

# EOSC 512: Advanced Geophysical Fluid Dynamics

Fall 2020

## Schedule

Lectures: Times TBD.

## Instructor

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## Philosophy

Geophysical fluid dynamics is a field in which observations and theory complement one another. We try to understand observations (in the ocean, atmosphere, within the fluid core of the earth, or even on other planets) in the context of an appropriate mathematical model, whose development is in turn guided by the observations. This interdependence may seem surprising - after all, the basic equations governing the complete motions of fluids can be written down in a few lines - isn't that the model? Unfortunately, these equations are so complicated to solve that it is impossible to make any progress unless simplifications are made. Theoretical advances basically boil down to learning which approximations are useful and understanding the consequences of those approximations. The mathematical "language" being developed is guided, of course, by the hope that the resulting equations may be useful in describing particular aspects of the workings of the ocean, atmosphere, or fluid core.

## Learning goals

The purpose of this course is to a) introduce the student to the dynamical principles governing the large-scale low-frequency motions in strongly rotating fluid systems (like the ocean, atmosphere, and liquid planetary core), and their consequences, and b) to develop the skills required to manipulate and use these equations to solve problems. At the end of this course, students should be able to

1. write down the 'standard equations' of GFD, identify the different terms, and explain how different dynamical features depend on these terms. Equations include:
  - geostrophic and quasi-geostrophic equations
  - shallow-water equations
  - boundary layer equations
  - thermodynamic relationships

2. Define the following terms (the “language” of GFD) and identify them when they arise in the context of dynamical interpretations:
  - Coriolis parameter
  - vorticity
  - boundary conditions
  - baroclinic and barotropic
  - Boussinesq approximation
  - Beta effect
  - potential and kinetic energy
  - eddy viscosity
  - group and phase velocity
  - entropy, enthalpy, and potential temperature
  - standard nondimensional numbers: Rossby, Ekman, Burger, Reynolds, Rayleigh
3. Use the following mathematical techniques to simplify complex equation sets:
  - linearization
  - scaling arguments and perturbation expansions (incl. WKBJ)
  - normal mode techniques
  - complex exponentials in wave and instability problems
4. Choose the appropriate mathematical technique to simplify and solve particular “canonical” GFD problems. Examples:
  - Taylor-Proudman theorem
  - Long Waves in Nonrotating systems (seiches, edge waves):
  - Long Waves in Rotating systems (Poincare, Sverdrup, Kelvin, Rossby)
  - Rossby Reflection and Adjustment problems
  - Rayleigh, baroclinic, and barotropic instability
  - Ekman boundary layers

## Requirements

This course is mathematical and assumes a working knowledge of vector calculus (div, grad, curl), partial differential equations (i.e. you can solve at least some of them), and some exposure to complex analysis (you know that  $z = x + iy$  and  $\exp(iz) = \cos(z) + i * \sin(z)$ ). Previous experience with fluid mechanics or knowledge of the basic characteristics of the ocean and/or atmosphere is useful but not required.

## Course Outline

1. Basic Equations
  - Typical oceanic scales
  - Equations of fluid motion (Mass and momentum conservation)
  - “Filtering” approximations
  - Rotating coordinate system
  - Mechanical Energy and Bernoulli equation
  - Boundary conditions
  - Equations of Thermodynamics (Heat Energy conservation and equation of state)
  - Vertical coordinates
2. Fundamentals of Rotating Flows

- Vorticity
  - Circulation
  - Vorticity equation
  - Taylor/Proudman theorem
  - Geostrophy, geostrophic degeneracy
  - Thermal wind
  - Quasi-geostrophy (QG) - the basis of GFD!
3. Concepts in Wave Propagation
- Wave equations
  - Plane waves
  - Dispersion
  - Group velocity/phase velocity
4. Characteristics of Inviscid flow - the Shallow-Water Equations (SWE)
- Shallow-water equations for a single layer
  - Potential vorticity for SWE
  - Small-amplitude motions
  - Seiches - dispersion, reflections and step boundaries
  - Effects of rotation - Sverdrup Waves
  - Effects of side boundaries - Poincaré and Kelvin Waves
  - Effects of earth's curvature - Rossby Waves
  - More Rossby waves: dispersion, energy transport, reflection
  - Adjustment problems
  - Initial disturbances
  - Two or more layers, continuous stratification, vertical modes
5. Friction and Viscosity
- Reynolds stresses
  - Eddy viscosity, mixing-length theory.
  - Ekman layer - bottom, top, and inertial oscillations.
  - ~~Realistic boundary layers~~
  - ~~Ekman pumping~~
  - ~~Spindown time~~
6. Instability in rotating flows
- Baroclinic instability - Eady and Phillips problem
  - ~~Barotropic instability~~
7. Convection
- ~~Rayleigh Instability~~

## Evaluation

1. Assignments (50%)
  - 5 Assignments over the term.
2. Midterm (20%) and Final (30%) Exams
  - The style of the mid-term and final is still TBD.

## Texts

I do not use any single text. However, you can find similar material (even the source) in many different places. These include:

Pedlosky, J., Geophysical Fluid Dynamics, Second Edition, Springer-Verlag, 1987.

This is a comprehensive and complete discussion of most of the basics in dynamical oceanography. It is, however, rather dense and relies a lot on nondimensionalizations which are not easy to find when dipping into it for knowledge on a specific topic. Other references:

Gill, A. E., Atmosphere-Ocean Dynamics, Academic Press, 1982.

covers much of the same ground as Pedlosky. However, the approach is much less formal and includes more observational data. A good (but decidedly opinionated) historical overview of “practical” GFD (including the observational basis and attempts to reconcile observations with theory) is given in

Wunsch, C. W., The Ocean Circulation Inverse Problem, Cambridge Univ. Press, 1996.

Need some nitty-gritty details (e.g., how to actually compute a geostrophic velocity from real data)? Read this:

Pond, S. and G. L. Pickard, Introductory Dynamic Oceanography, Pergamon, 1983

Much of the material on waves in this course is adapted from a course I took in graduate school, which was enormously influential on the way I approach thinking about the ocean. It turns out that the notes from this course were all put together by my instructors at that time as a tribute to the man from whom they learned it all...and you can get a copy of those notes from [http://oxbow.sr.unh.edu/ChapmanRizzoli/Wave\\_Motions\\_in\\_the\\_Ocean.html](http://oxbow.sr.unh.edu/ChapmanRizzoli/Wave_Motions_in_the_Ocean.html). Reading it will make you part of the 3rd generation of students learning about waves the Hendershott way.

A very good book reference for wave and wave motions is the out-of-print

LeBlond, P. H. and L. Mysak, Waves in the Ocean, Elsevier, 1978

Another reference for waves is the (somewhat dated) monograph

Turner, J. S., Buoyancy Effects in Fluids, Cambridge Univ. Press, 1973.

Finally, although we concentrate on the oceans here, many of the concepts carry over to studies of the atmospheric circulation (and vice versa); a good reference from that point of view is

Holton, J. R., An Introduction to Dynamic Meteorology, Second Edition, Academic Press, 1979.

Fluid dynamics in general is a very large field, of which GFD is only a small part. Many of the concepts and scalings used are much more general; good discussions can be found in

Batchelor, G. K., An Introduction to Fluid Dynamics, Cambridge Univ. Press, 1967.

A detailed derivation of the First Law of Thermodynamics in the context of the ocean is contained in the manual for the new seawater standard, TEOS-10 (available online at [www.teos-10.org](http://www.teos-10.org))

IOC, SCOR and IAPSO: The international thermodynamic equation of seawater - 2010: Calculations and use of thermodynamic properties. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. (2010)

There's also a good development in

Griffies, S. M., Fundamentals of Ocean Climate Models, Princeton University Press, 2004

or, in a more accessible set of Lecture Notes:

McDougall, T. MATH5185 Lectures on Thermodynamics, available at [www.teos-10.org](http://www.teos-10.org)

Finally, these are by no means the only books to look at. New texts come out every year, although I am often disappointed at different aspects of the way they present things. In addition, there is also a class of 'advanced' books that take you further into some of the things we will talk about in this course.

Huang, R.X., Ocean Circulation: Wind-drive and Thermohaline Processes, Cambridge, 2010

is one that covers a lot of more recent research in this area.