EOSC 512
Advanced Geophysical Fluid Dynamics

Problem Set 1

1. Consider the velocity field:

$$u = -\omega y,\; v = \omega x + \epsilon \sin(\alpha t),\; w = 0.$$

Answer the following questions first for $\epsilon = 0$ and then for ϵ not zero (we will be especially interested in the case when it is very small).

a) Find the streamlines for all t that pass through the point $x = x_o$, $y = y_o$, $z = 0$.

b) Find the trajectory of the fluid element that at $t = 0$ is at $x = x_o$, $y = y_o$, $z = 0$ and compare with the result of part (a). In both (a) and (b) sketch the streamlines and trajectory.

Contrast the results when $\epsilon \ll \omega$ especially in the limit $\alpha - \omega \ll \omega$. What does it tell you about using streamlines as particle trajectories? What is the behaviour of the streamlines for ϵ different from zero and small? What is the behaviour of the trajectories as $\omega \to \epsilon$?

2. Consider a property, for example temperature θ, that is conserved following the flow, *i.e.* $\frac{D\theta}{Dt} = 0$. Suppose the velocity field advecting the fluid is:

$$u = ax,\; v = -ay,\; w = 0.$$

a) Find the streamlines and particle trajectories for the fluid.

b) Find the form of the Eulerian description of the temperature field, *i.e.* $\theta(y, t)$, when at $t = 0$, $\theta(y, t = 0) = \theta_0 y/L$, where θ_0 and L are arbitrary constants. (Hint: At $t = 0$, the Lagrangian description of temperature, $\Theta(t = 0)$, is a function of Y where Y is the Lagrangian variable particle 'label'. Since temperature is conserved following a fluid particle, it remains a function only of Y for all time.)

c) Do the same if at $t = 0$, $\theta(y, t = 0) = \theta_0 \tanh(\frac{y}{L})$.

d) In each case discuss the gradient of temperature as a function of time at a fixed location.

3) For the example discussed in class:

$$u = U,\; v = V \cos\left(k(x - ct)\right),\; w = 0.$$

find an expression for the streakline that passes through $(x, y) = (0, 0)$. That is, find the trace at any time t of all the fluid elements that have passed through that point as if a dye release at that point was maintained with time. Discuss the shape of that trace as $U \to c$.