EOSC 512
Advanced Geophysical Fluid Dynamics

Problem Set 4

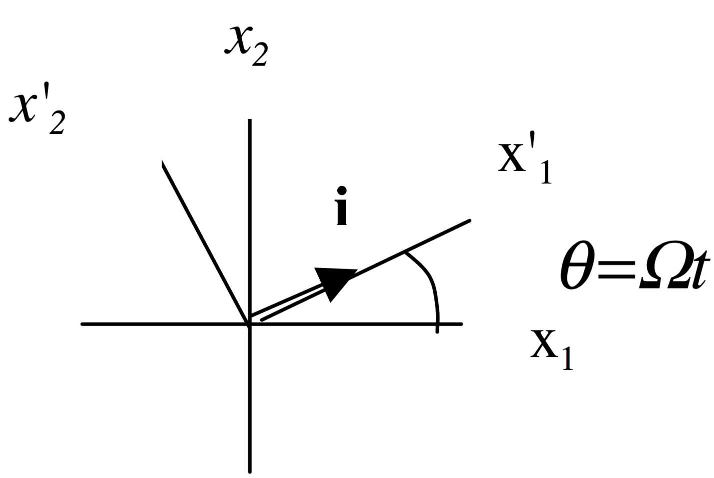
1. Consider the momentum equation and show, by taking the dot product with the velocity, that the equation for the kinetic energy can be written as:

$$\frac{\partial}{\partial t}\left(\rho\frac{u_i^2}{2}\right)+\frac{\partial}{\partial x_j}\left(\rho u_j\frac{u_i^2}{2}\right)=\rho u_i F_i-\frac{\partial}{\partial x_i}\left(p u_i\right)+p\frac{\partial}{\partial x_i}u_i+\frac{\partial}{\partial x_j}\left(u_i\tau_{ij}\right)-\tau_{ij}e_{ij} \quad (1)$$

a) The last term on the right hand side, $\tau_{ij}e_{ij}$, is called the dissipation function. Show that this term is positive so that the term represents a drain on the kinetic energy. Interpret the other terms as well. *Hint: It is easiest if you imagine you have already represented the strain tensor in its principal axis system, and if you use* $\lambda = \eta - \frac{2}{3}\mu$, $nu \geq 0$.

b) Consider an incompressible fluid, but with spatially variable density, and integrate the above equation over a volume of fluid contained in a container with rigid walls on which the fluid velocity vanishes. Discuss the net balance for the production and dissipation of kinetic energy. To make matters more concrete, think of $\vec{F} = -g\hat{k}$. Is this consistent with your intuition? Suppose the density were constant in space. Then what?

2. a) Consider two coordinate systems as shown below, one of which rotates with respect to the other at a constant rate so that $\theta = \Omega t$.



a) In terms of the angle θ, find the transformation of the coordinates x_1' and x_2' in terms of x_1 and x_2 and vice-versa.

b) Consider the momentum equations in a rotating, incompressible fluid with constant density. Suppose we can ignore friction. Show that the momentum equations for a strictly two-dimensional flow in the $x - y$ plane can be written :

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \Omega^2 x \tag{2}$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \Omega^2 y \tag{3}$$

where $f = 2\Omega$.

c) Suppose the pressure gradient has been arranged to balance the centrifugal force, *i.e.* such that $p = \rho\Omega^2\frac{(x^2+y^2)}{2}$. (This eliminates the centrifugal force from the horizontal equations of motion as in the dynamics of the atmosphere and ocean). Find the solution for u and v subject to the initial conditions $u(0) = U$, $v(0) = 0$.

d) Find the trajectory of fluid elements that satisfy $x = y = 0$ at $t = 0$ in the <u>rotating frame</u>. Use the results of part (a) to find the trajectory as seen in the <u>inertial frame</u>.

3. A fluid element in the Gulf Stream travels northward at about 1 m/s. Calculate the Coriolis acceleration acting on the element at 40^o N. If you walk northward at 2 miles/hr what is the Coriolis acceleration you would experience? Why is the Coriolis acceleration so much more important for the Gulf Stream than for you? *Hint: Consider the relative acceleration,* $\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}$*, and estimate those terms with respect to the Coriolis acceleration using scales* L*,* U *and* T *for length, velocity and time scales for you and for the Gulf Stream.*