EOSC 512

Advanced Geophysical Fluid Dynamics

Problem Set 5

1. Consider the Ekman layer over a flat horizontal plate in the presence of a uniform zonal flow, U, at great distances from the lower boundary as we discussed in class. Calculate the frictional stress *on the lower boundary* by the fluid. What is the corresponding stress exerted *on the fluid* by its interaction with the wall?

2. For this same Ekman layer problem, derive an equation that governs the balance between the generation and dissipation of kinetic energy.

Hint: Consider the Ekman layer momentum equations:

$$fu = fU + \nu v_{zz}$$
$$-fv = \nu u_{zz}$$

and multiply the first equation by v and the second equation by u and integrate over z.

You should obtain:

$$0 = -\int_0^\infty \frac{1}{\rho}\,\vec{u}\cdot\nabla p\,dz \,-\, \nu\int_0^\infty \left(\frac{\partial\vec{u}}{\partial z}\right)\cdot\left(\frac{\partial\vec{u}}{\partial z}\right)dz$$

Carefully discuss the sign of each term and interpret each term physically in terms of work done and viscous dissipation.

3. Consider the problem that Ekman solved for Nansen for his doctoral thesis to explain why icebergs drifted at an angle to the wind rather than directly downwind. Specifically show that for the case of an applied stress $\vec{\tau}$ generated by the wind on the sea surface, the motion driven by the stress at the sea surface (*i.e.* at $z = 0$) is given by:

$$\vec{u} = \delta\,\frac{\left[\vec{\tau} - \hat{k}\times\vec{\tau}\right]}{2\rho\nu} = \frac{\left[\vec{\tau} - \hat{k}\times\vec{\tau}\right]}{2\rho\sqrt{\Omega\nu}}$$

You can assume that the water occupies the infinite region $z < 0$ and that for large negative z the velocities must go to zero (implying that there is no pressure gradient for large negative z, although this could be added on later.)