8 Other Topics

There are many topics in this field that we don't have time to cover.

These include:

1. **thermodynamics, and particularly thermodynamic relations for an ideal gas and a liquid relevant to air and seawater**

   Thermodynamics yields the so-called equation of state that relates density to pressure needed to close the set of governing equations and completely describe the motion at all scales from planetary to molecular.

   For more information see this chapter of notes: `https://www.eoas.ubc.ca/~swaterma/512/LectureNotes/8_Thermodynamics_and_the_Equations_of_Motion.pdf`.

2. **wave motions relevant to geophysical flows**

   This was the subject of a whole different class when I was a graduate student.

   In this class, we talked briefly about barotropic Rossby waves, the potential vorticity waves whose restoring force is a background gradient of vorticity. This background gradient is typically supplied by the background gradient in planetary vorticity that exists in the latitudinal direction on a rotating sphere. It can also be supplied by a background gradient in "thickness PV" created by large-scale topographic gradients.

   There are many other wave motions that are important including:

   - baroclinic Rossby waves = a generalization of the 2D barotropic counterpart in which we include the stratification and the structure of the wave in 3 dimensions. These waves are governed by the QGPVE in a stratified fluid.

   - surface gravity waves = a wave motion in a single fluid layer whose restoring force is gravity. These are the waves we see on the air-sea interface for example.

   - internal gravity waves = the surface gravity wave counterpart that exists inside a stratified fluid/fluid whose density is variable.

- coastal-trapped Kelvin waves and more....

For more information, I recommend the textbook *Waves in the Ocean and Atmosphere: Introduction to Wave Dynamics* by Joseph Pedlosky available as an e-book via the UBC library. This book is a reworking of course notes for a course taught by Pedlosky for over 20-years to first-year graduate students in oceanography and meteorology.

3. **general circulation theory**

General circulation theory refers to the dynamical theories that describe the large-scale, steady general circulation of the ocean and atmosphere: features like the Hadley cells and jet streams in the atmosphere and the gyre circulations and western boundary currents in the ocean. These theories are based on the fundamental principles, theorems and conservation laws we covered here.

For more information I recommend the textbooks (all available as e-books via the UBC Library):

- *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation* by Geoff Vallis available here.
- *An Introduction to Dynamic Meteorology* by James Holton available here.
- *The theory of large-scale ocean circulation* by Rick Salmon available here.

4. **instabilities and turbulence, as applicable to geophysical flows**

Unstable solutions to geophysical wave problems that arise when we consider fluids with variable density and/or add the effects of a large-scale “mean” flow (like the westerly winds in the atmosphere or the Gulf Stream in the ocean) are the description of instabilities that give rise to the spontaneously generation of smaller-scale perturbations to the flow state, i.e. what we call ‘turbulence’ in geophysical flows.

There are many different types and classes of instability relevant to atmospheric and ocean flows that include:

- Kelvin-Helmhotz instability = an instability that results when heavier fluid ends up on top of light fluid and/or a vertical velocity shear becomes large enough. The propensity for Kelvin-Helmholtz (or K-H, for short) instability is measured by a non-dimensional number called the Richardson number: Ri =

$\frac{N^2}{U_z^2}$ which measures the stabilizing effect of stratification relative to the de-stabilizing effect of vertical velocity shear. The flow is unstable (and turbulence results) if Ri is small (i.e. vertical shear dominates over stratification) or Ri < 0 (a result of $N^2 < 0$ i.e. a fluid that is unstably stratified with heavy fluid over light). This is the instability that is primarily responsible for causing internal waves to break that then leads to turbulent mixing in the interior of the atmosphere and ocean.

- baroclinic instability = another important type of instability also involving velocity shear and buoyancy effects that occurs on much larger scales in the atmosphere and ocean for which the Earth's rotation is critical for the existence of the instability. Baroclinic instability results when the vertical slope of density surfaces in near-thermal wind balance with large vertical velocity shears becomes too steep such that the restoring force due to gravity becomes anti-restoring and a large-scale eddy overturning results. The net effect of the overturning is a slumping of the tilted density surfaces and a reduction of the large-scale vertical velocity shears to a stable state.

  Baroclinic instability is described fully by QGPVE dynamics. The criterion for its existence is a function of the meridional (north-south) gradient of q. If density stratification/vertical shear is large enough to make $\frac{\partial}{\partial}q$ change sign in the vertical, the flow satisfies the so-called necessary condition for baroclinic instability.

- Barotropic instability = the counterpart to baroclinic instability but for horizontal velocity shears instead of vertical velocity shears. Barotropic instability results when the horizontal shears of the large-scale flow become sufficient strong that horizontal gradients in the relative vorticity of the flow become sufficiently large to overcome the stabilizing effect of planetary vorticity. The strong shears in the horizontal flow "break-up" the large-scale flow structure.

  Barotropic instability is similarly fully described by QGPVE dynamics. The criterion for its existence is a function of the meridional (north-south) gradient of q. If horizontal shear is large enough to make $\frac{\partial}{\partial}q$ change sign in the horizontal, the flow satisfies the so-called necessary condition for barotropic instability.

For more information I recommend:

- Chapter 11 of the Notes from the course 12.800 Fluid Dynamics of the Atmosphere and Ocean taught by

Joesph Pedlosky at the Woods Hole Oceanographic Institution (the course on which this course is very closely based!): https://www.whoi.edu/fileserver.do?id=188989&pt=2&p=197629

- Course materials form the MIT OpenCourseWare course: *Turbulence in the Ocean and Atmosphere*, a 2nd-year graduate course that assumes a background in geophysical fluid dynamics that matches what we cover in EOSC 512: https://ocw.mit.edu/courses/12-820-turbulence-in-the-ocean-and-atmosphere-spring