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TRANSVERSE ISOTROPY OF THE UPPER MANTLE IN THE VICINITY OF PACIFIC FRACTURE ZONES

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ABSTRACT

Several recent investigations suggest that portions of the Earth's upper mantle behave anisotropically to seismic wave propagation. Since several types of anisotropy can produce azimuthal variations in P_n velocities, it is of particular geophysical interest to provide a framework for the recognition of the form or forms of anisotropy most likely to be manifest in the upper mantle. In this paper upper mantle material is assumed to possess the elastic properties of transversely isotropic media. Equations are presented which relate azimuthal variations in P_n velocities to the direction and angle of tilt of the symmetry axis of a transversely isotropic upper mantle. It is shown that the velocity data of Raitt and Shor taken near the Mendocino and Molokai fracture zones can be adequately explained by the assumption of transverse isotropy with a nearly horizontal symmetry axis.

Introduction

In the most general anisotropic elastic solid 21 independent constants are required to describe the equations of motion. Examples of materials with this type of behavior are crystals possessing triclinic symmetry. For many materials the existence of symmetry elements in the elastic properties leads to the vanishing of some elastic constants along with simple algebraic relations between others. In the limiting case of an isotropic solid only two independent constants are required for a complete description of elastic behavior. Isotropic elasticity, for which wave velocities are independent of propagation direction, is generally assumed for simplicity in seismological investigations of the Earth's interior. This assumption is reasonable if the material under investigation possesses no crystalline structure, or if it consists of an aggregate of crystals in which the anisotropic properties of each crystal are averaged out over sufficiently large volumes.

A number of recent papers (e.g., Hess, 1964; Backus, 1965; Cleary and Hales, 1966; Bolt and Nuttli, 1966; Cleary, 1967) have reported azimuthal variations in P_n velocities or apparent source terms from nuclear explosions and seismic refraction studies. It is possible that, at least in part, these azimuthal variations are related to anisotropic elastic properties of rock of the upper mantle.

Laboratory investigations have shown that many rocks are highly anisotropic to ultrasonic wave propagation. Birch (1960, 1961) found that compressional wave velocities vary with propagation direction in dunites and he related this anisotropy to preferred olivine orientation using the elastic properties of single crystal olivine measured by Verma (1960). Later Hess (1964) postulated that variations with azimuth of P_n velocities supplied by Raitt (1963) and Shor (1964) for the Mendocino and Maui areas of the Pacific were due to preferred olivine orientation.

Backus (1965) developed the theory for P_n and S_n propagation in the most general type of anisotropic upper mantle. He concluded that P_n velocity data of Raitt and Shor for the Mendocino and Molokai fracture zones of the Pacific (presented in both the Hess and Backus papers) are most readily explained by small anisotropy of the upper mantle. Using these data, Backus evaluated 5 of the 21 elastic tensor coefficients for the upper mantle in the vicinity of the fracture zones.

A number of authors have suggested that the upper mantle as well as possibly some regions of the crust may be transversely isotropic (e.g., Stoneley, 1949; Anderson, 1961; Helbig, 1966). Transversely isotropic media possess an axis of symmetry such that all planes containing the axis are equivalent. This type of anisotropy is perhaps the simplest of all forms owing to the axis of symmetry. Love (1944) and Musgrave (1954, 1959) among others have treated the theory of wave propagation in materials with transversely isotropic symmetry.

Transverse isotropy may result from the geometry of layering alone (Backus, 1962) or it may be an intrinsic rock property arising from preferred crystal orientation. Other structural characteristics can also be conceived which would produce transverse isotropy. Preferred crystal orientation which is usually thought to be a function of existing or previous stress or strain conditions may be directly related to layering as is the case with sediments which have been compacted under axial stress normal to layering. Most authors postulate transverse isotropy with a vertical symmetry axis, expressing or implying either a direct connection between crystal orientation and horizontal layering or assuming that the geometry of horizontal layering alone is responsible for transverse isotropy. The assumption of a vertical symmetry axis for transverse isotropy results in considerable mathematical simplification of the problem but it is obvious that azimuthal variation of seismic velocities cannot be explained by this type of model. However, Christensen and Crosson (1968) find evidence from both laboratory velocity measurements and petrofabric studies that transverse isotropy may be an intrinsic mode of elastic behavior for olivine-rich rocks, possibly reflecting the elastic properties of upper mantle rocks of similar composition. Thus the anisotropy (transverse isotropy) of the upper mantle need not be related to layering or other spherically symmetric earth properties. In a laterally inhomogeneous upper mantle, changes in anisotropy from region to region might be expected due to changes in orientation as suggested by Christensen and Crosson. It is therefore desirable to examine the implications of transverse isotropy in a more general framework which allows for a non-vertical symmetry axis.

In this paper the theory of Backus (1965) is adapted to the case where the mantle is transversely isotropic with unrestricted orientation of the symmetry axis. We show how certain parameters of the system such as the direction and angle of tilt of the symmetry axis can be estimated from the observed velocity distribution if reasonable assumptions are made regarding the symmetry axis velocity. Furthermore for certain cases, limits can be placed on these parameters without making prior assumptions. We find that the P_n velocity data of Raitt and Shor can be satisfactorily interpreted as being due to a transversely isotropic upper mantle. Some sample values are calculated for the angle of tilt of the symmetry axis for two different models corresponding to the symmetry axis as slow and fast propagations directions respectively. The data of Raitt and Shor are found to be consistent with the transversely isotropic model of the upper mantle presented by Christensen and Crosson.

Theory

Using a perturbation technique, Backus developed a first order theory for P_n velocity as a function of azimuth over a general anisotropic upper mantle. If the stress-strain relationship for a general elastic medium is (using summation convention)

$$T_{ij} = \rho \Gamma_{ijkl} \sigma_{kl} , \tag{1}$$

(generalized Hook's Law) where T_{ij} and σ_{ij} are the stress and strain tensors respectively, ρ is the density, and Γ_{ijkl} is the elastic tensor in cartesian coordinates, the problem of plane wave propagation may be reduced to the solution of the eigenvalue problem (Backus, 1965)

$$B_{il} s_l = v^2 s_i . \tag{2}$$

$B_{il} = \Gamma_{ijkl} \nu_j \nu_k$, v^2 is the squared-phase velocity and s_i is the particle motion vector. ν_j is a unit vector in the direction of the wave vector k_j, hence $\nu_j = k_j/|k_j|$. The three eigenvalues of equation (2) represent the squared phase velocities of the three principle modes of propagation for any given propagation direction specified by ν_j. The associated eigenvectors represent the particle motion for each mode.

Γ_{ijkl} has 21 independent coefficients for the most general type of anisotropic elastic body. As the symmetry of the material increases the number of independent coefficients diminishes correspondingly.

The essential part of Backus' theory is representation of the anisotropic elastic tensor, Γ_{ijkl}, as the sum of an isotropic tensor, $\Gamma^{(0)}_{ijkl}$, and a small anisotropic perturbation tensor, γ_{ijkl},

$$\Gamma_{ijkl} = \Gamma^{(0)}_{ijkl} + \gamma_{ijkl} . \tag{3}$$

Substituting equation (3) into equation (2) and expanding the eigenvalues and eigenvectors of (2) into isotropic terms plus higher order anisotropic correction factors, Backus showed that correct to first order in the anisotropy, the deviation of the squared phase velocity for P waves, v_p^2, from the square of an assumed isotropic velocity, c_p^2, is $\gamma_{ijkl} \nu_i \nu_j \nu_k \nu_l$ so that

$$v_p^2(\nu_j) = c_p^2 + \gamma_{ijkl} \nu_i \nu_j \nu_k \nu_l . \tag{4}$$

For a cartesian coordinate system in which the x_1x_2-plane is horizontal with x_3 vertically downward the expansion of equation (4), when ν_j lies in the horizontal plane at an angle ϕ from the x_1 axis (refraction shooting), results after some manipulation in

$$v_p^2(\phi) = c_p^2 + A + C \cos 2\phi + D \sin 2\phi + E \cos 4\phi + F \sin 4\phi. \tag{5}$$

The coefficients A, C, D, E, and F are given in terms of the elements of γ_{ijkl} by equations (22) of Backus.

The criterion of validity of this first order theory is that the anisotropy shall be sufficiently small so that the group and phase velocities are approximately equal, i.e., the direction of energy propagation is essentially normal to the "wave fronts". The observed velocity variations of 10 per cent or less are within the validity of the theory.

Suppose now that the medium is transversely isotropic, possessing an axis of symmetry about which the wave velocities are independent of direction. Since for a transversely isotropic medium the coefficients of the elastic tensor are most conveniently expressed in a material oriented coordinate system, we define a primed coordinate system with the x_3' axis parallel to the axis of material symmetry. Then P waves propagating in the $x_1'x_2'$-plane are pure mode with constant velocity. The perturbation elastic tensor expressed in the primed coordinate system is designated γ'_{ijkl} and the

non-zero elements are readily identified from the well-known theory of transversely isotropic solids (e.g., Anderson, 1961). Only five elements of γ'_{ijkl} are independent. Table 1 gives the relationships between the possible permutations of the non-zero elements of γ'_{ijkl} and the commonly used dual index notation for the elastic constants (c_{ij}).

Figure 1 shows the specified relationship between the primed and unprimed co-

TABLE 1

CORRESPONDENCE OF ELASTIC COEFFICIENTS

c_{11}	c_{12}	c_{13}	c_{11}	c_{13}	c_{33}	c_{44}	c_{44}	$\frac{1}{2}(c_{11}-c_{12})$
γ'_{1111}	γ'_{1122}	γ'_{1133}	γ'_{2222}	γ'_{2233}	γ'_{3333}	γ'_{2323}	γ'_{1313}	γ'_{1212}
	γ'_{2211}	γ'_{3311}		γ'_{3322}		γ'_{3223}	γ'_{3113}	γ'_{2112}
						γ'_{2332}	γ'_{1331}	γ'_{1221}
						γ'_{3232}	γ'_{3131}	γ'_{2121}

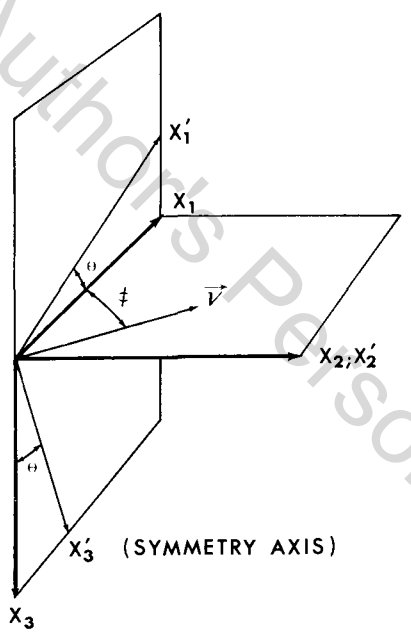

FIG. 1. Coordinate axes with relationships between material oriented system (primed), earth oriented system (unprimed), and the propagation vector $\vec{\nu}$.

ordinate systems. The direction of the x_1 axis is the direction of the vertical plane containing the symmetry axis. The x_1 and x_2 axes lie in a horizontal plane which is assumed parallel to the Moho beneath the oceans. The propagation vector ν_j for P_n waves is in the x_1x_2-plane and lies at an angle ϕ from the x_1 axis. The symmetry axis is tilted θ from the vertical. For this geometry, γ_{ijkl} is expressed in terms of γ'_{ijkl} through the tensor transformation,

$$\gamma_{ijkl} = a_{im}a_{jn}a_{ko}a_{lp}\gamma'_{mnop} , \tag{6}$$

where a_{ij} is the coordinate transformation given by

$$a_{ij} = \frac{\partial x_i}{\partial x_j'} .$$

From Figure 1 it is clear that

$$a_{ij} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}. \tag{7}$$

Substituting equation (6) into (4) we may write

$$\begin{aligned} Q(\phi) &= v_p^2(\phi) - c_p^2 \\ &= a_{im}a_{jn}a_{ko}a_{lp}\gamma'_{mnop}\nu_i\nu_j\nu_k\nu_l\,. \end{aligned} \tag{8}$$

Referring again to Figure 1 it is clear that the components of the unit vector in the direction of propagation, ν_j, are

$$\nu_j = (\cos\phi, \sin\phi, 0). \tag{9}$$

Equation (8) may be expanded out in a straightforward but somewhat lengthy operation. Using equations (7) and (9) in addition to the relationships of the coefficients summarized in Table 1,

$$\begin{aligned} Q(\phi) = {} & c_{11}\{\cos^4\theta\cos^4\phi + \sin^4\theta\} + c_{12}\{2\cos^2\theta\cos^2\phi\sin^2\phi\} \\ & + c_{13}\{2\cos^2\theta\sin^2\theta\cos^4\phi + 2\sin^2\theta\sin^2\phi\cos^2\theta\} \\ & + c_{33}\{\sin^4\theta\cos^4\phi\} + c_{44}\{4\sin^2\theta\sin^2\phi\cos^2\phi + 4\cos^2\theta\sin^2\theta\cos^4\phi\} \\ & + \tfrac{1}{2}(c_{11} - c_{12})\{4\cos^2\theta\cos^2\phi\sin^2\phi\} \end{aligned} \tag{10}$$

which upon rearranging may be written

$$Q(\phi) = A_1\cos^4\phi + A_2\sin^2\phi\cos^2\phi + A_3\sin^4\phi \tag{11a}$$

where

$$A_1 = c_{11}\cos^4\theta + 2(c_{13} + 2c_{44})\cos^2\theta\sin^2\theta + c_{33}\sin^4\theta \tag{11b}$$

$$A_2 = 2[c_{11}\cos^2\theta + (c_{13} + 2c_{44})\sin^2\theta] \tag{11c}$$

$$A_3 = c_{11}\,. \tag{11d}$$

Expansion of the cosine and sine terms in equation (11a) yields the further reduction to a truncated even Fourier series,

$$Q(\phi) = D_0 + D_2\cos 2\phi + D_4\cos 4\phi, \tag{12a}$$

where

$$\begin{aligned} D_0 &= \tfrac{1}{8}[3A_1 + A_2 + 3A_3] \\ &= c_{11} + [c_{13} + 2c_{44} - c_{11}]\sin^2\theta + \tfrac{3}{8}[c_{11} + c_{33} - 2(c_{13} + 2c_{44})]\sin^4\theta \end{aligned} \tag{12b}$$

$$D_2 = \tfrac{1}{2}[A_1 - A_3]$$

$$= [c_{13} + 2c_{44} - c_{11}] \sin^2 \theta + \tfrac{1}{2}[c_{11} + c_{33} - 2(c_{13} + 2c_{44})] \sin^4 \theta \tag{12c}$$

$$D_4 = \tfrac{1}{8}[A_1 - A_2 + A_3]$$

$$= \tfrac{1}{8}[c_{11} + c_{33} - 2(c_{13} + 2c_{44})] \sin^4 \theta. \tag{12d}$$

The square of the P-wave velocity as a function of azimuth is then written in the particularly simple form

$$v_p^2(\phi) = c_p^2 + D_0 + D_2 \cos 2\phi + D_4 \cos 4\phi. \tag{13}$$

Equation (13) is a special case of the function derived by Backus (equation (5)). The coefficients D_0, D_2, D_4, which can be obtained numerically by fitting (13) to the observational data, are related directly through equations (12b, c, and d) to the elastic constants of the system and the angle of tilt of the symmetry axis, θ. Note that only four of the five elastic constants appear in equations (12).

Investigation of $Q(\phi)$. Since $Q(\phi)$ has period π we need only investigate $Q(\phi)$ over the range $0 \leqq \phi \leqq \pi/2$. c_{11} governs the velocity of propagation normal to the axis of symmetry and may be calculated immediately from equations (12b, c, and d).

$$c_{11} = D_0 - D_2 + D_4 = Q(\pm\pi/2). \tag{14}$$

Equation (14) expresses the physical consideration that for a transversely isotropic solid, a distribution of velocities in any given plane will always include the velocity normal to the symmetry axis.

Multiplying equation (12c) by $\sin^2 \theta$ and combining with equation (12d) we obtain the quadratic equation in $\sin^2 \theta$,

$$(c_{11} - c_{33}) \sin^4 \theta + 2(D_2 - 4D_4) \sin^2 \theta + 8D_4 = 0, \tag{15}$$

which has the solution

$$\sin^2 \theta = \frac{-(D_2 - 4D_4) \pm [(D_2 - 4D_4)^2 - 8D_4(c_{11} - c_{33})]^{1/2}}{(c_{11} - c_{33})}. \tag{16}$$

In equation (16), c_{33} is the only unknown parameter. By assigning a value to c_{33}, which is equivalent to estimating the velocity along the symmetry axis, we may calculate $\sin^2 \theta$ and hence θ. For physically possible values of θ it is apparent that

$$(D_2 - 4D_4)^2 \geqq 8D_4(c_{11} - c_{33}) \tag{17a}$$

and

$$0 \leqq \frac{-(D_2 - 4D_4) \pm [(D_2 - 4D_4)^2 - 8D_4(c_{11} - c_{33})]^{1/2}}{(c_{11} - c_{33})} \leqq 1. \tag{17b}$$

In addition if $\theta = \pi/2$ it is clear that $Q(0) = Q(\pi) = c_{33} = D_0 + D_2 + D_4$; the sym-

metry axis velocity is observed directly in the distribution of velocities. Thus $c_{11} - c_{33} = -2D_2$ and (16) becomes

$$1 = \frac{-(D_2 - 4D_4) \pm [(D_2 - 4D_4)^2 + 16D_2D_4]^{1/2}}{-2D_2}$$

$$= \frac{-(D_2 - 4D_4) \pm |D_2 + 4D_4|}{-2D}. \tag{17c}$$

Equations (17) are used to determine the appropriate sign in equation (16) and also to limit the range of possible values for c_{33}.

For convenience in the ensuing discussion we first examine the extrema of the function $Q(\phi)$. $dQ/d\phi$ vanishes at $\phi = 0$, $\phi = \pi/2$, and ϕ defined implicitly by

$$\cos 2\phi = \frac{-D_2}{4D_4}, \; |4D_4| > |D_2|; \tag{18}$$

thus these points represent either maxima or minima. The sign of $d^2Q/d\phi^2$ at these points indicates the following characteristics for these extrema: (a) $\phi = 0$ (or π), maximum if $D_2 + 4D_4 > 0$, otherwise minimum; (b) $\phi = \pi/2$, maximum if $D_2 - 4D_4 < 0$, otherwise minimum; (c) extrema defined by equation (18), maximum if $D_4 < 0$, otherwise minimum.

The difference between the extreme values at $\phi = \pi/2$ and $\phi = 0$ is

$$Q\left(\frac{\pi}{2}\right) - Q(0) = -2D_2. \tag{19}$$

Similarly by straightforward substitution into equation (12a) the difference between the extreme value defined by equation (18) and the value at $\phi = 0$ is

$$D_0 - \frac{D_2^2}{8D_4} - D_4 - (D_0 + D_2 + D_4) = -\frac{1}{8D_4}(D_2 + 4D_4)^2, \tag{20}$$

and the difference between the extreme value at $\phi = \pi/2$ and the value defined by equation (18) is

$$-2D_2 + \frac{1}{8D_4}(D_2 - 4D_4)^2 = \frac{1}{8D_4}(D_2 - 4D_4)^2. \tag{21}$$

For selecting c_{33} in evaluating equation (16) there are two choices of practical interest, $c_{33} < c_{11}$ and $c_{33} > c_{11}$. The selection of one or the other of these alternatives will be coupled with the original selection of the plane containing the symmetry axis in fitting (13) to the observed data. There is no information in the observational data to indicate which choice of models is correct.

Model I. ($c_{33} < c_{11}$) For this model we assume the symmetry axis is the slow velocity direction and $Q(0) < Q(\pi/2)$. From (19) we see that $D_2 < 0$; however D_4 may be $>$ or < 0.

If $D_4 < 0$, equation (17a) is satisfied and the radical in (16) is never imaginary. In order that $\theta = \pi/2$ when $(c_{11} - c_{33}) = -2D_2$, we see from equation (17c) that the

positive sign must be chosen in (16). Furthermore in order that (17b) be satisfied with the positive sign selection,

$$-(D_2 - 4D_4) + [(D_2 - 4D_4)^2 - 8D_4(c_{11} - c_{33})]^{1/2} \leqq (c_{11} - c_{33})$$

or

$$-2D_2 \leqq (c_{11} - c_{33}). \tag{22}$$

The inequality (22) means physically that the observed difference in velocity between the symmetry axis azimuth and the azimuth normal to the symmetry axis is less than or equal to the true difference in velocity between the symmetry axis and the normal to the symmetry axis. The discrepancy is a function of the angle of tilt of the symmetry axis. It is of interest to determine if a lower limit exists for θ. Since $\sin^2\theta$ is uniformly decreasing with increasing $(c_{11} - c_{33})$, $(c_{11} - c_{33})$ has no a priori maximum value. In the limit as $(c_{11} - c_{33}) \to \infty$, $\sin^2\theta \to 0$, hence $0 \leqq \theta \leqq \pi/2$. Note that there always exists a sign ambiguity on θ, corresponding to the fact that the symmetry axis tipped in the positive direction is indistinguishable from the axis tipped an equal amount in the negative direction.

Taking now the case $D_4 > 0$, the quantity $(c_{11} - c_{33})$ must be restricted in magnitude to avoid imaginary values of the radical in equation (16).

$$(c_{11} - c_{33}) \leqq \frac{1}{8D_4}(D_2 - 4D_4)^2. \tag{23}$$

If $|4D_4| < |D_2|$, the positive sign must be adopted in equation (16) in order that (17c) is satisfied when $(c_{11} - c_{33}) = -2D_2$. For this choice of sign, equation (17b) implies that

$$(c_{11} - c_{33}) \geqq -2D_2. \tag{24}$$

Since $1/(8D_4)(D_2 - 4D_4)^2 \geqq -2D_2$, we see that combining (23) and (24) results in

$$-2D_2 \leqq (c_{11} - c_{33}) \leqq \frac{1}{8D_4}(D_2 - 4D_4)^2. \tag{25}$$

$\mathrm{Sin}^2\theta$ assumes a minimum value, $8D_4/|D_2 - 4D_4|$, at the upper limit of $(c_{11} - c_{33})$. Thus

$$\sin^{-1}\left[\frac{|8D_4|}{|D_2 - 4D_4|}\right]^{1/2} \leqq \theta \leqq \frac{\pi}{2}. \tag{26}$$

If $|4D_4| > |D_2|$ then (17c) requires that the negative sign be selected in (16). A secondary minimum defined by equation (18) now exists. Equation (17b) with the negative choice of sign implies that

$$(c_{11} - c_{33}) \leqq -2D_2 \tag{27}$$

and the minimum value of $\sin^2\theta$ is $4D_4/|D_2 - 4D_4|$, obtained by evaluating (16) in

the limiting case as $(c_{11} - c_{33}) \to 0$. Hence

$$\sin^{-1}\left[\frac{|4D_4|}{|D_2 - 4D_4|}\right]^{1/2} \leqq \theta \leqq \frac{\pi}{2}. \tag{28}$$

For certain values of the coefficients D_2 and D_4 we are restricted in our choice of c_{33}. Furthermore the angle of tilt of the symmetry axis may be restricted to a range of values strictly on the basis of observed data. All of these considerations have reason-

TABLE 2

SUMMARY OF RESULTS FOR COMPUTING TILT ANGLE FROM VELOCITY FUNCTIONS

$c_{11} - c_{33}$	D_2	D_4	$\lvert 4D_4 \rvert : \lvert D_2 \rvert$	Sign	Limits on $(c_{11} - c_{33})$	Limits on θ
>0	<0	<0	$\lvert 4D_4 \rvert < \lvert D_2 \rvert$	$+$	$(c_{11} - c_{33}) \geqq -2D_2$	$0 \leqq \theta \leqq \pi/2$
			$\lvert 4D_4 \rvert > \lvert D_2 \rvert$	$+$	$(c_{11} - c_{33}) \geqq -2D_2$	$0 \leqq \theta \leqq \pi/2$
(Model I)	<0	>0	$\lvert 4D_4 \rvert < \lvert D_2 \rvert$	$+$	$-2D_2 \leqq (c_{11} - c_{33}) \leqq \frac{1}{8D_1}(D_2 - 4D_4)$	$\sin^{-1}\left(\frac{\lvert 8D_4 \rvert}{\lvert D_2 - 4D_4 \rvert}\right)^{1/2} \leqq \theta \leqq \pi/2$
			$\lvert 4D_4 \rvert > \lvert D_2 \rvert$	$-$	$(c_{11} - c_{33}) \leqq -2D_2$	$\sin^{-1}\left(\frac{\lvert 4D_4 \rvert}{\lvert D_2 - 4D_4 \rvert}\right)^{1/2} \leqq \theta \leqq \pi/2$
<0	>0	<0	$\lvert 4D_4 \rvert < \lvert D_2 \rvert$	$-$	$\frac{1}{8D_4}(D_2 - 4D_4)^2 \leqq (c_{11} - c_{33}) \leqq -2D_2$	$\sin^{-1}\left(\frac{\lvert 8D_4 \rvert}{\lvert D_2 - 4D_4 \rvert}\right)^{1/2} \leqq \theta \leqq \pi/2$
			$\lvert 4D_4 \rvert > \lvert D_2 \rvert$	$+$	$(c_{11} - c_{33}) \geqq -2D_2$	$\sin^{-1}\left(\frac{\lvert 4D_4 \rvert}{\lvert D_2 - 4D_4 \rvert}\right)^{1/2} \leqq \theta \leqq \pi/2$
(Model II)	>0	>0	$\lvert 4D_4 \rvert < \lvert D_2 \rvert$	$-$	$(c_{11} - c_{33}) \leqq -2D_2$	$0 \leqq \theta \leqq \pi/2$
			$\lvert 4D_4 \rvert > \lvert D_2 \rvert$	$-$	$(c_{11} - c_{33}) \leqq -2D_2$	$0 \leqq \theta \leqq \pi/2$

able physical interpretations in terms of the geometry of the velocity surface for a transversely isotropic medium.

Model II. $(c_{11} < c_{33})$ The symmetry axis is the fast velocity direction. This model is associated with $D_2 > 0$, the horizontal projection of the symmetry axis is a fast velocity direction. Again depending on the sign and magnitude of the coefficient D_4, several alternative cases arise. Applying the same reasoning as outlined for model I, a similar set of restrictions is derived. These are summarized, along with those for model I, in Table 2.

Comparison with Observation. The observational data of Raitt and Shor on directional dependence of P_n velocities from near the Mendocino fracture zone and Maui were fitted by Backus (1965) to the following truncated Fourier series, Mendocino $v_p^2(\psi) = 67.722 + 2.336 \sin 2\psi - 3.806 \cos 2\psi - 2.163 \sin 4\psi + 0.492 \cos 4\psi$; Maui $v_p^2(\psi) = 67.900 + 1.698 \sin 2\psi - 4.786 \cos 2\psi - 0.876 \sin 4\psi + 3.130 \cos 4\psi$; and

Mendocino combined with Maui $v_p^2(\psi) = 67.663 + 2.100 \sin 2\psi - 3.796 \cos 2\psi - 1.677 \sin 4\psi + 1.205 \cos 4\psi$. ψ is the azimuth measured from north, v_p is in km/sec and Backus assumes $c_p^2 = 67.750$ km^2/sec^2.

Letting $\phi = \psi - \epsilon$ in equation (13), we may find an ϵ, D_0, D_2, and D_4, for each

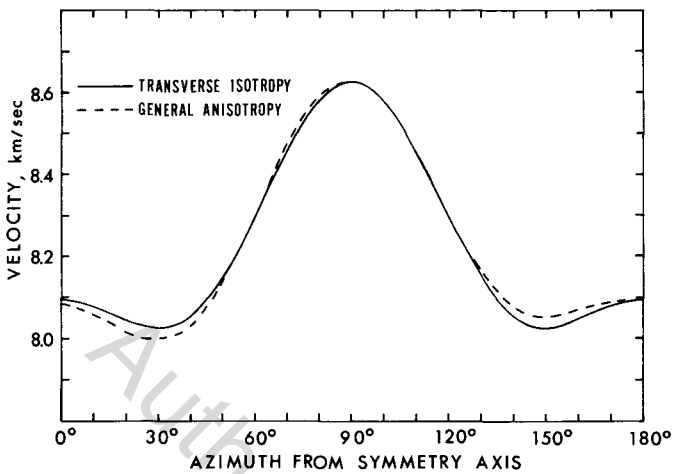

FIG. 2. Comparison between Backus' fit of Mendocino data, shifted 17.5 degrees (dashed line), and transversely isotropic model: $v_p^2 = 67.722 - 4.414 \cos 2\theta + 2.218 \cos 4\theta$.

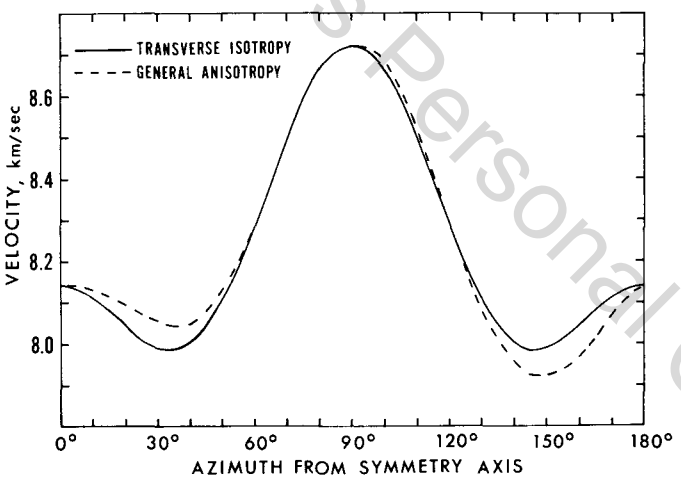

FIG. 3. Comparison between Backus' fit of Maui data, shifted by 6.8 degrees (dashed line), and transversely isotropic model: $v_p^2 = 67.90 - 4.898 \cos 2\theta + 3.258 \cos 4\theta$.

of these three cases which makes equation (13) a very close fit to the functions given above. Thus there exists a change of coordinates which reduces the antisymmetric coefficients in Backus' curves to relatively small values. Alternatively, equation (13) could have been fitted directly to the original data by ordinary least squares procedures. In this event, ϵ should be made an additional parameter in minimizing the sums of the squared error. The velocity functions resulting from matching Backus' curves are given for the Mendocino, Maui, and combined cases in Figures 2, 3, and 4 respectively, and compared to the original curves. The shift of coordinates, ϵ, required to

achieve a fit is indicated in each case. The new coefficients D_0, D_2 and D_4, assuming $c_p^2 = 67.75$ in each case, are presented in Table 3. Figures 2, 3, and 4 are appropriate only for model I of the previous discussion since the symmetry axis direction ($\phi = 0$) is a slow velocity direction. The curves for model II are obtained by merely translating the abscissae of these diagrams through 90 degrees such that the maximum velocity

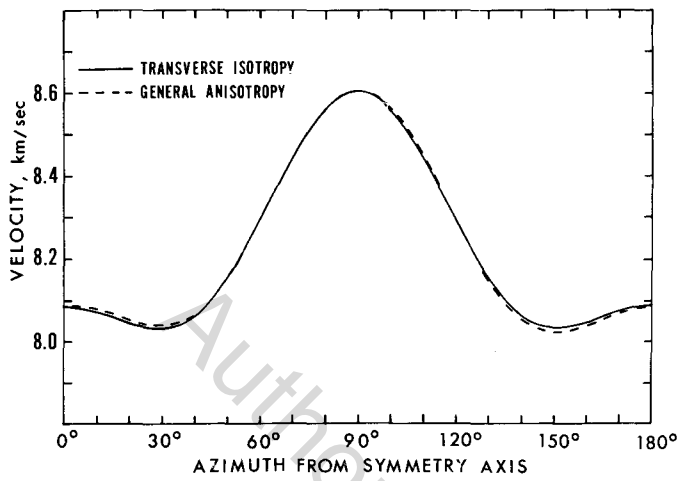

FIG. 4. Comparison between Backus' fit of Mendocino and Maui data combined, shifted by 14.0 degrees (dashed line), and transversely isotropic model: $v_p^2 = 67.663 - 4.307 \cos 2\theta + 2.052 \cos 4\theta$.

TABLE 3

NUMERICAL PARAMETERS AND RESULTS FOR VARIOUS AREAS

	Mendocino Model I	Mendocino Model II	Maui Model I	Maui Model II	Combined Model I	Combined Model II
Direction of symmetry axis from North	17.5°W	72.5°E	6.8°W	83.2°E	14.0°W	76.0°E
D_0	−0.028	−0.028	0.150	0.150	−0.087	−0.087
D_2	−4.414	4.414	−4.898	4.898	−4.307	4.307
D_4	2.218	2.218	3.258	3.258	2.052	2.052
c_{11}	6.604	−2.224	8.306	−1.490	6.272	−2.342
Limits on θ	$55° \leqq \theta \leqq 90°$	$0° \leqq \theta \leqq 90°$	$58° \leqq \theta \leqq 90°$	$0° \leqq \theta \leqq 90°$	$54° \leqq \theta \leqq 90°$	$0° \leqq \theta \leqq 90°$
Assumed value of c_{33}	−1.627	7.653	0.075	8.387	−1.959	7.535
Calculated θ	76°	79°	74°	87°	78°	78°

falls at $\phi = 0$. This coordinate shift results only in changing the sign of D_2 in equation (13).

From the derived coefficients D_0, D_2, and D_4 we may proceed immediately to evaluate the range of coefficients and tilt angle, θ, using the considerations developed in the previous section and summarized in Table 2. For model I, in each case we have $D_2 < 0$, $D_4 > 0$, and $|4D_4| > |D_2|$, hence $(c_{11} - c_{33}) \leqq -2D_2$ and the negative sign must be adopted in equation (16). Accordingly the angle of tilt, θ, must have a range of possible values lying between about 55 and 90 degrees for all cases. Figure 5 is a plot of θ as a function of $(c_{11} - c_{33})$ for the three cases considered. In each case the 90 degree tilt is reached when $(c_{11} - c_{33})$ is equal to the observed difference in squared velocities. Near the maximum angle of tilt the calculated angle is of course extremely

sensitive to the selection of c_{33}. Assuming a reasonable value of c_{33} in each case, such that for the examples considered the difference between maximum and minimum velocities is 0.5 km/sec, we arrive at the angles of tilt listed in Table 3.

Turning to model II, $D_2 > 0$ and we see from Table 2 that $| c_{11} - c_{33} | \geqq | 2D_2 |$ and no restrictive limits can be placed on the angle of tilt, θ. Again the results have been tabulated in Table 3 and tilt angles have been calculated by choosing c_{33} such that the difference between maximum and minimum velocities (i.e., propagation normal to the symmetry axis and parallel to the symmetry axis) is approximately 0.6 km/sec. These assumptions for both models result in tilt angles approximately 80 degrees from vertical.

Note that model II gives us more freedom in selecting the symmetry axis velocity. However if model I is correct we have valuable constraints on the selection of c_{33} and corresponding limits on the angle of tilt. With this exception, the choice of models and

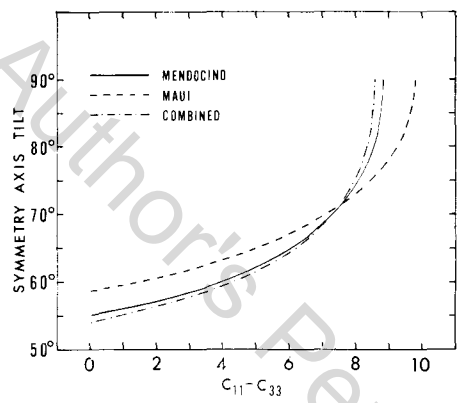

FIG. 5. Calculated values of symmetry axis tilt as a function of choice of $c_{11} - c_{33}$ using model I.

selection of a reasonable compressional wave velocity for propagation parallel to the symmetry axis must to a large extent rely on laboratory measurements of elastic wave velocities and related petrofabric studies. This velocity will be dependent upon the degree of orientation of olivine axes as well as which axis concentrates in the direction of the transverse isotropic symmetry axis. Measurements by Christensen (1966) of compressional wave velocities for a dunite from Addie, N. C. show a low velocity for propagation parallel to the symmetry axis (type I transverse isotropy) and a maximum velocity difference of 0.5 km/sec at 5 kb. Reported petrofabric analyses also suggest that type I transverse isotropy may be the most important for olivine-rich rocks (Christensen and Crosson, 1968).

Discussion and Conclusions

We await further data and more extensive analysis before attempting to adequately correlate orientation data with geotectonic parameters of an area. However it is significant that the symmetry axis is nearly horizontal in the vicinity of the fracture zones (see Hess, 1964, for locations of the profiles). This is in accord with the postulates of Christensen and Crosson (1968) and earlier by Hess (1964) that fracture zones represent anomalously stressed areas which may result in orienting or reorienting the material of the upper mantle.

Several conclusions may be drawn at this stage:

(a) The data of Raitt and Shor as presented by Hess (1964) and Backus (1965) is adequately explained by a "tilted" transversely isotropic model such as postulated by Christensen and Crosson (1968). However within this framework, sub-models are possible.

(b) Under the assumption of transverse isotropy, the data of Raitt and Shor indicate a nearly horizontal symmetry axis.

(c) The transversely isotropic model is comparatively simple, with only 5 parameters as contrasted to 21 for the completely general case. Consequently where the model is valid we may obtain more complete knowledge of the elasticity from limited observational data than is possible for more complex models.

(d) For some sub-models the observational data is sufficient to place valuable restraints on the range of axis tilt and symmetry axis velocity under the assumption of transverse isotropy.

(e) Much additional field and laboratory data is necessary to determine the validity of a transversely isotropic model of the upper mantle and to distinguish between the various alternative sub-models.

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