.13	3.17	3.26	3.32	3.38
	YX	3.1	3.3	3.3	3.30	3.33	3.35	3.42	3.52	3.55	3.57	3.59
	ZX, *ZY*	2.6	2.8	2.9	3.01	3.05	3.08	3.15	3.23	3.30	3.36	3.40
Mean		2.8	2.9	3.0	3.09	3.14	3.16	3.24	3.31	3.37	3.42	3.46
Gneiss 2	*XZ*	2.4	2.5	2.7	2.78	2.85	2.92	3.23	3.30	3.39	3.44	3.48
Torrington, Conn.	*XY*	2.8	3.0	3.2	3.31	3.37	3.42	3.53	3.57	3.59	3.61	3.62
	YZ	2.5	2.7	2.9	3.06	3.16	3.22	3.35	3.38	3.40	3.43	3.45
	YX	2.8	3.0	3.2	3.29	3.37	3.39	3.49	3.53	3.56	3.59	3.61
	ZX, *ZY*	2.8	3.0	3.0	3.24	3.27	3.35	3.39	3.42	3.43	3.45	3.46
Mean		2.7	2.8	3.0	3.13	3.21	3.26	3.40	3.44	3.47	3.51	3.53
Gneiss 4	*XZ*	2.9	3.1	3.2	3.25	3.30	3.32	3.35	3.38	3.39	3.46	3.47
Torrington, Conn.	*XY*	2.8	3.0	3.2	3.29	3.34	3.37	3.45	3.52	3.60	3.67	3.70
	YZ	3.3	3.3	3.4	3.37	3.38	3.39	3.41	3.43	3.49	3.52	3.54
	YX	3.3	3.4	3.5	3.57	3.60	3.63	3.66	3.72	3.77	3.82	3.83
	ZX, *ZY*	2.7	2.9	3.1	3.29	3.35	3.40	3.49	3.55	3.58	3.60	3.60
Mean		3.0	3.1	3.3	3.35	3.39	3.42	3.47	3.52	3.57	3.61	3.63
Gneiss 5	*XZ*	3.1	3.2	3.3	3.29	3.32	3.34	3.36	3.39	3.46	3.52	3.57
Torrington, Conn.	*XY*	3.4	3.5	3.5	3.58	3.62	3.65	3.68	3.71	3.73	3.76	3.78
	YZ	2.7	2.8	2.9	3.00	3.07	3.12	3.20	3.33	3.38	3.45	3.50
	YX	3.0	3.2	3.3	3.35	3.41	3.45	3.51	3.55	3.60	3.62	3.68
	ZX, *ZY*	3.1	3.2	3.3	3.33	3.36	3.37	3.41	3.45	3.48	3.49	3.52
Mean		3.1	3.2	3.3	3.31	3.36	3.39	3.43	3.48	3.53	3.57	3.61
Gneiss 6	*XZ*	2.8	2.9	2.9	2.99	3.04	3.12	3.22	3.35	3.40	3.44	3.47
Goshen, Conn.	*XY*	3.6	3.7	3.7	3.76	3.80	3.84	3.92	3.99	4.05	4.08	4.11
	YZ	2.8	2.9	3.0	3.11	3.15	3.21	3.33	3.41	3.45	3.48	3.51
	YX	3.4	3.5	3.6	3.64	3.70	3.73	3.85	3.95	3.98	4.01	4.03
	ZX, *ZY*	2.6	2.8	2.9	2.96	3.03	3.11	3.22	3.36	3.41	3.46	3.50
Mean		3.0	3.2	3.2	3.29	3.35	3.40	3.51	3.61	3.66	3.69	3.72
Quartzite	*X*	3.5	3.7	3.8	3.87	3.88	3.89	3.95	3.99	4.01	4.04	4.07
Clarendon Springs,	*Y*	3.7	3.8	3.8	3.90	3.92	3.94	3.99	4.02	4.03	4.03	4.04
Vermont	*Z*	3.6	3.7	3.8	3.92	3.95	3.97	4.01	4.05	4.07	4.08	4.09
Mean		3.6	3.7	3.8	3.90	3.92	3.94	3.98	4.02	4.04	4.05	4.07
Feldspathic mica	*XZ*	3.5	3.6	3.6	3.58	3.58	3.59	3.61	3.66	3.69	3.71	3.73
quartzite	*XY*	3.6	3.6	3.7	3.73	3.74	3.75	3.78	3.81	3.84	3.85	3.87
Thomaston, Conn.	*YZ*	3.5	3.6	3.6	3.66	3.67	3.68	3.71	3.75	3.78	3.79	3.81
	YX	3.6	3.8	3.8	3.85	3.88	3.89	3.92	3.96	3.98	3.99	4.01
	ZX, *ZY*	3.4	3.5	3.5	3.54	3.56	3.59	3.62	3.64	3.66	3.68	3.70
Mean		3.5	3.6	3.6	3.67	3.69	3.70	3.73	3.76	3.79	3.81	3.83

TABLE 1. (Continued)

Rock	Orientation	0.1	0.2	0.4	0.6	0.8	1.0	2.0	4.0	6.0	8.0	10.0
		Pressure, kb										
Slate	*XZ*	2.7	2.7	2.7	2.74	2.75	2.78	2.85	2.93	2.97	3.03	3.09
Poultney, Vermont	*XY*	3.8	3.9	3.9	3.89	3.90	3.92	3.93	3.98	4.02	4.05	4.07
	YZ	2.7	2.8	2.8	2.82	2.84	2.85	2.87	2.98	3.05	3.09	3.11
	YX	3.8	3.9	3.9	3.90	3.91	3.93	3.95	4.01	4.05	4.08	4.10
	ZX, ZY	2.7	2.7	2.7	2.77	2.79	2.80	2.83	2.92	3.00	3.05	3.08
Mean		3.1	3.2	3.2	3.22	3.24	3.26	3.29	3.36	3.42	3.46	3.49
Amphibolite 2	*XY*	3.5	3.7	3.8	3.98	4.05	4.09	4.25	4.33	4.34	4.35	4.37
Bantam, Conn.	*XZ*	2.7	3.0	3.3	3.41	3.51	3.54	3.68	3.75	3.78	3.80	3.83
	YX	3.8	3.9	4.0	4.07	4.11	4.13	4.24	4.29	4.32	4.33	4.36
	YZ	3.1	3.3	3.5	3.58	3.62	3.66	3.74	3.82	3.85	3.88	3.90
	ZX	2.9	3.1	3.3	3.44	3.50	3.55	3.68	3.75	3.78	3.81	3.85
	ZY	3.3	3.4	3.5	3.58	3.63	3.73	3.75	3.80	3.83	3.86	3.89
Mean		3.2	3.4	3.6	3.67	3.74	3.78	3.89	3.96	3.98	4.01	4.03

Anisotropy

Shear velocity differences expressed as a percentage of the mean velocity are given in Table 2. The large anisotropies at low pressure are related to porosity. At pressures above a few kilobars most of the pore spaces in the rocks have been eliminated, and the anisotropies are primarily a consequence of preferred mineral orientation. Thus the anisotropies at 4 and 10 kb for a given rock are similar.

The data in Tables 1 and 2 allow certain generalizations to be made regarding shear anisotropies in the micaceous rocks. Low velocities are characteristic of propagation normal to the schistosity, cleavage, or banding. This propagation direction corresponds to the maximum concentration of normals to the {001} planes of mica. The velocities do not vary significantly with displacement direction for propagation normal to the planar elements; therefore only one velocity is reported in Table 1 for orientations *ZX* and *ZY*. For propagation parallel to the schistosity, cleavage, or banding, however, the displacement direction is critical. A displacement direction normal to the planar elements produces a relatively low velocity which is similar in magnitude to velocities

TABLE 2. Shear Velocity Anisotropy in Metamorphic Rocks

Rock	P = 0.1 kb			P = 4.0 kb			P = 10.0 kb		
	V_s, km/sec		Diff., % of mean	V_s, km/sec		Diff., % of mean	V_s, km/sec		Diff., % of mean
	Highest	Lowest		Highest	Lowest		Highest	Lowest	
Garnet schist	3.7	2.6	34	3.91	3.06	24	3.96	3.17	22
Kyanite schist 1	3.1	2.6	17	3.52	3.15	11	3.60	3.32	8
Staurolite garnet schist	3.8	2.5	40	4.08	2.89	34	4.17	3.04	31
Gneiss 2	2.8	2.5	15	3.57	3.30	8	3.62	3.45	5
Gneiss 4	3.3	2.7	20	3.72	3.38	10	3.83	3.47	10
Gneiss 5	3.4	2.7	23	3.71	3.33	11	3.78	3.50	8
Gneiss 6	3.6	2.6	32	3.99	3.35	17	4.11	3.47	17
Quartzite	3.7	3.5	6	4.05	3.99	1	4.09	4.04	1
Feldspathic mica quartzite	3.6	3.4	6	3.98	3.66	8	4.01	3.70	8
Slate	3.8	2.7	33	4.01	2.92	31	4.10	3.09	28
Amphibolite 2	3.8	2.7	33	4.33	3.75	14	4.37	3.83	13

TABLE 3. Shear Wave Velocities in Mica [*Alexandrov and Ryzhova*, 1961*b*] (km/sec)

Propagation Direction	Displacement Direction	Muscovite	Biotite
001	100	2.03	1.38
001	010	2.05	1.38
010	001	2.06	1.34
010	100	5.01	5.06
100	001	2.19	1.40
100	010	4.95	5.00
110	001	2.16	1.29
110	110	4.86	5.08

obtained for propagation normal to the planar elements. On the other hand, high velocities are characteristic of propagation and displacement directions parallel to the planar elements. Thus the highest shear velocities in the metamorphic rocks are obtained for propagation and displacement directions perpendicular to the maximum concentration of normals to the {001} planes in mica. Measurements of velocities of shear waves in mica reported by *Alexandrov and Ryzhova* [1961*b*] are reproduced in Table 3. Their data show that the single-crystal velocities are consistent with the anisotropies of the micaceous rocks.

It is of interest to compare shear anisotropies with compressional wave anisotropies [*Christensen*, 1965, Table 5] at high pressure. For both compressional and shear waves the staurolite-garnet schist, slate, and garnet schist have relatively high anisotropies, whereas gneiss specimens 2, 4, and 5, the feldspathic mica quartzite, and the kyanite schist have relatively low anisotropies. The quartzite is nearly isotropic for both compressional and shear waves. As with compressional wave anisotropies, the shear wave anisotropies correlate with the abundance and orientation of mica in the specimens. Rocks with the greatest percentage of mica and the highest degree of preferred mica orientation have the largest anisotropies.

For many of the micaceous metamorphic rocks the shear wave anisotropies are approximately twice the compressional wave anisotropies. This feature is again related to the elastic properties of the mica single crystals. Both muscovite and biotite have anisotropies of approximately 60% for compressional waves, whereas shear wave anisotropies are 85% and 119% for muscovite and biotite, respectively. Thus a rock with predominantly biotite as its micaceous mineral will have a shear anisotropy approximately twice that of its compressional wave anisotropy. The influence of the lower shear anisotropy of muscovite is evident in a comparison of the anisotropies of the garnet schist and the staurolite-garnet schist. The garnet schist has no muscovite [*Christensen*, 1965, Table 2] and hence has a shear wave anisotropy (22%) that is approximately twice that of its compressional wave anisotropy (10%). The staurolite-garnet schist, with 13% muscovite and 23% biotite, has a shear wave anisotropy of 31% compared with a compressional wave anisotropy of 21%.

The shear anisotropy in the amphibolite is a consequence of preferred orientation of hornblende and the elastic anisotropy of hornblende single crystals. Measurement of shear wave velocities by *Alexandrov and Ryzhova* [1961*a*] in single crystals of hornblende are given in

TABLE 4. Shear Wave Velocities in Amphibolite and Single Crystals of Hornblende (km/sec)

Propagation Direction	Displacement Direction	Hornblende I[a]	Hornblende II[a]	Amphibolite 2[b]
c axis	*b* axis	4.29	4.40	4.37
c axis	{100}	3.16	3.03	3.83
b axis	*c* axis	4.52	4.45	4.36
b axis	{100}	3.53	3.72	3.90
{100}	*c* axis	3.18	3.46	3.85
{100}	*b* axis	3.43	3.78	3.89

[a] *Alexandrov and Ryzhova* [1961*a*.]
[b] Velocities correspond to propagation and displacement directions parallel to maximum concentrations of hornblende directions at 10 kb.

TABLE 5. Calculated Shear Velocities in Aggregates [*Simmons*, 1965]

Mineral	V_s, km/sec Voigt	Reuss	Mean
Augite	4.25	4.11	4.18
Hornblende II	4.03	3.60	3.82
Microcline[a]	3.58	3.09	3.34
Oligoclase[b]	3.48	2.98	3.23
Labradorite[c]	3.70	3.40	3.55
Olivine[d]	4.98	4.89	4.94
Biotite	3.73	2.02	2.88
Muscovite	3.84	2.82	3.33
Quartz	4.21	3.94	4.08
Garnet	4.77	4.77	4.77
Staurolite	4.79	4.53	4.66
Magnetite	4.20	4.20	4.20

[a] $Or_{78.5}Ab_{19.4}An_{2.1}$.
[b] An_{15-16}.
[c] An_{57-60}.
[d] Fo_{92}.

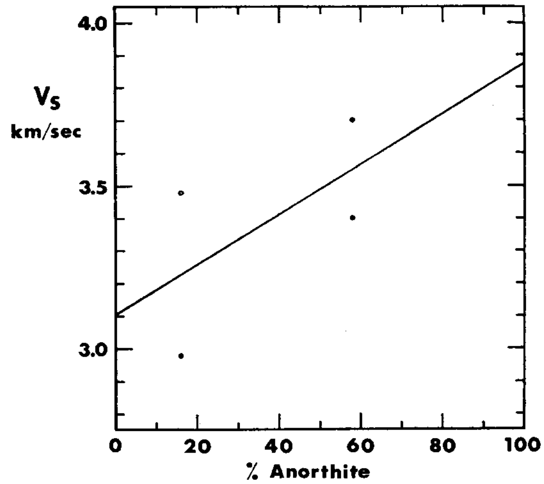

Fig. 1. Shear velocity versus plagioclase composition calculated from aggregate theory. Circles represent Voigt and Reuss values.

Table 4 along with the corresponding 10-kb velocities in amphibolite 2. A petrofabric diagram illustrating the relationships of the hornblende orientation in amphibolite 2 to measured directions of velocity is given elsewhere [*Christensen*, 1965, Figure 8]. For a given propagation direction in amphibolite the dependence of velocity on displacement direction is consistent with the relative velocities in single crystals of hornblende (Table 4). Departures of the rock velocities from those of the single hornblende crystals are presumably related to the abundant plagioclase in the amphibolite, the lack of perfect crystallographic orientation of hornblende, and the variability of chemical composition of hornblende.

Variations in composition among specimens cut from the same rock may be partially responsible for the observed velocity differences. This effect is probably significant only in the kyanite schist, which contains relatively large kyanite crystals, and the more coarsely banded gneiss specimens. Conceivably some anisotropy may also be related to the shape of the mineral components.

Mineral Composition

Elastic constants for most of the common rock-forming minerals have recently been determined. Most of the minerals are highly anisotropic with respect to elasticity, but it is possible to estimate mean values. Velocities for a randomly oriented aggregate are obtained from single-crystal data by averaging the elastic constants (c_{ij}) [*Voigt*, 1928] and the elastic compliances (s_{ij}) [*Reuss*, 1929] over all directions. Velocities calculated from the Voigt and Reuss averages are upper and lower limits, respectively, of the velocity for an isotropic aggregate [*Hill*, 1952]. The true velocity is presumably near the mean of the extreme estimates.

Voigt and Reuss shear velocities calculated

TABLE 6. Calculated and Observed Shear Velocities in Metamorphic Rocks

Rock	Calculated Velocity, km/sec	Observed Mean Velocity, km/sec 1 kb	2 kb
Gneiss 2	3.4	3.26	3.40
Gneiss 4	3.5	3.42	3.47
Gneiss 5	3.4	3.39	3.43
Gneiss 6	3.5	3.40	3.51
Garnet schist	3.4	3.35	3.38
Staurolite garnet schist	3.6	3.32	3.36
Amphibolite 2	3.7	3.78	3.89
Quartzite	4.0	3.94	3.98
Feldspathic mica Quartzite	3.7	3.70	3.73

by *Simmons* [1965] are given in Table 5 for the more common minerals. A curve of aggregate shear velocity versus composition for the plagioclase feldspar series is given in Figure 1. The curve was constructed from the Voigt and Reuss velocities of the single crystals of oligoclase and labradorite reported by *Alexandrov and Rhyzova* [1962].

Calculated mean velocities (V_c) for the metamorphic rocks were determined (Table 6) from the Voigt-Reuss mineral averages by using the relationship

$$1/V_c = x_1/V_1 + x_2/V_2 + \cdots x_i/V_i$$

where $x_1, x_2, \ldots, x_i$ are the percentages by volume of the minerals in the rocks as determined from modal analyses, and $V_1, V_2, \ldots, V_i$ are Voigt-Reuss shear velocity averages of the minerals. A slightly higher calculated velocity can be obtained by using the relationship

$$V_c = x_1 V_1 + x_2 V_2 + \cdots x_i V_i$$

Calculations by both methods generally agree with the two values which are given in Table 6. The elastic constants of kyanite have not been determined. Also, the fine grain size of the slate does not permit an accurate modal analysis. Therefore the kyanite schist and slate are not included in Table 6.

Since the Voigt-Reuss mineral averages were determined from elastic constants at atmospheric pressure, the calculated velocities should be compared with measured velocities which are high enough to produce solid contact of the minerals. Furthermore, the pressure should not be sufficiently high to give high velocities related to the intrinsic pressure effects on the mineral components. The slow increase in velocity for pressures above 2 kb, coupled with projections of the nearly linear portions of velocity-pressure curves to 1 atm, indicate that pressures in the range of 1 to 2 kb are the most favorable for comparisons of calculated and observed velocities. Referring to Table 6, we see that the calculated shear velocities are within a few per cent of the mean observed velocities for most of the rock specimens. The discrepancy between the calculated and observed mean velocities of the staurolite-garnet schist may be a consequence of the schist's extreme anisotropy and relatively loose texture.

Elastic Constants

It is customary to apply to rocks the elastic parameters developed for isotropic elastic

TABLE 7. Poisson's Ratio from Mean V_p and V_s
$R = V_p/V_s$, $2\sigma = (R^2 - 2)/(R^2 - 1)$

Rock	P (kb) =	1	2	4	6	10
Gneiss 2	R	1.79	1.78	1.80	1.80	1.80
	σ	0.273	0.269	0.277	0.277	0.277
Gneiss 4	R	1.77	1.80	1.82	1.81	1.81
	σ	0.266	0.277	0.284	0.280	0.280
Gneiss 5	R	1.77	1.79	1.80	1.81	1.81
	σ	0.266	0.273	0.277	0.280	0.280
Gneiss 6	R	1.70	1.71	1.72	1.72	1.73
	σ	0.235	0.240	0.245	0.245	0.249
Garnet schist	R	1.85	1.87	1.88	1.88	1.88
	σ	0.294	0.300	0.303	0.303	0.303
Staurolite garnet	R	1.89	1.90	1.91	1.91	1.91
	σ	0.306	0.308	0.311	0.311	0.311
Kyanite schist	R	2.05	2.09	2.13	2.14	2.14
	σ	0.344	0.352	0.360	0.360	0.360
Quartzite	R	1.54	1.54	1.54	1.54	1.55
	σ	0.136	0.136	0.136	0.136	0.144
Feldspathic mica	R	1.65	1.65	1.66	1.66	1.67
quartzite	σ	0.210	0.210	0.215	0.215	0.220
Slate	R	1.82	1.82	1.81	1.80	1.80
	σ	0.284	0.284	0.280	0.277	0.277
Amphibolite 2	R	1.75	1.77	1.78	1.78	1.78
	σ	0.258	0.266	0.269	0.269	0.269

bodies. With the formulas of isotropic elasticity [*Birch,* 1960, p. 2206] any two elastic constants can be used to calculate all others. The elastic constants also exhibit a functional relationship to the velocities of compressional and shear waves and the rock density. *Simmons and Brace* [1965] have recently shown that elastic constants calculated from velocities are in good agreement with static values.

Values of Poisson's ratio (σ) at high pressure are given in Table 7. The mean V_p for three directions has been combined with the mean V_s of the specimens to obtain the calculated values of σ. Since the accuracy of the velocities is 1%, the calculation of σ leads to an accuracy of approximately 8% [*Simmons and Brace,* 1965]. The calculations are more reliable for the quartzite and gneiss specimens with relatively low anisotropy. For the highly anisotropic rocks the data are presumably representative of rocks with similar mineralogy but a random mineral orientation.

The quantity K/ρ and the adiabatic compressibility β are given in Table 8. Corrections have not been applied for the increased density of the specimens at high pressure in the calculation of β. A comparison of the calculated values of β with actual measurements is available for gneiss 2. Linear compressibility measurements by *Brace* [1965] from two cores of gneiss 2 are given in Table 9. The specimens were the same as those used in determining V_p and V_s. The volume compressibility, $\beta = -(1/V_0)\ (dV/dp)$, given in Table 9 is assumed to be the sum of three linear compressibilities, two parallel and one perpendicular to the banding. The calculated compressibility of gneiss 2 is within 5% of that measured by Brace for pressures above 4 kb, which is within the experimental error.

The preceding treatment of elastic properties is strictly valid only for isotropic rocks. Physical measurements of properties such as compressibility and velocity show that rocks are highly anisotropic at low pressures. This anisotropy at low pressures is generally caused by an anisotropic crack distribution [*Birch,* 1960; *Walsh,* 1965]. Furthermore, measurements at pressures which are high enough to eliminate the effects of porosity show that most metamorphic rocks and many igneous rocks are in fact anisotropic. Thus, for a valid presentation of the elastic properties of many rocks, it may be more appropriate to use the formulas of anisotropic elasticity.

The anisotropy at high pressure of the

TABLE 8. Compressibility and Ratio of Bulk Modulus to Density from Mean V_p and V_s
Units are $(km/sec)^2$ for K/ρ and mb^{-1} for β.

Rock	P(kb) =	1	2	4	6	10
Gneiss 2	K/ρ	20.0	21.3	22.4	22.9	23.5
	β	1.88	1.77	1.68	1.65	1.60
Gneiss 4	K/ρ	20.8	23.0	24.5	24.9	25.7
	β	1.70	1.54	1.45	1.42	1.38
Gneiss 5	K/ρ	21.3	22.0	23.2	24.1	25.0
	β	1.65	1.59	1.51	1.46	1.41
Gneiss 6	K/ρ	18.1	19.6	21.0	21.7	22.8
	β	2.01	1.85	1.73	1.67	1.59
Garnet schist	K/ρ	23.5	24.7	25.6	26.4	27.0
	β	1.54	1.47	1.42	1.37	1.34
Staurolite garnet	K/ρ	24.6	25.8	27.0	27.5	28.2
schist	β	1.48	1.41	1.35	1.32	1.29
Kyanite schist	K/ρ	28.7	32.0	35.1	36.7	39.0
	β	1.16	1.04	0.95	0.91	0.86
Quartzite	K/ρ	15.9	16.3	16.8	17.2	17.6
	β	2.39	2.33	2.26	2.21	2.16
Feldspathic mica	K/ρ	19.0	19.5	20.1	20.6	21.1
quartzite	β	1.97	1.92	1.86	1.82	1.78
Slate	K/ρ	21.1	21.4	21.9	22.2	23.3
	β	1.72	1.69	1.66	1.63	1.56
Amphibolite 2	K/ρ	24.9	27.0	28.7	29.3	29.9
	β	1.33	1.22	1.15	1.13	1.10

TABLE 9. Compressibilities of Gneiss 2 [*Brace*, 1965]
Units are megabars^{-1}.

P(kb) =	1	2	3	5	7	9
⊥	0.76	0.67	0.65	0.63	0.61	0.59
‖	0.80	0.65	0.61	0.56	0.54	0.53
β	2.36	1.97	1.87	1.75	1.69	1.65

micaceous metamorphic rocks is a result of preferred orientation of highly anisotropic mica crystals. The micas deviate only slightly from hexagonal symmetry and are more conveniently treated as pseudo-hexagonal with regard to elasticity [*Alexandrov and Ryzhova*, 1961*b*]. This simplification results in the reduction of the thirteen independent monoclinic elastic constants to only five. The resulting error is less than 2%.

Wave propagation in hexagonal crystals is characterized by three displacement components which form an orthogonal set. In hexagonal crystals the three velocity surfaces have circular symmetry around the crystallographic c axis. Furthermore, the two shear velocity surfaces are tangent to one another at the c axis. A similar directional dependence of velocity is characteristic of the micaceous metamorphic rocks. The direction normal to the banding, foliation, or cleavage corresponds to the crystallographic c axes of hexagonal minerals. In this direction a single shear wave is transmitted with a velocity which is independent of vibration direction. The splitting of the original pulse into two pulses for propagation parallel to the planar elements is similar to the acoustic birefringence observed in crystals. Both shear waves can usually be identified on the oscilloscope trace if the displacement directions of the sending and receiving transducers are oriented at 45° to the planar elements.

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