Classical Tidal Harmonic Analysis with Error Analysis in MATLAB using T_TIDE

Rich Pawlowicz*
Department of Earth and Ocean Sciences,
University of British Columbia,
Vancouver, Canada, V6T 1Z4

Bob Beardsley, Steve Lentz,
Woods Hole Oceanographic Institution,
Woods Hole, MA, 02543, USA

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*Dept. Of Earth and Ocean Science, University of British Columbia 6270 University Blvd., Vancouver, B.C., Canada, V6T 1Z4. email: rich@ocgy.ubc.ca

Abstract

A standard part of any oceanic pressure gauge or current meter analysis is the separation of tidal from non-tidal components of the signal. The tidal signal can either be discarded, or its characteristics described in some fashion useful for further analysis. Although tidal signals can be removed by standard high or band-pass filtering techniques, their relatively deterministic character and large amplitude make special techniques more effective. In classical harmonic analysis the tidal signal is modelled as the sum of a finite set of sinusoids at specific frequencies related to astronomical parameters. A set of programs has been written in MATLAB to a) perform classical harmonic analysis for periods of about one year or shorter, b) account for (some) unresolved consituents using nodal corrections, and c) compute confidence intervals for the analyzed components.

As the earth rotates on its axis spatially varying gravitational forces from the moon and the sun act on the ocean, generating a forced elevation and current response primarily (but not solely) at diurnal and semi-diurnal frequencies. Body forces act directly on deep oceanic waters. Tidal effects in coastal regions are not directly forced by these astronomical forces. Instead they arise as a side-effect of deep oceanic variability, propagating through shallower coastal waters as a wave. In a typical oceanic time series tidal variability is often the largest signal. Power spectra for such time series are often characterized by a broad hump with a low-frequency maximum and a decline at higher frequencies. Superimposed are a number of sharp tidal peaks near diurnal and semi-diurnal frequencies, and sometimes a broader peak associated with Coriolis or inertial effects. Dynamical analysis requires the separation of the tidal signal from sub or super-tidal variations, or in some cases that we separate tidal effects from inertial effects at a nearby frequency. The tidal information information is either discarded kept for further analysis.

Standard high/low/bandpass filtering techniques (Jackson, 1986) can be used but in general these are inefficient because fairly narrow filters with a great deal of rejection are needed. Also, although these are useful in analyzing non-tidal variability they provide no compression of the tidal information. Specialized techniques have been devised to take advantage of the "deterministic" nature of tidal processes. In classical harmonic analysis, the tidal forcing is modelled as a set of spectral lines, i.e., the sum of a finite set of sinusoids at specific frequencies. These frequencies are related directly to astronomical parameters (Godin, 1972). They can be described as arising from interactions of 6 basic periods: the lunar day (24.8 hour), the lunar month (27 days), the tropical year (1 year), lunar perigee (8.85 years), the precession of the lunar nodes (18.6 years), and perihelion (21000 years) and are specified by various combinations of sums and differences of integer multiples of the 6 frequencies. The set of 6 signed integers required to describe a particular frequency are then called *Doodson numbers*. Many of the more important frequencies have names such as "M_2", "K_1", etc. From astronomical considerations an "equilibrium" response can be predicted; this is the phase and amplitude that would be observed if the response of the earth was fast enough that the surface deformation was effectively in equilibrium with the forcing at all times. The real ocean is very definitely not in equilibrium with the tidal forcing. However, as tidal amplitudes are small compared with total ocean depth the dynamics are very nearly linear, implying that the forced response contains only those frequencies present in the forcing. A least squares fit can be used to determine the relative phase and amplitude of each frequency in the response. This phase/amplitude data thus provides a compression of the data in the complete time series, which can then be compared with similar data at other locations to understand the characteristics of tidal dynamics, or can be used to synthesize time series of tidal effects at other times for predictive purposes.

There are several drawbacks to classical harmonic analysis. The first is that, ignoring the modulation of perihelion which is effectively constant over historical time, an ≈ 18.6 year time series is required to resolve all of the listed frequencies (that is, the number of wavelengths of each constituent in the record is at least 1 different from all other constituents). In practise record lengths are generally one year or shorter. In order to handle this issue an assumption is made that the relative phase/amplitudes of sinusoids with similar frequencies (i.e. those whose first three Doodson numbers are identical) are identical to predicted in the equilibrium response. In such a cluster large equilibrium peaks are surrounded by small subsidiary peaks in frequency-space which provide "nodal modulations" (or more correctly "satellite modulations") to the main peak. The appearance of the total signal will be a sinusoid whose phase and amplitude varies slowly with time. These changes are slow enough to be considered effectively constant for record lengths of up to one year. At much shorter record lengths another problem arises. The frequency resolution further degrades until even dissimilar constituents are unresolveable. The best solution is to apply inference. This technique for finding the absolute phase/amplitude requires that the relative differences in phase/amplitude between the two unresolved constituents is known from other nearby data. If this is not the case, it is thought best to discard the smaller constituents and fit only to the largest in a given frequency interval.

Another drawback of classical analysis is that it provides no easy way to determine whether the resulting phase/amplitude of a given sinusoid is meaningful in a deterministic way (i.e. it is truly a tidal line), or whether it results from fitting to a component of the broad-spectrum variability. In general it is likely to include both and some kind of confidence interval would be useful. In order to address this issue the "response" method was invented (Munk and Cartwright, 1966). Although this provides better results than classical harmonic analysis it has not found widespread use.

Further problems with classical harmonic analysis arise in coastal regions where the tidal response is in the form of a wave propagating onshore. In large estuaries the seasonal change in salinity and flow may change the dynamic response but as these changes can vary from year to year this is not really deterministic. Instead spectral peaks are broadened so that they are no longer pure lines. Within smaller estuaries the tidal height variations are significant compared to water column depth and a variety of nonlinear effects can occur. For example, flood periods shorten and intensify and ebbs lengthen. As long as these effects are resonably deterministic they may be handled by adding extra "shallow water" constituents. More problematic in these regions are the effects of internal variability. Tidal interactions with varying topography can produce large internal waves and bores whose characteristics are highly sensitive to ambient stratification. In such cases the assumption of "line" frequencies becomes questionable and other techniques such as wavelet analysis have been suggested (Jay and Flinchem, 1999).

Here we describe T_TIDE, a package of routines that can be used to perform classical harmonic analysis with nodal corrections, inference, and a variety of user-specified options. There are several novel features. First, although the harmonic analysis algorithm with nodal corrections etc. itself is not original (other than conversion to complex algebra), being heavily based on (Foreman, 1977) and Foreman (1978), it is implemented in MATLAB, an analysis package widely used by oceanographers. This allows for easy use within the framework of a complete analysis involving plotting of raw data, scatter plots, and so forth. Second, the code is written directly in matrix terms and thus relatively easy to understand and modify if required. Finally, in order to differentiate between true deterministic (line) frequencies and broad-spectrum variability, confidence intervals for the estimated tidal parameters are computed using one of several user-selectable algorithms. The package is made up of a number of files each of which contains one or more functions. User-callable functions generally have a "*t_*" prefix to prevent namespace collisions.

The paper is composed of four parts. In the first the form of the equilibrium potential is described. In the second part the mathematical basis of the technique for making phase/amplitude estimates is described. In the third part the generation of confidence intervals is outlined. Finally an illustrative example is discussed.

1 Tidal potential

The effect of gravitational force vectors from the sun and moon, $\mathbf{F}$, can be written as the gradient of a scalar potential V, $\mathbf{F} = -\nabla V$. The magnitude of this potential at the earth's surface at any time obviously is dependent on the relative positions of the earth, moon, and sun. In Doodson's development (described in /citegodin:72) the potential is described as a function of lunar time τ, and other astronomical variables (which are also functions of time):

$$\begin{aligned} s &= \text{mean longitude of moon} \\ h &= \text{mean longitude of sun} \\ p &= \text{longitude of perigee} \\ N' &= \text{negative of the longitude of the ascending node} \\ p' &= \text{longitude of perihelion} \end{aligned}$$

where all terms are in units of cycles. These variables can be evaluated for a given Julian date in subfunction *astron* of file *t_vuf.m*. Their effects are combined with the integer Doodson number set $\{i', j', k', l', m', n'\}$ into the astronomical argument $V_a = i'\tau + j's + k'h + l'p + m'N' + n'p'$. Sets with a common i' are called a species (thus the slow, diurnal, and semidiurnal species for $i' = 0, 1, 2$ respectively), and sets with common $i'j'k'$ are called a subgroup. The constituent frequency σ is defined as $\sigma = 2\pi dV_a/dt$. The tidal potential is then be written in the form

$$V = \sum_{i'=0}^{3} \left[G_{i'}(\theta) \sum_{j'k'l'm'n'} A'_{i'j'k'l'm'n'} \cos(2\pi V_a) + G'_{i'}(\theta) \sum_{j'k'l'm'n'} B'_{i'j'k'l'm'n'} \sin(2\pi V_a) \right] \tag{1}$$

For a given Doodson number set either A' or B' is nonzero, but not both. These constants are tabulated and stored in data structures that can be loaded using *t_getconsts*. The geodetic functions $G_{i'}$ and $G'_{i'}$ vary with species i' and latitude θ, and also depend on such constants as the radius of the earth and the masses and separations of the earth, moon, and sun. The equilibrium amplitude for a constituent is defined as GA'/g or $G'B'/g$ and can be generated for a particular latitude using *t_equilib*.

2 Phase/Amplitude Estimates

The algorithm used here is generally as described by Godin (1972), Foreman (1977), and Foreman (1978). Unlike these authors we use complex algebra directly rather than deal with sine and cosine fits separately. This has the advantage of unifying the treatment for scalar (e.g. pressure) and vector (e.g. horizontal currents) time series which are represented as $u + iv$.

Consider a time series of observations $y(t), t = t_1, t_2, ...t_M$ arranged in a vector, where the observation times are regularly spaced at an interval Δt (default 1 hour) and M is an odd number (an endpoint is discarded if required). The time axis is defined such that the origin (or central time) is at $t_{(M+1)/2}$. Some missing observations can be handled by using a "missing data" marker in the input vector (by MATLAB convention this is NaN, the IEEE arithmethic representation for Not-a-Number). This regular interval restriction does not arise from the least-squares fit itself but rather from the automated constituent-selection algorithm and is also a requirement when spectra are estimated in one of the confidence interval algorithms. This time series may be be composed of either real or complex numbers. The time series is passed to the analysis program *t_tide* along with a variety of (mostly optional) parameters.

The tidal response is be modelled as

$$x(t) = b_0 + b_1 t + \sum_{k=1...N} a_k e^{i\sigma_k t} + a_{-k} e^{-i\sigma_k t} \tag{2}$$

where N constituents (each with unique Doodson number sets) are being used. Each constituent has a frequency σ_k which is known from the development of the potential, and a complex amplitude a_k which is not known, although if $y(t)$ is a real time series $a_k = \overline{a_{-k}}$. A possible offset and (optional) linear drift is handled by the first two terms. The traditional approach uses real sinusoids:

$$x(t) = b_0 + b_1 t + \sum_{k=1...N} A_k \cos(\sigma_k t) + B_k \sin(\sigma_k t) \tag{3}$$

and is related to (2) by $A_k = a_k + a_{-k}$ and $B_k = i(a_k - a_{-k})$. The real representation is more convenient for the linear error analysis described later.

Constituents can be chosen from a list of 45 astronomical and 101 shallow-water consituents. Data structures containing information about these constituents is loaded using *t_getconsts*. There are several alternatives for selecting constituents. For general use an automated selection algorithm (following Foreman (1977)) is in place, which works as follows. A basis of all astronomical and 24 of the most important shallow-water constituents are gathered together. All constituents are listed in order of predefined importance. Less important constituents whose frequencies are less than a Rayleigh resolution limit $\alpha(N\Delta t)^{-1}$ (with default $\alpha = 1$) apart from more important constituents in frequency are discarded. Additional shallow-water constituents can be specified if required. If the relative phase/amplitude of two constituents that are otherwise unresolveable is known from other sources than an inference procedure can be carried out. Alternatively, constituent lists can be explicitly specified.

The least squares fit are the coefficients minimizing

$$E = \sum_m |x(t_m) - y(t_m)|^2 = ||Ta - y||^2 \tag{4}$$

where $y = [y(t_1), y(t_2), ...y(t_M)]'$, $a = [b_0, b_1, a_1, a_{-1}, a_2, a_{-2}, ...a_{-N}]'$, and T is an $M \times 2N+2$ matrix of sinusoids evaluated at observation times. The solution is found using the Matlab "\" matrix operator.

Once the fit has been performed various corrections are applied. These are generated in *t_vuf*. First, the phase of the constituent response is usually reported as "Greenwich phase" g_k, that is, phase referenced to the phase of the equilibrium response at 0° longitude (the Greenwich meridian). Alternatively this can be interpreted as reporting the phase of the response at the time when the equilibrium forcing is at its largest positive value at 0° longitude. It is simplest to find the fitted phase at the central time of the record ($t = 0$); the equilibrium phase v_k is then just V_a for the given constituent computed at the julian date corresponding to this central time, with possible adjustments of $1/4, 1/2$, or $3/4$ cycle depending on whether A or B is nonzero, and their signs.

Second, if a latitude is specified then nodal or satellite corrections are computed as follows. Consider a main peak of index k with satellites with indices kj. The effect of the different satellites will be to slowly modulate the phase/amplitude of the main peak over various periods, all more than 8 years. Our fitted response $\hat{a}_k$ over some period can then be written as a modification of the "true" response a_k, $\hat{a}_k = f_k \exp(iu_k) a_k$ where f_k and u_k are called the nodal correction amplitude and phase respectively. Then

$$\begin{aligned} f_k e^{iu_k} a_k e^{i\sigma_k t} &= a_k e^{i\sigma_k t} + \sum_j a_{kj} e^{i\sigma_{kj} t} \\ f_k e^{iu_k} &= 1 + \sum_j \frac{a_{kj}}{a_k} e^{i(\sigma_{kj} - \sigma_k)t} \\ f_k e^{iu_k} &\approx 1 + \sum_j \frac{a_{kj}}{a_k} \end{aligned} \tag{5}$$

The final approximation will hold as long as $(\sigma_{kj} - \sigma_k)t$ remains "small" (i.e. $N\delta t << 8$ years). In general the true phases and amplitudes of the satellites are not known. However since their frequencies are very similar to that of the main peak it is standard to assume the ratio of true amplitudes is the same as the ratio of amplitudes in the equilibrium response, and the difference in true phases will be equal to the difference in equilibrium phases. The nodal corrections are thus computed from the equilibrium response (1). A latitudinal dependence arises from the geodetic functions. G_1' is zero at the equator and a crude limiting is used to prevent some diurnal corrections from getting overly large. The validity of using the latitude-dependent equilibrium response to predict an aspect of the dynamic behavior in one part of an ocean basin in such a simple way is not clear.

The products of the analysis are now a pair of complex values (possibly corrected for nodal modulation) for each constituent k, $\{a_k, a_{-k}\}$. These are now converted into standard parameters:

$$U_k = |a_k| + |a_{-k}| \tag{6}$$

$$u_k = |a_k| - |a_{-k}| \tag{7}$$

$$\theta_k = \frac{ang(a_k) + ang(a_{-k})}{2} \tag{8}$$

$$g_k = v_k - \frac{ang(a_k) - ang(a_{-k})}{2} \tag{9}$$

For horizontal currents these parameters describe the features of an ellipse traced out by the tip of the velocity vector: the length of the semi-major and semi-minor axis of the ellipse (U_k, u_k respectively), the inclination of the northern semi-major axis counter-clockwise from due east θ_k, and the Greenwich phase g_k. For scalar time series the parameter U_k is merely called the amplitude, and $u_k, \theta_k \equiv 0$. Note that the restriction of the definition of inclination to use the northern axis means that analyses of constituents whose ellipses are aligned in an east/west direction may have inclinations that fluctuate between close to

0° and nearly 180° due to noise. These apparently large jumps are an artifact of the restriction but do not represent similarly large changes in the appearance of the ellipse.

Once ellipse parameters are found these can be used for further analysis. They can also be used to generate predictions at other times using *t_predic*. Nodal corrections in *t_predic* are computed at the time series mid-point so that it is an exact inverse of *t_tide*.

3 Confidence Intervals

One drawback of classical harmonic analysis is that the degree to which a given constituent represents true tidal energy as opposed to the energy of a broad-band nontidal process is not determined. This is useful information for two reasons: first, it allows one to make better estimates of the tidal behavior, and second it can allow one to quantitatively compare different analyses. There are two steps to producing confidence intervals. First, we must form an estimate of the characteristics of non-tidal or residual "noise" affecting the a_k (or A_k, B_k). Second, we must convert these estimates into confidence intervals for the standard parameters through a nonlinear mapping. We discuss the case of real time series first.

3.1 Residual noise (real)

After the harmonic analysis for an N-point real time series $x(t)$ is performed we examine the structure of the residual series. In the simplest case the noise is gaussian and uncorrelated in time. If this is the case then the total power $P_T = \sigma_x^2 = P/\Delta t$, where P is a spectral density. The amplitude of the fit to sine and cosine terms (A and B respectively) will be contaminated by errors arising from unresolved noise components within a frequency interval of $\Delta f = (N\Delta t)^{-1}$ around the line. Thus $\sigma_A^2 = \sigma_B^2 = P\Delta f = \sigma_x^2/N$. It is unlikely that a geophysical series will be spectrally flat, and a more sophisticated approach used in *t_tide* is to find a local value of P by making a spectral estimate from the residual time series (i.e. after the removal of all fitted constituents) and averaging the power over frequency bins in a window around the frequency of any constituent, neglecting bins in which fitted constituents reside. Here we chose a sequence of windows of width 0.4 cpd centered on $1, 2, 3, \ldots$ cpd.

3.2 Conversion to standard parameters (real)

A conversion from errors in the cos/sine amplitudes to errors in standard parameters (amplitude and phase) can be done using a linearized analysis. Consider a constituent k. Let Let $\xi = F(A_k, B_k)$ be a nonlinear function of these parameters, either the amplitude or the Greenwich phase. Then if $\{A_k, B_k\}$ are independent random variables, we can find a linearized estimate of the standard error of ξ in terms of the standard errors of the sinusoid amplitudes:

$$\sigma_\xi^2 = \left(\frac{\partial F}{\partial A_k}\right)^2 \sigma_A^2 + \left(\frac{\partial F}{\partial B_k}\right)^2 \sigma_B^2 \tag{10}$$

where the partial derivatives can be derived exactly (but tediously) from (6-9).

Alternatively the nonlinear mapping can be handled directly using a "parametric bootstrap" (Efron and Tibshirani, 1993). In this case we assume that the noise contaminating A, B is gaussian with a known variance. We then simulate a number of realizations or replicated of the analysis by taking our estimate of the sinusoid amplitudes and adding noise to them. All of these realizations are then converted nonlinearly to standard parameters using (6-9) and an estimate of the standard error computed from this replicate dataset directly.

Once a standard error is found, 95% confidence intervals can be estimated using standard techniques. Alternatively a signal-to-noise power ratio (SNR) can be computed. Simulations performed in *t_synth* (and described in the text file *t_errors*) show that the linear procedure appears to be adequate for real time series (e.g. tidal height), as long as the SNR < 10, and is probably not bad for SNR as low as 2 or 3. The nonlinear procedure gives similar results to the linearized procedure at high SNR, and is more accurate at low SNR.

3.3 Residual noise (complex)

A complex residual time series $u + iv$ can be modelled as bivariate white noise and variances σ_u^2, σ_v^2 and covariance σ_{uv} computed. If we assume further that the noise in both components is not correlated ($\sigma_{uv} \approx 0$) then a coloured bivariate noise model can be used and variances assigned to real and imaginary parts of constituent amplitude separately on the basis of local spectral densities as described above. If it is suspected that ($\sigma_{uv} \neq 0$) then it is recommended that the time series be rotated into a coordinate system in which this is true (e.g. into along/across channel axes).

3.4 Conversion to standard parameters (complex)

The linearized analysis now involves functions of 4 variables, since both $A_k = A_r + iA_i$ and $B_k = B_r + iB_i$ have real and imaginary parts, and analytic expressions for partial derivatives

$$\sigma_\xi^2 = \left(\frac{\partial F}{\partial A_r}\right)^2 \sigma_{A_r}^2 + \left(\frac{\partial F}{\partial A_i}\right)^2 \sigma_{A_i}^2 + \left(\frac{\partial F}{\partial B_r}\right)^2 \sigma_{B_r}^2 + \left(\frac{\partial F}{\partial B_i}\right)^2 \sigma_{B_i}^2 \tag{11}$$

become large. Some analytical simplification is possible by assuming that the all 4 are independent.

The bootstrap approach is now applied to the complex coefficients a_k. One complication that arises is that unless the noise is circular ($\sigma_u^2 = \sigma_v^2, \sigma_{uv} =$) the errors in a_k and a_{-k} are correlated with each other. The bootstrap process now requires generating correlated noise replicates.

4 Example

The analysis of an example dataset provided in Foreman (1977) is shown in Figure 1 and table 1. This example is included in datafile *t_example.mat*. The example dataset consists of 66 days of hourly elevations with a 3 day gap. The automated constituent selection algorithm selected 35 constituents. One shallower water constituent (M10) was added and two other constituents (P1 and K2) analyzed via inference. The coloured bootstrap analysis was used to determine significance and confidence intervals. The time series can be loaded and analyzed using the demonstration script *t_demo*. Table 1 gives the output of the program. In the first column the name of the constituent is given. Significant constituents (those with SNR in the last column > 1) are marked with a "*". Frequencies are listed in cph and phases in degrees. 11 constituents were judged to be significant (only 6 would be so at an SNR cutoff of 2). Figure 1a shows the results of the analysis and the residual time series after removal of these significant constituents. In Figure 1b are shown the analyzed phases. Note that the significant constituents generally have reasonably small phase errors. One would expect insignificant constituents have large errors but some of them are still reasonably small.

5 Summary

Separation of tidal and nontidal energy in oceanic time series is an important task in any analysis. Here we discuss the theoretical foundation and some implementation details of a MATLAB package for classical harmonic analysis. The package can also compute confidence intervals for the tidal parameters using one of three different sets of assumptions about the structure of residual noise. An example is provided to show typical results. The code is available at `http://www.ocgy.ubc.ca/~rich`.

References

Efron, B. and R. J. Tibshirani 1993: *An Introduction to the Bootstrap*, Volume 57 of *Monogr. on Stat. and Appl. Probab.* New York: Chapman and Hall.

Foreman, M. G. G. 1977: Manual for tidal heights analysis and prediction. Technical Report Pacific Marine Science Report 77-10, Institute of Ocean Sciences, Patricia Bay, Sidney, B.C.

Foreman, M. G. G. 1978: Manual for tidal currents analysis and prediction. Technical Report Pacific Marine Science Report 78-6, Institute of Ocean Sciences, Patricia Bay, Sidney, B.C.

Godin, G. 1972: *The Analysis of Tides*. University of Toronto Press.

Jackson, L. E. 1986: *Digital Filters and Signal Processing*. Kluwer Academic Publishers.

Jay, D. A. and E. P. Flinchem 1999: A comparison of methods for analysis of tidal records containing multi-scale non-tidal background energy. *Cont. Shelf. Res. 19*, 1695–1732.

Munk, W. H. and D. E. Cartwright 1966: Tidal spectroscopy and predication. *Phil. Trans. Roy. Soc. London, Ser. A 259*, 533–581.

```
file name: PAPEROUT.txt
date: 17-Aug-2001
nobs = 1584,  ngood = 1510,  record length (days) = 66.00
start time: 06-Jul-1975 01:00:00
rayleigh criterion = 1.0
Greenwich phase computed with nodal corrections applied to amplitude
and phase relative to center time

x0= 1.98, x trend= 0

var(x)= 0.82196   var(xp)= 0.21224   var(xres)= 0.60972
percent var predicted= 25.8 %

     tidal amplitude and phase with 95% CI estimates

tide   freq       amp     amp_err    pha    pha_err     snr
 MM    0.00151    0.2121    0.503   263.34   161.41     0.18
 MSF   0.00282    0.1561    0.526   133.80   188.82    0.088
 ALP1  0.03440    0.0152    0.044   334.95   150.82     0.12
 2Q1   0.03571    0.0246    0.044    82.69   106.21     0.31
 Q1    0.03722    0.0158    0.045    65.74   160.30     0.12
*O1    0.03873    0.0764    0.055    74.23    43.35      1.9
 NO1   0.04027    0.0290    0.035   238.14    74.68     0.69
*P1    0.04155    0.0465    0.045    71.88    70.96      1.1
*K1    0.04178    0.1405    0.059    64.81    23.49      5.7
 J1    0.04329    0.0253    0.050     7.32   129.76     0.25
 OO1   0.04483    0.0531    0.059   235.75    72.96     0.81
 UPS1  0.04634    0.0298    0.055    91.73   137.06     0.29
 EPS2  0.07618    0.0211    0.030   184.59   104.65     0.51
*MU2   0.07769    0.0419    0.034    83.23    48.82      1.5
*N2    0.07900    0.0838    0.035    44.52    25.54      5.9
*M2    0.08051    0.4904    0.035    77.70     4.51  1.9e+02
 L2    0.08202    0.0213    0.037    35.22   113.22     0.33
*S2    0.08333    0.2197    0.038   126.72     9.14       34
*K2    0.08356    0.0598    0.043   149.12    46.60        2
 ETA2  0.08507    0.0071    0.033   246.05   207.25    0.048
*MO3   0.11924    0.0148    0.014   234.97    67.38      1.1
 M3    0.12077    0.0123    0.014   261.57    62.11     0.81
 MK3   0.12229    0.0049    0.012   331.60   144.92     0.18
 SK3   0.12511    0.0023    0.010   237.69   219.86    0.054
 MN4   0.15951    0.0092    0.011   256.47    69.76     0.68
*M4    0.16102    0.0126    0.011   291.78    65.09      1.4
 SN4   0.16233    0.0083    0.011   270.85    91.22     0.54
 MS4   0.16384    0.0010    0.008   339.35   248.82    0.015
 S4    0.16667    0.0047    0.010   299.56   142.32     0.23
 2MK5  0.20280    0.0013    0.005   310.10   181.03    0.067
 2SK5  0.20845    0.0045    0.006   104.00    99.71     0.64
 2MN6  0.24002    0.0035    0.007   271.24   133.30     0.22
 M6    0.24153    0.0017    0.006   158.88   197.43    0.093
 2MS6  0.24436    0.0056    0.008   306.10    90.03     0.54
 2SM6  0.24718    0.0023    0.007   298.92   175.13     0.11
*3MK7  0.28331    0.0086    0.006   212.25    44.21      2.1
 M8    0.32205    0.0030    0.004    42.43    75.29     0.55
 M10   0.40256    0.0009    0.003   198.23   209.99    0.089
```

Table 1: Analysis for Tuktoyuktuk datset

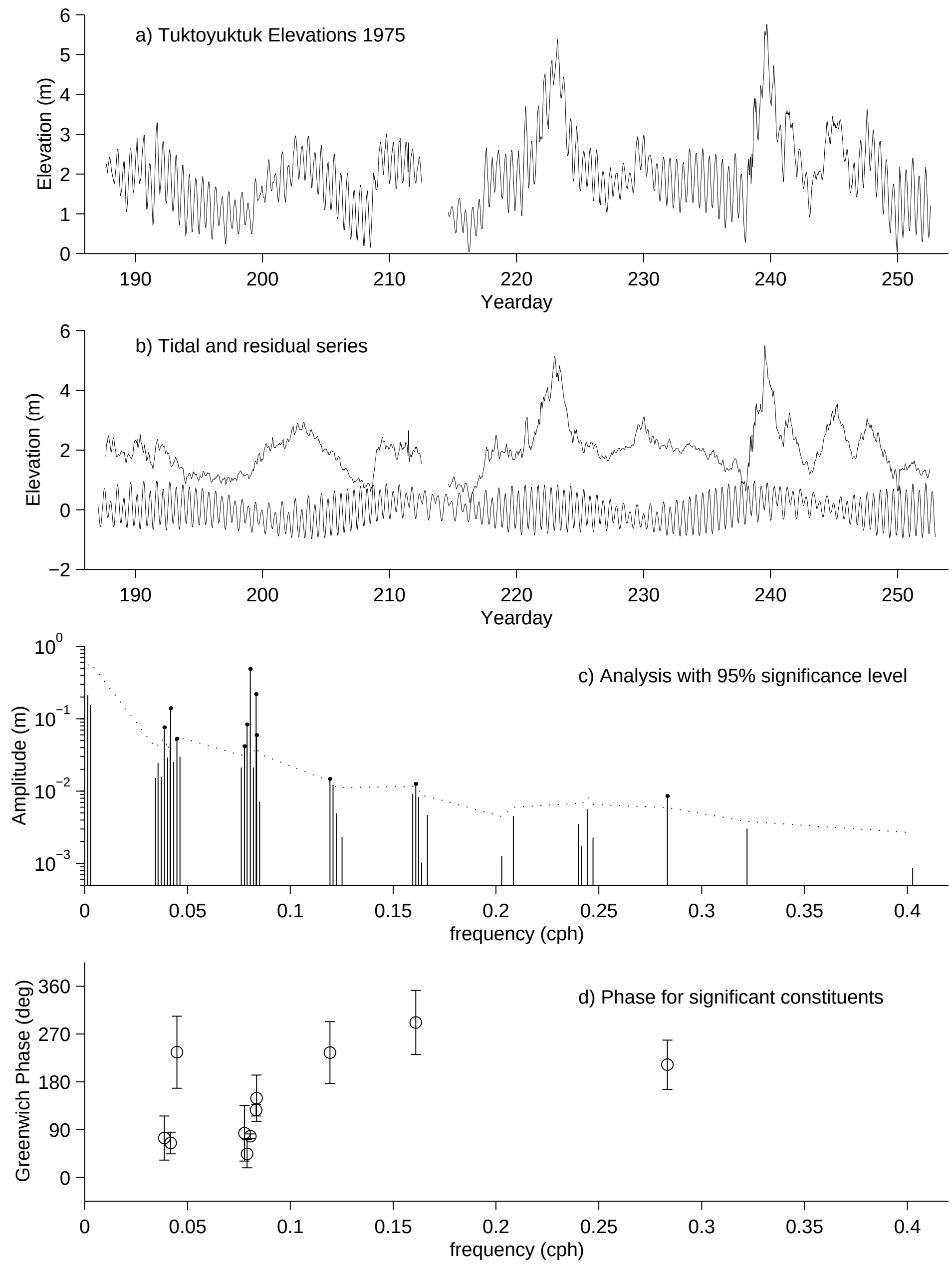

Figure 1: Tuktoyuktuk analysis example. (a) Raw time series. (b) Upper curve is residual time series after removal of tidal signal. Lower curve is a synthesized tidal series using significant constituents. (c) Amplitude of all analyzed components with 95% significance level. Note frequency dependence. Significant constituents are marked with a solid circle. (d) The phase of significant constituents with a 95% confidence interval.