EOSC 512
Advanced Geophysical Fluid Dynamics

Fall 2024

Course Description

The purpose of this course is to a) introduce the student to the dynamical principles governing the large-scale, low-frequency motions in strongly rotating fluid systems (like the ocean, atmosphere, and liquid planetary core) and their consequences; and b) to develop the skills required to manipulate and use these principles to solve problems.

Prerequisites

Formally none. However, this course is mathematical and assumes a working knowledge of vector calculus (e.g. div, grad, curl), partial differential equations (i.e. you can solve at least some of them), and some exposure to complex analysis (e.g. you know that if $z = x + iy$, then $e^z = e^x \cos(y) + ie^x \sin(y)$). A background in fluid dynamics, geophysics, atmospheric sciences, and/or oceanography is not required.

Instructor

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Course Learning Goals

At the end of this course, students should be able to:

1. write down the 'standard equations' of geophysical fluid dynamics (GFD), identify the different terms, evaluate their relative importance based on scaling arguments, and explain how different dynamical features depend on these terms. Examples include the geostrophic and quasi-geostrophic equations, boundary layer equations, and thermodynamic relationships.

2. define standard terms and concepts used in GFD (the "language" of GFD), and identify them when they arise in the context of dynamical interpretations. Examples include Eulerian, Lagrangian, hydrostatic, Boussinesq, the Coriolis acceleration/force, Ekman layers, vorticity, geostrophic, barotropic, and baroclinic.

3. use standard mathematical techniques to simplify complex equation sets relevant to GFD. Examples include linearization, scaling arguments, normal mode techniques, and complex exponentials in wave and instability problems.

4. use the appropriate approximations and mathematical techniques to simplify and solve particular "canonical" GFD problems. Examples include a description of Taylor columns, a description of Ekman layers, spin-down problems, Rossby adjustment problems, wave problems in non-rotating and rotating systems, and instability problems.

Course Outline

This is an ambitious plan. We'll see how fast we want to go and modify accordingly.

1. **Some Basics**
   - Definition of a fluid
   - The continuum hypothesis
   - The fluid element
   - Kinematics
     - The Eulerian approach
     - The Lagrangian approach
     - The fluid trajectory
   - Rates of change
   - Streamlines & streak lines

2. **The Continuum Equations**
   - The conservation of mass
   - The momentum equation
   - Diversion: The stress tensor
   - The momentum equation in differential form

3. **The Stress Tensor for a Fluid & the Navier-Stokes Equations**
   - The symmetry of the stress tensor
   - Putting the stress tensor in diagonal form
   - The static pressure
   - Analysis of fluid motion at a point
   - The vorticity
   - The rate of strain tensor
   - Principal strain axes & the decomposition of the motion
   - The relation between stress and rate of strain
   - The coefficient of viscosity μ
   - The Navier-Stokes equations

4. **The Equations of Motion in a Rotating Coordinate System**
   - Rates of change of vectors to a fixed vs. rotating observer
   - The Coriolis acceleration/force

- The Navier-Stokes equations in a rotating frame

5. **Boundary Conditions & Frictional Boundary Layers**
   - Boundary conditions at a solid surface
   - Boundary conditions at a fluid surface
   - Ekman layers over a solid surface
   - Ekman layers below a free surface (Nansen's problem)
   - Example applications to coastal upwelling and the ocean's gyre circulations
   - The Ekman spin-down time

6. **Fundamental Theorems: Vorticity & Circulation**
   - Kelvin's circulation theorem
   - Example application: The Rossby wave
   - Example application: Ocean gyre circulation
   - Example application: The bathtub vortex
   - Further consequences of Kelvin's theorem: Vortex tube stretching
   - The vorticity equation
   - Example application: Thermal wind
   - Example application: The Taylor-Proudman theorem
   - Ertel's theorem and definition of potential vorticity

7. **Geostrophy, Quasi-Geostrophy & the Quasi-Geostrophic Potential Vorticity Equation**
   - Synoptic-scale dynamics: Geostrophy scaling & geostrophic balance
   - Important consequences of geostrophy
   - The quasi-geostrophic potential vorticity equation
   - Example application: Baroclinic Rossby waves

8. **Waves, Instability & Turbulence**
   - Irrotational gravity waves
   - Waves in the presence of a mean flow
   - Introduction to instability
     - Convective instability
     - Shear instability
     - Baroclinic instability
   - Introduction to geophysical turbulence

9. **Thermodynamics & the Equations of Motion**
   - The 1^{st} law of thermodynamics for a fluid & the equation of state
   - Simplifications for a perfect gas (air)
   - Simplifications for a liquid (water)

Meeting Times

EOSC 512 will be held Tuesdays and Thursdays from 9:30 AM - 10:50 AM in ORCH 4052.

Cuiyi Fei, the class TA, will hold office hours on Mondays from 4:00 PM - 5:00 PM in ESB 3031. She is also willing to meet with students by appointment. In this case, please email her directly (cfei@eoas.ubc.ca) to arrange a time. She will hold review sessions in advance of the midterm and final exams (details to come).

Course Website

We will use a course website: `https://www.eoas.ubc.ca/~swaterma/512/index.html` to distribute notes and problem sets.

Texts

I will post copies of lecture notes on the course website in advance of each unit. These should act as a supplement to your own notes taken during class.

The class does not follow any specific text, but the texts listed below can be helpful for additional reading. Physical copies of these texts will be made available for a shortened, in-library loan period via Course Reserves at the Woodward Library. Items marked with an * are also available online as e-books through the UBC Library website.

1. *Kundu, P.K. *Fluid Mechanics*. Academic Press. (Any edition 1990 or later.)
2. Batchelor, G.K. *An Introduction to Fluid Dynamics*. Cambridge Univ. Press. (Any edition 1967 or later.)
3. *Aris, R. *Vectors, Tensors and the Basic Equations of Fluid Mechanics*. Prentice Hall. (Any edition 1962 or later.)
4. Cushman-Roisin, B. *Introduction to Geophysical Fluid Dynamics*. Prentice-Hall. (Any edition 1994 or later.)
5. Gill, A.E., *Atmospheric-Ocean Dynamics*. Academic Press. 1982.
6. *Pedlosky, J. *Geophysical Fluid Dynamics*. Springer Verlag. (Any edition 1979 or later.)
7. *Marshall, J. and R.A. Plumb. *Atmosphere, Ocean and Climate Dynamics: An Introductory Text*. Academic Press. 2008.
8. *Vallis, G.K. *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation*. Cambridge Univ. Press. (Any edition 2006 or later.)

Evaluation

1. **Problem Sets (40%-50%)**

   We place a lot of value in applying the concepts we are learning to the exercise of solving problems. As such, a problem set consisting of 1-3 problems will be assigned every 1-2 weeks. Problem sets will be posted on the course website. They will be graded on a 1 point per question basis, with the possibility for partial credit or extra merit via a score of 1+. I encourage you to discuss the problems with your peers. Please however write-up your problem solutions individually.

2. **Midterm Exam (0%-20%)**

   Per your request, we will hold a midterm exam in early November. This exam is optional. If you elect to take it, your final grade will be the higher of a 40%-20%-40% weighting for your problem set grades, midterm exam, and final exam, respectively vs. a 50%-50% weighting for your problem set grades and final exam, respectively (thus, you can't loose by taking the exam!). If you elect to skip it, your final grade will have a 50%-50% weighting for your problem set grades and final exam, respectively. The midterm will cover modules 1-5 of the course material (up to the end of Boundary

Conditions and Frictional Boundary Layers) and will consistent of $\sim$3 problems to solve similar to those on Problem Sets.

3. **Final Exam (40%-50%)**

   The final exam will be held in the formal exam period December 10 - 21. The date and time will be scheduled by UBC later in the term.

Important Dates

Date	**Event**
Tue Sep 10	First class
Mon Sep 16	Last day to withdraw from course **without** a 'W' appearing on transcript
Fri Oct 15	Last day to withdraw from course **with** a 'W' appearing on transcript
Tue Nov 5 (2p-4p)	Midterm exam
Tue Nov 12	No class (UBC Midterm Break)
Thu Dec 5	Last class
Thu Dec 12 (9a-12p)	Final exam

Final Notes

Comments on anything to do with the course (content, format, pace, level, lecture style, problem sets, *etc.*) are always very welcome. There will be a formal course evaluation at the end of the term, but if you tell me about any issues/suggestions earlier, we can try to make improvements this term as well.

I think geophysical fluid dynamics is a beautiful subject. I look forward to learning about it with you this term.

V4 *Revised October 29 2024*