EOSC 512
Advanced Geophysical Fluid Dynamics

Fall 2019

Course Description

The purpose of this course is to a) introduce the student to the dynamical principles governing the large-scale, low-frequency motions in strongly rotating fluid systems (like the ocean, atmosphere, and liquid planetary core) and their consequences, and b) to develop the skills required to manipulate and use these principles to solve problems.

Prerequisites

Formally none. However, this course is mathematical and assumes a working knowledge of vector calculus (e.g. div, grad, curl), partial differential equations (i.e. you can solve at least some of them), and some exposure to complex analysis (e.g. you know that if $z = x + iy$, then $e^z = e^x \cos(y) + ie^x \sin(y)$). A background in fluid dynamics, geophysics, atmospheric sciences, and/or oceanography is not required (although undoubtedly will be helpful).

Instructor

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Meeting Times

EOSC 512 will be held Wednesdays and Fridays at a time that can accommodate the majority of students. Please contact Stephanie Waterman at `swaterman@eoas.ubc.ca` to register your interest and indicate your timetable availability. There will be an organizational meeting on Friday September 6 at 1:30p in ESB 3064.

Course Learning Goals

At the end of this course, students should be able to:

1. write down the 'standard equations' of geophysical fluid dynamics (GFD), identify the different terms, evaluate their relative importance based on scaling arguments, and explain how different dynamical features depend on these terms. Examples include the geostrophic and quasi-geostrophic equations, boundary layer equations, and thermodynamic relationships.

2. define standard terms and concepts used in GFD (the "language" of GFD), and identify them when they arise in the context of dynamical interpretations. Examples include Eulerian, Lagrangian, hydrostatic, Boussinesq, the Coriolis acceleration/force, Ekman layers, vorticity, geostrophic, barotropic, and baroclinic.

3. use standard mathematical techniques to simplify complex equation sets relevant to GFD. Examples include linearization, scaling arguments, normal mode techniques, and complex exponentials in wave and instability problems.

4. use the appropriate approximations and mathematical techniques to simplify and solve particular "canonical" GFD problems. Examples include a description of Taylor columns, a description of Ekman layers, spin-down problems, Rossby adjustment problems, wave problems in non-rotating and rotating systems, and instability problems.

Course Outline

This is an ambitious plan. We'll see how fast we want to go and modify accordingly.

1. **Some Basics**
   - Definition of a fluid
   - The continuum hypothesis
   - The fluid element
   - Kinematics
     - The Eulerian approach
     - The Lagrangian approach
     - The fluid trajectory
   - Rates of change
   - Streamlines & streak lines

2. **The Continuum Equations**
   - The conservation of mass
   - The momentum equation
   - Diversion: The stress tensor
   - The momentum equation in differential form

3. **The Stress Tensor for a Fluid & the Navier-Stokes Equations**
   - The symmetry of the stress tensor
   - Putting the stress tensor in diagonal form
   - The static pressure
   - Analysis of fluid motion at a point
   - The vorticity

- The rate of strain tensor
- Principal strain axes & the decomposition of the motion
- The relation between stress and rate of strain
- The coefficient of viscosity μ
- The Navier-Stokes equations

4. **The Equations of Motion in a Rotating Coordinate System**
   - Rates of change of vectors to a fixed vs. rotating observer
   - The Coriolis acceleration/force
   - The Navier-Stokes equations in a rotating frame

5. **Boundary Conditions & Frictional Boundary Layers**
   - Boundary conditions at a solid surface
   - Boundary conditions at a fluid surface
   - Ekman layers over a solid surface
   - Ekman layers below a free surface (Nansen's problem)
   - Example applications to coastal upwelling and the ocean's gyre circulations
   - The Ekman spin-down time

6. **Thermodynamics & the Equations of Motion**
   - The 1^{st} law of thermodynamics for a fluid & the equation of state
   - Simplifications for a perfect gas (air)
   - Simplifications for a liquid (water)

7. **Fundamental Theorems: Vorticity & Circulation**
   - Kelvin's circulation theorem
   - Example application: The Rossby wave
   - Example application: Ocean gyre circulation
   - Example application: The bathtub vortex
   - Further consequences of Kelvin's theorem: Vortex tube stretching
   - The vorticity equation
   - Example application: Thermal wind
   - Example application: The Taylor-Proudman theorem
   - Ertel's theorem and definition of potential vorticity

8. **Geostrophy, Quasi-Geostrophy & the Quasi-Geostrophic Potential Vorticity Equation**
   - Synoptic-scale dynamics: Geostrophy scaling & geostrophic balance
   - Important consequences of geostrophy
   - The quasi-geostrophic potential vorticity equation
   - Example application: Baroclinic Rossby waves

9. **Waves, Instability & Turbulence**
   - Irrotational gravity waves
   - Waves in the presence of a mean flow
   - Introduction to instability
     - Convective instability
     - Shear instability
     - Baroclinic instability
   - Introduction to geophysical turbulence

Texts

I will post copies of my own notes at `https://www.eoas.ubc.ca/~swaterma/512/index.html` in advance of each unit. These should act as a supplement to your own notes taken during class.

The class does not follow any specific text, but the texts listed below can be helpful for additional reading. Physical copies of these texts will be made available for a shortened, in-library loan period via Course Reserves at the Woodward Library. Items marked with an * are also available online as e-books through the UBC Library website.

1. *Kundu, P.K. *Fluid Mechanics.* Academic Press. (Any edition 1990 or later.)
2. Batchelor, G.K. *An Introduction to Fluid Dynamics.* Cambridge Univ. Press. (Any edition 1967 or later.)
3. *Aris, R. *Vectors, Tensors and the Basic Equations of Fluid Mechanics.* Prentice Hall. (Any edition 1962 or later.)
4. Cushman-Roisin, B. *Introduction to Geophysical Fluid Dynamics.* Prentice-Hall. (Any edition 1994 or later.)
5. Gill, A.E., *Atmospheric-Ocean Dynamics.* Academic Press. 1982.
6. *Pedlosky, J. *Geophysical Fluid Dynamics.* Springer Verlag. (Any edition 1979 or later.)
7. *Marshall, J. and R.A. Plumb. *Atmosphere, Ocean and Climate Dynamics: An Introductory Text.* Academic Press. 2008.
8. *Vallis, G.K. *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation.* Cambridge Univ. Press. (Any edition 2006 or later.)

Class Website

I will post copies of my lecture notes, problem sets and problem set solutions at `https://www.eoas.ubc.ca/~swaterma/512/index.html`.

Evaluation

1. **Problem Sets (50%)**

   I place a lot of value in applying the concepts we are learning to the exercise of solving problems. As such, a problem set consisting of 1-3 problems will be assigned every week or so. It will be available online at `https://www.eoas.ubc.ca/~swaterma/512/index.html`. Problem sets will be graded on a 1 point per question basis, with the possibility for partial credit or extra merit via a score of 1+. I encourage you to discuss the problems with your peers. Please however write-up your problem solutions individually. Problem solutions will be posted online also at `https://www.eoas.ubc.ca/~swaterma/512/index.html`. I obviously can't accept problem sets after this date. Please review these solutions and come speak to me if you have questions about the problems remaining.

2. **Final Exam (50%)**

   The final exam will be held in the formal exam period December 3 - 18. The date and time will be scheduled by UBC later in the term.

Important Dates

Date	Event
Fri Sep 6	Organizational meeting at 1:30p
Wed Sep 11	First class
Tue Sep 17	Last day to withdraw from course without a 'W' appearing on transcript
Fri Nov 29	Last class: Review and pre-exam Q & A
TBA	Final exam

Final Note

Comments on anything to do with the course (content, format, pace, level, lecture style, problem sets, *etc.*) are always very welcome. There will be a formal course evaluation at the end of the term, but if you tell me about any issues/suggestions earlier, we can try to make improvements this term as well.