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Compressional to Shear Velocity Ratios in Sedimentary Rocks

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INTRODUCTION

Poisson's ratio is an important parameter in both rock mechanics, where it is used to describe behavior of a material under stress, and in seismological studies, where it is used to identify and constrain properties of subsurface lithologies. In geophysical studies, compressional and shear wave velocities of a rock sample are used to determine Poisson's ratio (σ), where

$$\sigma = \frac{0.5r^2 - 1}{r^2 - 1} \qquad (1)$$

r being the ratio of compressional to shear wave velocity (Vp/Vs).

Literature of the past two decades contains many examples of the importance of Vp/Vs in the geosciences. Tatham [1] was one of the first to emphasize potential importance of Vp/Vs to exploration seismologists as an indicator of sandstone porosity and pore content. Eastwood and Castagna [2] examined Vp/Vs ratios from well log data and investigated effects of lithology, porosity, and pore aspect ratio on this parameter. Robertson [3] discovered a correlation between changes in average porosity (and pore content) and lateral changes in Vp/Vs in a hydrocarbon producing formation in Oklahoma.

Interpretation of Vp/Vs in carbonate lithologies is believed to be fairly straightforward, as it appears to be indicative of lithology and relatively insensitive to porosity [2,4]. Interpretation of Vp/Vs in other sedimentary lithologies is relatively more difficult as both porosity and clay content affect this parameter [2]. In addition, Vp/Vs has been found to increase significantly with increasing pore pressure in a wide variety of sedimentary rocks [5].

One potential problem which has not been previously addressed involves effects of anisotropy on Vp/Vs. In this paper, velocity ratios obtained from two highly anisotropic, well-indurated shales from the Thorn Hill sedimentary section of eastern Tennessee are compared with data from laboratory studies of Vp/Vs in low porosity, isotropic rocks. It is shown that anisotropy of Vp/Vs in shales is of greater magnitude than changes in Vp/Vs produced by variations in lithology.

EXPERIMENTAL TECHNIQUE AND DATA

Three main sources of laboratory data on Vp/Vs in low porosity sedimentary rocks were used in this summary. The study by Wilkens *et al.* [4] provides valuable velocity data for a suite of silty limestones. In addition, Christensen and Szymanski [6], and Johnston and Christensen [7] published compressional and shear wave data, respectively, for a wide variety of sedimentary rocks collected from the Thorn Hill sedimentary section of Eastern Tennessee. Data from these studies is summarized in Table 1. Carbonates in the form of dolostones, limestones, and silty limestones are well represented, as are low porosity sandstones. In addition to these samples, data from a quartzite [8,9] have been added as an analog for an extremely low porosity sandstone. Velocities are at 100 MPa confining pressure and under unsaturated conditions. At this pressure, low aspect ratio pores are closed (e.g., [4,7]).

Although shales comprise a great percentage of the total rock volume in sedimentary basins, laboratory velocity studies of these rocks are rare due to difficulty in obtaining cores in fissile and/or poorly consolidated samples. A few notable papers have been published. Jones and Wang [10] measured Vp and Vs in two unconsolidated shales from the Williston basin and found that the samples behaved as transversely isotropic solids with the main symmetry axis perpendicular to bedding. Lo *et al.*, [11] measured Vp and Vs in the Chicopee shale as a function of confining pressure and used these values to calculate elastic constants of the sample.

Shales examined in this study were collected from the Millboro (TH-26) and Braillier (TH-29) members of the Devonian-Mississippian Chattanooga formation. These shales are extremely well indurated, with porosities of approximately one percent. Since earlier velocity studies of shales have demonstrated transverse isotropy [10], four cores 2.54 cm in diameter and from 2 to 4 cm long were taken from each sample (Figure 1). The cores were trimmed and polished into right circular cylinders, jacketed with copper to isolate pore spaces from high pressure oil, and subjected to ultrasonic

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Table 1. Sandstone (SS), limestone (LS), silty limestone (SLS), dolostone (DOL), and quartzite (QU) data used in this study.

Sample	Lithology	Vp (km/s)	Vs (km/s)	Vp/Vs	Porosity (%)	Carbonate (%)
TH-55	SS	5.40	3.78	1.43	2.4	0
TH-25	SS	5.29	3.45	1.53	1.9	0
C65	QU	6.05	3.94	1.54	0	0
TH-57	SS	5.37	3.45	1.56	1.2	0
W2625a	SLS	5.76	3.62	1.59	0.2	80
W2625b	SLS	5.72	3.59	1.59	0.2	6
TH-38	SS	5.32	3.31	1.61	1.7	0
W2635c	SLS	5.48	3.40	1.61	1.1	39
W2625c	SLS	5.40	3.35	1.61	0.4	13
W2623a	SLS	5.82	3.60	1.62	0.3	13
W2622c	SLS	5.76	3.51	1.64	0.2	27
W2637c	SLS	5.90	3.55	1.66	1.1	31
W2169c	SLS	5.48	3.29	1.67	1.2	40
TH-58	SS	5.50	3.30	1.67	1.5	0
W2624c	SLS	5.95	3.50	1.70	0.5	35
W2623c	SLS	5.93	3.47	1.71	0.4	26
W2633c	SLS	5.40	3.15	1.71	3.7	37
W2636c	SLS	6.01	3.46	1.74	1.1	41
W2620c	SLS	5.70	3.18	1.79	2.1	56
TH-47	DOL	6.59	3.67	1.80	0.9	95
TH-12	DOL	6.77	3.75	1.81	0.8	99
TH-44	DOL	6.82	3.77	1.81	1.2	99
W2621c	SLS	6.03	3.32	1.82	1.6	64
TH-11	DOL	6.94	3.82	1.82	0.4	99
TH-45	DOL	6.89	3.79	1.82	0.7	99
TH-39	DOL	6.81	3.70	1.84	0.3	97
W2621a	SLS	6.34	3.43	1.85	2.1	40
W2627c	SLS	5.77	3.12	1.85	0.6	80
W2169a	SLS	5.65	3.04	1.86	0.8	50
W2636b	SLS	6.36	3.37	1.89	1.8	63
TH-35	LS	6.09	3.22	1.89	0.9	100
W2636a	SLS	6.38	3.37	1.89	0.8	60
TH-14	LS	6.41	3.36	1.91	0.2	99
TH-17	LS	6.19	3.23	1.92	0.3	99
TH-19	LS	6.38	3.32	1.92	0.1	99
TH-42	LS	6.36	3.32	1.92	0.1	99
TH-3	LS	6.34	3.29	1.93	0.3	98
W2626c	SLS	5.93	3.07	1.93	2.2	99
W2626a	SLS	5.92	3.06	1.93	2.1	92
TH-7	LS	6.57	3.38	1.94	0.1	99
TH-37	LS	6.60	3.40	1.94	0.5	99
W2621b	SLS	6.56	3.38	1.94	0.3	81
TH-6	LS	6.50	3.34	1.95	0.1	95
TH-4	LS	6.32	3.22	1.96	0.4	99
TH-5	LS	6.54	3.33	1.96	0.1	99
TH-16	LS	6.24	3.19	1.96	0.7	95
TH-22	LS	6.31	3.22	1.96	0.2	99
TH-15	LS	6.43	3.26	1.97	0.1	99
W2626b	SLS	6.16	3.09	1.99	1.7	90

velocity measurements to 200 MPa using a Harwood pressure generating system and the mercury delay line technique [12]. Shear and compressional transducers with resonant frequencies of 1 MHz provided the seismic sources and receivers.

To obtain the complete elastic constants for a transversely isotropic medium, a minimum of five measurements are required. As a check of symmetry, nine velocities were measured per rock (Figure 1). Two Vp measurements were made from cores taken in the bedding plane (Vp_{bb} and Vp_{cc}, where the first subscript indicates propagation direction and the second, vibration direction). Velocities of shear waves propagating in the bedding plane and vibrating parallel (Vs_{bc}) and perpendicular (Vs_{ca}) to bedding were also obtained from

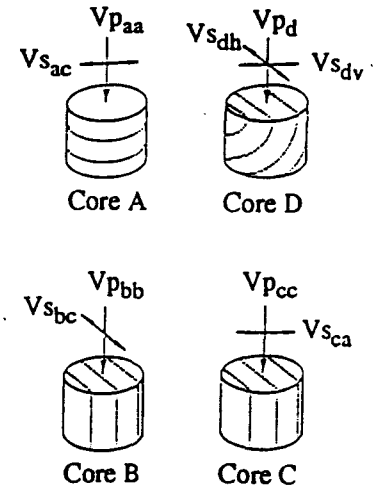

Fig. 1. Diagram showing shear and compressional wave velocities (including vibration directions) measured in shale samples TH-26 and TH-29.

these cores. Wave velocities were also measured for propagation direction perpendicular to bedding (Vp_{aa} and Vs_{ac}). The velocity of a quasi-compressional wave propagating at 45° to bedding was measured (Vp_d), as well as those for two shear waves, one vibrating parallel to the bedding plane (Vs_{dh}), and one vibrating perpendicular to the plane (Vs_{dv}) (Figure 1).

The velocity data, compiled in Table 2, indicate that both TH-26 and TH-29 are to first approximation transversely isotropic, ($Vp_{bb}=Vp_{cc}>Vp_{aa}$ and $Vs_{ac}=Vs_{ca}<Vs_{bc}$), with the main symmetry axis perpendicular to bedding. At pressures of 100 MPa and above, most rock microcracks are closed and anisotropy can be attributed to clay minerals aligned parallel to bedding [7, 10, 11]. At 100 MPa, TH-26 exhibits 37% Vp anisotropy and 45% Vs anisotropy. TH-29 has slightly greater Vp and Vs anisotropies, at 49% and 34%, respectively.

The five independent elastic constants necessary to describe a tranversely isotropic medium are also listed as a function of pressure in Table 2. These stiffness constants were calculated from velocities and densities using the equations given in Lo *et al.*, [11]. Calculation of these constants is relatively simple. Difficulties can arise in the calculation of C_{13}, however, if the quasi compressional velocity at 45° is not measured with extreme accuracy as this value is raised both to the second and fourth power in the velocity-stiffness relation.

Using the elastic constants of Table 2, the Voigt-Reuss-Hill (VRH) velocities [13] of the two shales at 100 MPa were calculated and are presented in Table 3. The VRH averaging scheme is commonly used to define the isotropic elastic properties of anisotropic single crystals. Also given in Table 3 are Vp/Vs values for propagation directions normal, parallel, and at 45° to bedding for both shales at 100 MPa. Low values of Vp/Vs are found for propagation directions at 45° to bedding with the quasi-shear wave vibrating perpendicular to bedding

Table 2. Velocities and elastic constants of shale samples as a function of pressure. See Figure 1 for explanation of velocity measurements.

TH-26:
density = 2.341 g/cc

Pressure	Velocities (km/s)									Elastic Constants ($x10^{10}$ dynes/cm^2)				
(MPa)	Vp aa	Vp bb	Vp cc	Vp d	Vs ac	Vs bc	Vs ca	Vs dh	Vs dv	C11	C12	C33	C44	C13
20	2.930	4.496	4.395	3.645	1.760	2.755	1.760	2.324	2.056	46.3	10.7	20.1	7.3	10.1
30	3.022	4.530	4.442	3.710	1.792	2.767	1.722	2.335	2.091	47.1	11.3	21.4	7.5	11.2
40	3.086	4.559	4.480	3.754	1.815	2.779	1.786	2.346	2.115	47.8	11.7	22.3	7.7	11.7
50	3.132	4.586	4.511	3.790	1.833	2.791	1.797	2.356	2.132	48.4	12.0	23.0	7.9	12.1
60	3.172	4.612	4.540	3.820	1.848	2.803	1.809	2.365	2.145	49.0	12.2	23.6	8.0	12.4
70	3.204	4.636	4.566	3.847	1.860	2.814	1.820	2.373	2.157	49.6	12.5	24.0	8.1	12.7
80	3.233	4.659	4.589	3.871	1.870	2.825	1.830	2.382	2.170	50.1	12.7	24.5	8.2	12.9
90	3.261	4.680	4.610	3.892	1.878	2.835	1.840	2.390	2.179	50.5	12.9	24.9	8.3	13.2
100	3.286	4.701	4.630	3.912	1.886	2.845	1.850	2.397	2.189	51.0	13.1	25.3	8.3	13.4
150	3.400	4.793	4.725	4.001	1.927	2.890	1.895	2.423	2.232	53.0	13.9	27.1	8.7	14.1
200	3.496	4.872	4.806	4.077	1.965	2.927	1.930	2.445	2.259	54.8	14.7	28.6	9.0	14.6

TH-29:
density = 2.356 g/cc

Pressure	Velocities (km/s)									Elastic Constants ($x10^{10}$ dynes/cm^2)				
(MPa)	Vp aa	Vp bb	Vp cc	Vp d	Vs ac	Vs bc	Vs ca	Vs dh	Vs dv	C11	C12	C33	C44	C13
20	2.975	4.690	4.612	3.737	1.772	2.853	1.738	2.354	2.139	51.0	12.6	20.9	7.4	9.3
30	3.096	4.723	4.649	3.772	1.795	2.870	1.775	2.366	2.152	51.7	12.9	22.6	7.6	9.3
40	3.174	4.755	4.676	3.798	1.812	2.882	1.796	2.376	2.162	52.4	13.3	23.7	7.7	9.2
50	3.226	4.786	4.703	3.818	1.826	2.895	1.813	2.383	2.172	53.0	13.5	24.5	7.9	9.0
60	3.270	4.815	4.725	3.836	1.837	2.905	1.825	2.391	2.180	53.6	13.8	25.2	8.0	8.8
70	3.306	4.840	4.746	3.852	1.846	2.915	1.836	2.398	2.189	54.1	14.1	25.8	8.0	8.7
80	3.340	4.861	4.766	3.868	1.855	2.925	1.846	2.404	2.198	54.6	14.3	26.3	8.1	8.7
90	3.372	4.880	4.786	3.881	1.862	2.932	1.855	2.410	2.205	55.0	14.5	26.8	8.2	8.6
100	3.402	4.897	4.805	3.893	1.867	2.942	1.863	2.416	2.212	55.4	14.7	27.3	8.2	8.5
150	3.540	4.968	4.892	3.949	1.897	2.986	1.906	2.435	2.245	57.3	15.2	29.5	8.5	8.1
200	3.650	5.031	4.966	4.000	1.920	3.025	1.945	2.451	2.275	58.9	15.7	31.4	8.7	8.0

Table 3. Variation of Vp/Vs with propagation direction for TH-26 and TH-29. Calculated Voigt-Reuss-Hill velocities also shown.

Propagation		TH-26			TH-29		
Direction	Velocities	Vp	Vs	Vp/Vs	Vp	Vs	Vp/Vs
Normal	Vpaa, Vsac	3.29	1.89	1.74	3.40	1.87	1.82
45 Degrees	Vpd, Vsdh	3.91	2.40	1.63	3.89	2.42	1.61
	Vpd, Vsdv	3.91	2.19	1.79	3.89	2.21	1.76
Parallel	Vpbb, Vsbc	4.70	2.85	1.65	4.90	2.94	1.66
	Vpcc, Vsca	4.63	1.85	2.50	4.81	1.86	2.58
	Voigt	4.15	2.35	1.76	4.22	2.49	1.70
	Reuss	3.88	2.17	1.79	3.86	2.24	1.72
	Hill	4.01	2.26	1.78	4.04	2.36	1.71

(Vp_d/Vs_{dv}), and for propagation in the bedding plane with the shear wave vibrating parallel to bedding (Vp_{aa}/Vs_{ac}). The highest value is found for propagation in the bedding plane with the shear wave vibrating perpendicular to bedding (Vp_{cc}/Vs_{ca}). The Vp/Vs values calculated from VRH averages fall between the maximum and minimum values for both rocks (Table 3).

DISCUSSION

Wilkens *et al.* [4] demonstrated that velocity ratios in low porosity, impure sedimentary rocks can be related to elastic properties of the pure end members. In Figure 2, ranges of Vp/Vs (and Poisson's Ratio) for several ideal lithologic combinations are presented. End member

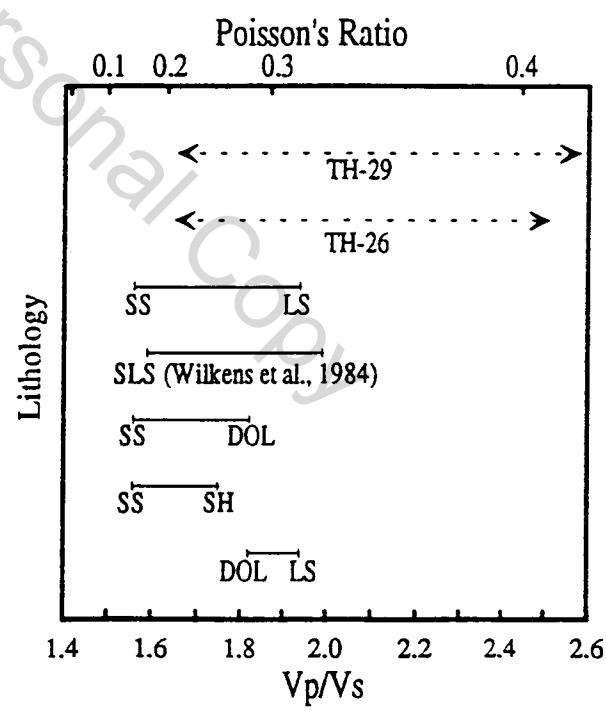

Fig. 2. Variation of Vp/Vs (Poisson's Ratio) expected between ideal end member lithologies. Sandstone (SS), dolostone (DOL), shale (SH), limestone (LS), and silty limestone (SLS) data shown. Range of TH-26 and TH-29 Vp/Vs due to anisotropy represented by dashed lines.

Vp/Vs values for sandstones, limestones, and dolostones are averages taken from Table 1, while the end member

shale Vp/Vs was calculated from the VRH averages presented in Table 3. Silty limestone data from Wilkens *et al.* [4] is also presented which shows fairly good agreement with range of Vp/Vs produced by sandstone-limestone end members.

The ranges of velocity ratios shown as solid lines in Figure 2 are for isotropic mineral assemblages. Clay-rich rocks, on the other hand, likely possess significant anisotropy due to preferred mineral orientation, thus complicating Vp/Vs - lithology systematics. This is illustrated in Figure 2, where ranges of possible Vp/Vs values due to anisotropy for the two shale samples are shown as dashed lines. Based on the results shown in Figure 2, we conclude:

1). Low values of Vp/Vs (less than ~ 1.65) are unique to quartz rich lithologies.

2). Values above 2.0 can originate from anisotropy in clay-rich sedimentary rocks.

3). Values of Vp/Vs between approximately 1.65 and 2.00 can be attributed to anisotropic shale or mixtures of calcareous, arenaceous, and argillaceous rock.

REFERENCES

1. Tatham, R. H. Vp/Vs and lithology. *Geophysics* 47, 336-344 (1982).
2. Eastwood, R. L., and Castagna, J. P. Interpretation of Vp/Vs ratios from sonic logs. *Shear Wave Exploration* (Edited by S. H. Danboom and S. N. Domenico), pp. 139-154. Society of Exploration Geophysicists Geophysical Development Series 1 (1986).
3. Robertson, J. D. Carbonate porosity from S/P traveltime ratios. *Geophysics* **52**, 1346-1354 (1987).
4. Wilkens, R., Simmons, G. and Caruso, L. The ratio Vp/Vs as a discriminant of composition for siliceous limestones. *Geophysics* **49**, 1850-1860 (1984).
5. Christensen, N. I. Pore pressure, seismic velocities, and crustal structure. *Geophysical Framework of the Continental United States* (Edited by L. C. Pakiser and W. D. Mooney), pp. 783-798. Geol. Soc. Am. Mem. 172 (1989).
6. Christensen, N. I. and Szymanski, D. I. Seismic properties and the origin of reflectivity from a classic Paleozoic sedimentary sequence, Valley and Ridge province, southern Appalachians. *Geol. Soc. Am. Bull.* **103**, 277-289 (1991).
7. Johnston, J. E., and Christensen, N. I. Shear wave reflectivity, anisotropies, Poisson's ratios, and densities of southern Appalachian Paleozoic sedimentary sequence. *Tectonophysics* **210**, 1-20 (1992)
8. Christensen, N. I. Compressional wave velocities in metamorphic rocks at pressures to 10 kilobars. *J. Geophys. Res.* **70**, 6147-6164 (1965).
9. Christensen, N. I. Shear wave velocities in metamorphic rocks at pressures to 10 kilobars. *J. Geophys. Res.* **71**, 3549-3556 (1966).
10. Jones, L. E. A. and Wang, H. F. Ultrasonic velocities in Cretaceous shales from the Williston basin. *Geophysics* **46**, 288-297 (1981).
11. Lo, T. W., Coyner, K. B. and Toksoz, M. N. Experimental determination of elastic anisotropy of Berea sandstone, Chicopee shale, and Chelmsford granite. *Geophysics* **51**, 164-171 (1986).
12. Christensen, N. I. Measurement of dynamic properties of rock at elevated temperatures and pressures. *Measurements of Rock Properties at Elevated Pressures* (Edited by H. J. Pincus and E. R. Hoskins), pp. 93-107. American Society for Testing and Materials, Philadelphia (1985).
13. Hearmon, R. F. S. *An Introduction to Applied Anisotropic Elasticity*. Oxford University Press, London (1961).