EOSC 512
Advanced Geophysical Fluid Dynamics

Problem Set 3

1. Consider the two-dimensional flow in the x-y plane given by the stream function[1]:

$$\psi = \alpha xy \tag{1}$$

a) Find the deformation tensor.

b) Is it diagonal?

c) What is the vorticity associated with the flow?

2. a) Consider the two-dimensional flow in the x-y plane with constant density given by the stream function:

$$\psi = -Uy + \frac{V}{k}\cos(kx)\sin(ly), \; l = \frac{\pi}{L}, \; 0 \leq y \leq L, \; -\infty \leq x \leq \infty \tag{2}$$

Calculate
(i) the divergence of the velocity
(ii) the vorticity
(iii) the rate of strain tensor.
In (iii) consider the case when $k = l$. Find the principal axes of the rate of strain tensor and the associated strain rates. Where are they a maximum? What does the geometry of the streamline pattern look like in the neighbourhood of those points?

b) For a general two-dimensional flow with stream function $\psi(x, y)$:
(i) find the rate of strain tensor in terms of ψ
(ii) find the vorticity in terms of ψ
(iii) show that the trace, e_{jj}, must be identically zero.

[1] We didn't get to it in class, but one can show that it is possible to construct a solution to the streamline equation, $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \lambda$, that consists of a surface, let's call it $\psi(x, y, z)$, composed of streamlines. By definition, $\vec{u} \cdot \nabla\psi = 0$. For two-dimensional (for example if z is constant for each fluid element), the velocity field can then be specified in terms of the single scalar ψ that satisfies $\vec{u} = \hat{k} \times \nabla\psi$. The two velocity components u and v are then $u = -\frac{\partial\psi}{\partial y}$, and $v = \frac{\partial\psi}{\partial x}$.