Anisotropic effects of non-axial seismic wave propagation in foliated crustal rocks

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1. Introduction

[1] Seismic wave propagation within the crust is usually in directions not parallel to inherent rock symmetry axes due to geologic dips of structures and/or oblique-to-arcuate raypaths. We use Christoffel equations and petrophysically measured velocities of crustal metamorphic rocks to examine anisotropic velocities and traveltime effects as functions of non-axial propagation. P- and S-wave velocities are trigonometrically weighted between fast and slow axial velocities. Averaging over a range of non-axial angles yields effective velocities as encountered by through-going crustal raypaths. Axial velocities cannot be simply used to quantify crustal anisotropic effects. The angular behavior of P-wave delay and shear wave splitting are sensitive to Vp measured diagonal to the material symmetry axes; this diagonal velocity controls the presence or absence of non-axial shear wave singularities. These behavior help quantify the contributions of crustal rocks to observations of seismic anisotropy which can serve as proxies for crustal deformation. INDEX TERMS: 7203 Seismology: Body wave propagation; 7260 Seismology: Theory and modeling; 7205 Seismology: Continental crust (1242); 5102 Physical Properties of Rocks: Acoustic properties


[4] P- and S-wave behavior in non-axial directions of propagation can be better understood with the aid of Christoffel equations [Christoffel, 1877]. These equations can be used to obtain P- and S-wave velocities at a relative propagation angle, γ, inclined to material symmetry axes (Figure 1) provided axial and diagonal velocities are known or measured. For hexagonal symmetries these equations reduce to [Mangrave, 1970]:

\[ qV_p^2(\gamma) = (1/2\rho) \left[ c_{11} \sin^2 \gamma + c_{33} \cos^2 \gamma + c_{44} + M^{1/2} \right] \]

\[ qV_s^2(\gamma) = (1/2\rho) \left[ c_{11} \sin^2 \gamma + c_{33} \cos^2 \gamma + c_{44} - M^{1/2} \right] \]

where \( M = (c_{11} + c_{44}) \sin^2 \gamma - 2(c_{13} - c_{44}) \cos^2 \gamma + (c_{13} - c_{44})^2 \sin^2 \gamma \) and the elastic constants are derived from four axial and one diagonal petrophysical measurements (Table 1):

\[ c_{11} = \rho V_{P90}^{-2}, \quad c_{33} = \rho V_{P90}, \quad c_{44} = \rho V_{SH90}^{-2} = (c_{11} - c_{12})/2, \]

\[ c_{13} = -c_{44} + \left[ (4pV_{P45}^2 - 2pV_{P45}^2) (c_{11} + c_{33} + 2c_{44}) \right]^{1/2}. \]

[5] An example of a strongly foliated crustal anisotropic terrane is the Haast schist of South Island, New Zealand [e.g., Okaya et al., 1995; Godfrey et al., 2000]. Laboratory velocity measurements indicate this schist is hexagonal in symmetry. Petrophysically measured principal velocities at 600 MPa (Table 1) may be inserted into the above equations in order to obtain non-axial qVp, qVsv, and qVsh velocities between 0° and 90° as measured from the schist’s hexagonal symmetry axis (Figure 2a). The resulting P-wave velocity is not linear but varies trigonometrically between 0° and 90° and is uniformly slow between 0°–40°. The SH-wave, whose pure mode has both propagation and vibration directions in the plane of foliation, has velocity behavior with similar angular variation. The SV-wave, whose pure mode has either propagation or vibration normal to the foliation, reaches a local maximum at approximately 40° before decreasing to its original axial value at
Figure 1. Foliated rock with foliation (planes) and lineation (dashed lines and circles in cross-section) tilted with respect to geographical axes $x$-$y$-$z$. For a hexagonal symmetry approximation, the symmetry axis, $c$, is normal to foliation while velocities are the same in all directions within the $a$-$b$ plane. Arrow represents direction of a propagating seismic wave inclined at relative angle $\gamma$ to the (tilted) symmetry axis.

90°. The SV-wave exhibits 4\(^{\circ}\) angular variation while the P- and SH-waves exhibits 2\(^{\circ}\) variation.

[6] Traveltime effects in non-axial directions are shown in Figures 2b-c for the Haast schist. A P-delay is up to 0.02 sec per propagation km for angles within 40° relative to the fast direction at 90°. Shear wave splitting is maximized at 90° where SH is faster than the SV wave. However, the SV wave exhibits “crossover” behavior in that between 0–50° the SV wave is faster, and at 50° a singularity (“crossover point”) exists where both S waves have the same velocity with no splitting (Figure 2a). Due to the geometry of hexagonal symmetry, in three dimensions the singularity is conical.

[7] For waves traveling in the crust, a single angle of propagation, $\gamma$, is not common. Rather, a range of angles relative to material symmetry axes may be encountered due to directional changes in either raypath dip and/or geological structure. With the use of equations (1) and (2), an effective P-wave velocity can be calculated by averaging over the range of angles which may be encountered upon propagation. Figure 2d illustrates effective P-wave velocities for all possible linear distributions of dips for the Haast schist. For example, a seismic wave encountering a foliated terrain whose dips range uniformly between 10–50° will travel at an averaged, effective P-velocity of 5790 m/s, whereas if the dips ranged from 20–70° the effective velocity is 5945 m/s (Figure 2d). Approximately half of all possible dip ranges result in slow effective velocities (<6000 m/s); only one-third of all dip ranges have an effective velocity greater than the average velocity of the axial P-wave velocities, $(V_{p90} + V_{p45})/2$ or 6125 m/s. The broad set of low effective velocities is due to the uniformly low P-wave velocities within 0–50° (Figure 2a). A seismic wave will

Figure 2. Anisotropic velocity behavior for the Haast schist. (a) Velocities as a function of propagation angle $\gamma$ relative to hexagonal symmetry axis. $qV_p$, $qV_{SV}$, and $qV_{SH}$ velocities are shown for $\gamma = 0–90°$. The five petrophysically measured values are overlain (triangles). For this schist, a $qV_{SV}-qV_{SH}$ crossover exists at 50°. (b) P-wave traveltime delay (sec) per km of propagation with respect to the fastest P at 90°. (c) Amount of shear wave splitting (sec) per km of propagation created by the $qV_{SV}$ and $qV_{SH}$ angular behavior. At 50° the SV and SH switch as to being the earlier-arriving phase. (d) Effective $qV_p$ (m/sec) for a range of propagation angles or geological dips calculated by averaging curves in (a) over a linear distribution of angles. For a given range of dips, e.g., 10–50°, the lower dip is identified on the horizontal axis and the resulting effective velocity can be obtained at height of the higher dip (vertical axis). The effective velocities are in general low and reflect the low qVp velocities as seen in Figure 2a. (e) Effective shear wave splitting (sec) times per propagation km. Maximum splitting of 0.05 sec per propagation km is obtained for a narrow range of angles (70–90°). At intermediate angles the splitting is neutralized since at low angles the SV and SH velocities are similar.

Table 1. Petrophysical velocity measurements of representative foliated crustal rocks. Measurements at 600 MPa.

<table>
<thead>
<tr>
<th>Rock Type (Locality)</th>
<th>$V_{p90}$ m/s</th>
<th>$V_{p45}$ m/s</th>
<th>$V_{SV90}$ m/s</th>
<th>$V_{SV45}$ m/s</th>
<th>$V_{SH90}$ m/s</th>
<th>$V_{SH45}$ m/s</th>
<th>Density kg/m$^3$</th>
<th>%Anisotropy$^{b}$</th>
<th>$qV_{p}$ ratio</th>
<th>Ref$^{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haast Schist (South Island, New Zealand)</td>
<td>5750</td>
<td>6501</td>
<td>5881</td>
<td>3332</td>
<td>3931</td>
<td>2718</td>
<td>12.3</td>
<td>16.5</td>
<td>1.35</td>
<td>A,B</td>
</tr>
<tr>
<td>Chugach phyllite (Chugach terrane, Alaska)</td>
<td>5994</td>
<td>6547</td>
<td>6084</td>
<td>3456</td>
<td>3931</td>
<td>2723</td>
<td>8.8</td>
<td>12.9</td>
<td>1.46</td>
<td>C,B</td>
</tr>
<tr>
<td>Poultney slate (Vermont, USA)</td>
<td>5403</td>
<td>6639</td>
<td>5826</td>
<td>3294</td>
<td>3948</td>
<td>2665</td>
<td>9.2</td>
<td>18.1</td>
<td>1.97</td>
<td>D,B</td>
</tr>
<tr>
<td>Coldfoot schist (Brooks Range, Alaska)</td>
<td>5800</td>
<td>6357</td>
<td>6059</td>
<td>3351</td>
<td>3685</td>
<td>2656</td>
<td>7.8</td>
<td>9.5</td>
<td>1.22</td>
<td>E,B</td>
</tr>
<tr>
<td>Nanga Parbat Gneiss (N.P., Pakistan)</td>
<td>5856</td>
<td>6331</td>
<td>6230</td>
<td>3332</td>
<td>3931</td>
<td>2718</td>
<td>8.8</td>
<td>12.9</td>
<td>1.46</td>
<td>F</td>
</tr>
</tbody>
</table>

$^{a}$ $V_{p90}$ = normal to foliation (slower P-wave), $V_{p45}$ = within planes of foliation (faster P-wave), $V_{SV90}$ = at 45° angle between normal and planes of foliation (intermediate value of $V_p$). $V_{SV90}$ = $V_{SV90}$ = $V_{SH90}$ = either propagation or particle motion is normal to foliation (slower S-wave). $V_{SV90}$ ranges have an effective velocity between 0–100%. $V_{p45}$ = propagation and particle motion both in the foliation planes (fastest S-wave). $^{b}$ Normalized $V_{p45}$ is $V_{p45} = 100 \times \frac{(V_{max} - V_{min})}{(V_{max} + V_{min})}$ for both P and S velocities. Ratio is VP%-to-VS%.

References: A = Okaya et al. [1995], B = Godfrey et al. [2000], C = Brocher et al. [1989], D = Christensen [1965; 1966], E = Fuis et al. [1997], F = Meltzer and Christensen [2001]. Descriptions of mineral assemblages provided in these references.
encounter the fastest P-wave velocities only if propagating at 70–90° from the symmetry axis (within 20° of propagation to the foliation planes). Similar calculations may be made for geologically more-plausible (non-linear or weighted) ranges of dips.

[8] Due to the crossover behavior in the velocities of the SV- and SH-waves, a uniform value of shear-wave splitting is not common. Effective shear-wave splitting times for linear ranges of propagation angles are calculated for the Haast schist (Figure 2c). Maximum splitting is maintained at high propagation angles (70–90°). At moderate angles splitting is negated by the SV-SH crossover. At low angles SV is the earlier-arriving phase but the splitting amounts are small (<0.015 sec per propagation km). There is no splitting for propagation parallel to the symmetry axis. Figures 2d and 2e both indicate that for most directions of propagation the slower Haast velocities are encountered; observations of fast Haast velocities indicate propagation in the narrow range of 70–90° from the symmetry axis.

3. Non-Axial Propagation Velocities for Representative Foliated Crustal Rocks

[9] We next compare our results for the Haast schist with a suite of representative crustal foliated rocks for which petrophysical velocity measurements have been obtained. This suite includes the Chugach phyllite, Alaska [Brocher et al., 1989]; Poultney slate, Vermont, [Christensen, 1965, 1966]; Coldfoot schist, Brooks Range, Alaska [Fiú et al., 1997]; and Nanga Parbat gneiss, Nanga Parbat, Pakistan [Meltzer and Christensen, 2001]. Laboratory measured velocities at 600 MPa (equivalent to a crustal depth of ~20 km) for hexagonal symmetry are listed in Table I. As with the Haast schist, these measurements are inserted into equations (1) and (2) to produce non-axial velocity curves. Figure 3a illustrates the behavior of qVp, qVsv, and qVsh for these samples. In all cases, qVp and qVsh vary not linearly but trigonometrically between their slow and fast velocities at 0 and 90°, respectively. qVsv exhibits differing behavior. The Haast schist and Chugach phyllite exhibit S-wave crossover behavior and have faster qVsv at 0 to ~50°. The Poultney slate qVsv nearly equals qVsh between 0–35° (e.g., little S-splitting), above which qVsh becomes the fast S. For the Coldfoot schist and Nanga Parbat gneiss, qVsv is always slower than qVsh, and in the case of the gneiss it is slowest not at 0° or 90° but at 50°. The shear-wave splitting traveltime effects in Figure 3C reflect these relative behavior of qVsv and qVsh. The Poultney slate exhibits a maximum of 0.125 sec splitting per propagation km while the phyllites, schists, and gneiss exhibit 0.25–0.50 sec per propagation km.

[10] A traditional calculation of anisotropy percentage as defined by Birch [1960] is %Anisotropy = (Vmax – Vmin)/Vave where Vave = (Vmax + Vmin)/2. Calculations of %Anisotropy for P and S velocities for all samples are presented in Table 1. These values show that the ratio of Vp45°-to-Vp90° anisotropy is not uniform but ranges between 1.22 to 1.97 (Table 1). We attribute this to compositional differences in the anisotropic mineral constituents and is thus rock sample specific; one cannot apply a generalized ratio to obtain the percent anisotropy of a P- or S-wave even if the other wave's percentage is known. We also note that this %Anisotropy calculation uses velocities measured in axial directions, but loses meaning in non-axial directions, particularly with SV and SH waves exhibiting different angular behavior.

4. SV Velocity Behavior and the Role of Vp45°

[11] As can be seen in Figure 3, the qVsv-qVsh crossover behavior does not necessarily vary according to metamorphic grade. Examination of equations (1) and (2) reveal that qVp and qVsv have similar derivation and are both sensitive to not only stiffness elements defined by axial velocities, but also to the element c13 which itself is sensitive to the diagonal velocity Vp45°. In order to determine the effects onto the qVp and qVsv velocities, we calculate for the representative rock samples hypothetical velocity curves using the principal axial velocity measurements (Table 1) and different values of the diagonal velocity based on the normalization formula vp45° = (hVp45° - Vp90°)/(Vp90° - Vp45°) × 100% where vp45° is a percentage between 0 and 100% so that hypothetical diagonal velocity, hVp45°, ranges between Vp90° and Vp45°.

[12] For the Haast schist in Figure 3a, we compute a series of hypothetical qVp and qVsv velocity curves where hVp45° varies every 10% between the slow and fast axial Vp (cold to warm color curves in Figure 3). At vp45° = 0%, hVp45° = Vp90° and for angles of 20–50° qVp is actually slower than VP90°. Similarly, at vp45° = 100%, hVp45° = Vp45° and qVp is faster than VP90° above 45°. At vp45° = 50%, hVp45° is the average of the two axial P-velocities but qVp
does not vary linearly between the two axial velocities. Travel time P-delays mimic the hypothetical qVp curves (color curves in Figure 3b).

13 qVsv curves also exhibit sensitivity to normalized vP45. At vP45 = 0%, qVsv reaches a maximum at 45° before returning to VSV(90°) = VSV(90°). For the Haast schist, this maximum is slightly larger than the largest SH velocity at VSH(0°). At vP45 = 50%, qVsv is essentially constant with no angular variation. At vP45 = 100%, qVsv reaches a large velocity low at 45°. These large magnitude variations in qVsv velocities when compared to qVSH produce a large degree of shear wave splitting over a broad range of angles and Vp45 percentages (color curves in Figure 3c). The actual Haast schist sample has a measured Vp45 which is at 17.4% above Vp0 (Table 1), producing the qVp and qVsv (bold) curves in Figure 3a and the (bold) S-splitting curve in Figure 3c.

14 The five representative samples in Figure 3a are arranged in essentially increasing Vp45; examination of the hypothetical velocity curves in relation to the actual rock sample velocities (Table 1) reveals conditions for qVsv crossover behavior. The Haast schist and Chunagch phyllite have low Vp45 (17.4% and 16.3%, respectively) and a qVsv which exhibits crossover behavior. The Poulney slate at vP45 = 34.2% has a qVsv which is nearly the same as qVSH between 0°–45° with minor crossover effects and minimal S-splitting. The Coldfoot schist at vP45 = 46.5% has a qVsv which is near constant and at all angles is slower than qVSH. The Nanga Parbat gneiss at vP45 = 78.7% has a qVsv with decreased velocity in off-directions. In this latter case, the shear wave splitting is maximized not at 90° but at 50° and has a large contribution over a wide range (30°–90°) of propagation angles.

5. Discussion and Summary

15 The magnitudes of fast and slow P- and S-velocities for each foliated crustal rock are unique and are related to mineral assemblages which are controlled by metamorphic grade and rock chemistry. Vp45 controls the non-axial behavior of qVp and qVsv between 0°–90° and thus the existence of non-axial propagation shear wave singularities. Failure to detect shear wave splitting in anisotropic crustal sections does not necessarily imply propagation parallel to a hexagonal symmetry axis (e.g., vertical waves through horizontal foliation). Vp45 is not necessarily the simple average of the P-velocities measured at 0° and 90°, thus an accurate assessment of this diagonal P-wave velocity is essential. A more comprehensive database of petrophysical measurements including the diagonal velocities is needed in order to identify a relationship between Vp45, mineral constituents and orientations, and metamorphic grades.

16 While we have illustrated these angular effects using a hexagonal symmetry approximation, many foliated crustal rocks are more realistically orthorhombic [e.g., Christensen, 1965]. In such cases, the three axial planes (a-b, a-c, b-c) have separate angular velocity behavior, requiring three independent Vp45 measurements. In addition, the solution for propagation velocities in any non-axial/non-axial plane direction requires the orthorhombic Christoffel equations as functions of both azimuth and inclination wave directions.

17 The ability to map the presence and extent of anisotropic terranes is related to the ability of a propagating seismic wave to resolve zones of rock fabrics. Factors which affect this resolution are frequency content and propagation direction of the seismic wavefield with respect to the scale length, continuity, and orientation of the fabrics (trend plane and depth plus orientation and continuity of internal structures).

18 Although complex to analyze, multi-component seismic refraction and reflection experiments may be designed to explicitly seek these terranes using P- or S-wave time delays or polarization or by using perpendicular transects. Polarization and splitting behavior of crustal Ps converted phases which are commonly used in receiver function studies [e.g., Jones and Phinney, 1998; Levin and Park, 1997] may be used to isolate crustal anisotropy in a manner analogous to mantle SKS shear wave splitting methods. Dense recording arrays used within such experiments may provide sufficient spatial resolution to identify with confidence lateral variations and terminations of an anisotropic terrane. Mantle S- and SKS-splitting results may contain a second-order crustal contribution or alteration to direction and lag if the upcoming seismic waves contain sufficiently high bandwidth to respond to crustal layers.

19 The need for anisotropic velocity estimation in non-axial directions is critical in crustal wave propagation due to structural heterogeneity (changes in rock orientation) plus changing raypath directions for common crustal seismic waves. Interpretations of crustal seismic data cannot simply use vertical or horizontal orientations of both material symmetry axes and seismic waves unless such orientations exist based on geological and seismic evidence. Such anisotropy when defined through carefully designed field investigations and petrophysical laboratory measurements will provide valuable information on the magnitudes and extent of crustal tectonic deformations.

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