Seasonal evolution of subglacial drainage and ice motion in a glacio-hydrodynamic flow-band model

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Numerical Model
- Ice Dynamics
- Basal Sliding
- Subglacial Hydrology

Examples
- Seasonal transition
- Subglacial flood

Future Applications
- Belcher Glacier
- Russell Glacier
Ice-flow Model  (Pimentel et al., *JGR*, 2010)

- **Flow-band**: 2-D flowline model with flow-unit width parameter

- **Higher-order stresses**: 1st-order approximation of the Stokes equation (Blatter, 1995; Pattyn, 2002), includes longitudinal stress gradients

\[
\frac{\partial}{\partial x} \left( 2\sigma_{xx}' + \sigma_{yy}' \right) + \frac{\partial \sigma_{xz}}{\partial z} + F_{lat} = \rho g \frac{\partial s}{\partial x}
\]

- **Lateral Drag**: lateral shear stress parameterization, includes sliding at the side walls and glacier basin shape

\[
F_{lat} = F_{lat}(x, z, u(x, z))
\]
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Basal Sliding

\[ \tau_b = C \left( \frac{u_b}{u_b + C^n N^n \Lambda} \right)^{1/n} N, \quad \Lambda = \frac{\lambda_{max} A}{m_{max}} \]

- The hydrology will be coupled to the ice mechanics by use of a regularized Coulomb friction law (Schoof, 2005; Gagliardini et al., 2007)

- This is a pressure dependent sliding rule utilizing the spatial and temporal variations in basal water pressure from the hydrology model

- Overcomes problem of standard sliding laws that allow arbitrarily large basal shear stresses regardless of effective pressure

- Implemented as a non-linear Robin-type boundary condition which cannot be solved independently but forms part of the solution to the ice-flow problem
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A mixed subglacial drainage network which includes dynamic switching between drainage components

- **Distributed**
  - macroporous water sheet
  - low capacity and efficiency
  - characteristic of winter

- **Channelized**
  - ice-walled conduits
  - high capacity and efficiency
  - characteristic of summer

- **Uplift**
  - When large amounts of water impinge on the glacier bed high water pressures are generated and cause flexure of the overlying ice
  - Elastic uplift is parameterized by treating the glacier as a uniform static beam
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Subglacial Hydrology Model

Distributed System

Conservation equation:
\[ \frac{\partial h^s}{\partial t} + \frac{\partial q^s}{\partial x} = \frac{Q_G + u_b \tau_b}{\rho L} + \dot{b}^s + \phi^{s:c} \]

Water flux:
\[ q^s = -\frac{Kh^s}{\rho_w g} \frac{\partial \psi^s}{\partial x} \]

Fluid potential:
\[ \psi^s = P_w^s + \rho_w gb \]

Basal water pressure:
\[ P_w^s = P_w^s (h^s) \]
Subglacial Hydrology Model

Channelized System

Conservation equation:

\[
\frac{\partial S}{\partial t} = -\frac{Q^c}{\rho L} \left( \frac{\partial \psi^c}{\partial x} - c_p \rho_w \Phi \frac{\partial P_w^c}{\partial x} \right) \\
- 2AS \left( \frac{P_i - P_w^c}{n} \right)^n
\]

Conduit discharge:

\[
Q^c = - \left( \frac{8S^3}{P_{wet} \rho_w f_R} \right)^{1/2} \frac{\partial \psi^c}{\partial x} \left| \frac{\partial \psi^c}{\partial x} \right|^{-1/2}
\]
\[ \phi^{s:c} = \chi^{s:c} \frac{K h^{s:c}}{\rho_w g d_c^2} (P_w^s - P_c^w) \]
A Test Case

An idealized mountain glacier

Glacier profile

Seasonal and diurnal cycle
Conduit cross-sectional area, m$^2$

Subglacial water pressure, flotation fraction

Distance downglacier, km

Time, days

Conduit cross-sectional area, m$^2$

Subglacial water pressure, flotation fraction

Distance downglacier, km
Pimentel

Glacio-Hydrodynamic Modelling

Basal speed, m a$^{-1}$

Basal shear stress, kPa
Model captures seasonal and diurnal cycles as well as the spring-transition

Such features have been well observed in Alpine glacier systems (e.g. Haut Glacier d’Arolla)

Increasing evidence of similar behaviour on Arctic and Greenland glaciers (e.g. Bartholomew et al., Nat. Geo., 2010)

Suggesting a unified treatment of basal processes across a range of scales
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Supraglacial Lake Drainage Event

- Das et al. Science, 2008
- Supraglacial lake of volume 0.044 km$^3$
- Drains through 980 m of ice in 1.4 h
- 1.2 m of vertical uplift and 0.8 m of horizontal displacement
- Rapid response followed by subsidence and deceleration over 24 hrs

Supraglacial lake on Belcher Glacier, Devon Island Ice Cap. Photo by A. Garner.
Pimentel Glacio-Hydrodynamic Modelling
• Pre-existing channel network needed to dissipate flood response as quickly as observed

• “Regular” seasonal melt as well as lake tapping events condition subglacial system

• Model limitations - multi-directional flow of flood water

• Other processes - horizontal turbulent hydraulic fracture for basal crack propagation (Tsai & Rice, *JGR*, 2010)
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Belcher Glacier, Canadian Arctic

A large, fast-flowing tidewater outlet glacier
Belcher Glacier, Canadian Arctic

Image from Angus Duncan (University of Alberta)
Russell Glacier, Greenland

MODIS Mosaic
67° N, -50° W

Landsat image
16th June 2010

Image from Andrew Fitzpatrick (University of Aberystwyth)
Russell Glacier, Greenland

Image from Andrew Fitzpatrick (University of Aberystwyth)
2009 velocities using TerraSAR-X images and speckle tracking algorithms

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